

Løsningsforslag – ekstraoppgaver 3

Oppgave 1

a)

	Alternativ 1:	
[1]	(1) $(\forall x) (Rx \supset \neg Gx)$	P
[2]	(2) $(\forall y) (By \vee Gy)$	P
[3]	(3) $(\forall x) Rx$	P
[1]	(4) $Rx \supset \neg Gx$	1, UI
[2]	(5) $Bx \vee Gx$	2, UI
[3]	(6) Rx	3, UI
[1,3]	(7) $\neg Gx$	4,6 mp
[1,2,3]	(8) Bx	5,7 v-elim
[1,2,3]	(9) $(\forall y) By$	8, UG

	Alternativ 2:	
[1]	(1) $(\forall x) (Rx \supset \neg Gx)$	P
[2]	(2) $(\forall y) (By \vee Gy)$	P
[3]	(3) $(\forall x) Rx$	P
[1]	(4) $Rx \supset \neg Gx$	1, UI
[2]	(5) $Bx \vee Gx$	2, UI
[3]	(6) Rx	3, UI
[1,2,3]	(7) Bx	4,5,6 TF
[1,2,3]	(8) $(\forall y) By$	8, UG

b)

[1]	(1) $(\forall x) (Hx \supset Gx)$	P
[2]	(2) Ha	P
[1]	(3) $Ha \supset Ga$	1, UI
[1,2]	(4) Ga	2,3 mp (eller TF)
[1,2]	(5) $(\exists x) Gx$	4, EG

c)

[1]	(1) $(\forall x) (\forall y) (Mxa \supset Fay)$	P
[1]	(2) $Mba \supset Fay$	1, UI
[1]	(3) $Mba \supset Fab$	2, UI
[1]	(4) $(\forall y) (Mya \supset Fay)$	3, UG

d)			
[1]	(1)	$(\forall x)(\forall y)(Gxy \supset Fy)$	P
[1]	(2)	$(\forall y)Gay \supset Fy$	1, UI
[1]	(3)	$Gab \supset Fb$	2, UI
[4]	(4)	$\neg Fb$	P
[1,4]	(5)	$\neg Gab$	3,4 TF (eller mt)
[1,4]	(6)	$(\forall x)\neg Gxb$	5, UG
[1]	(7)	$\neg Fb \supset (\forall x)\neg Gxb$	[4](6) D
[1]	(8)	$(\exists y)(\neg Fy \supset (\forall x)\neg Gxy)$	7, EG

Oppgave 2

a)

[1]	(1)	$(\exists x)(\forall y)Fxy$	P
[1,2]	(2)	$(\forall y)Fay$	(1)a EII
[1,2]	(3)	Fab	(2) UI
[1,2]	(4)	$(\exists x)Fxb$	(3) EG
[1,2]	(5)	$(\forall y)(\exists x)Fxy$	(4) UG
[1]	(6)	$(\forall y)(\exists x)Fxy$	[2](5) EI ϵ

b)

[1]	(1)	$(\forall x)(\forall y)Lxy$	P
[2]	(2)	$(\exists x)(\forall y)(Lxy \supset Gay)$	P
[2,3]	(3)	$(\forall y)(Lay \supset Gay)$	(2)a EII
[1]	(4)	$(\forall y)Lay$	1, UI
[2,3]	(5)	$Lab \supset Gab$	3, UI
[1]	(6)	Lab	4, UI
[1,2,3]	(7)	Gab	5,6 TF
[1,2,3]	(8)	$(\forall y)Gay$	7, UG (OK, for b er ikke fri i noen av premissene til ln. 7)
[1,2,3]	(9)	$(\exists x)(\forall y)Gxy$	8, EG
[1,2]	(10)	$(\exists x)(\forall y)Gxy$	[3](9) EI ϵ

c)

[1]	(1)	$(\exists x)(\forall y)Gxy$	P
[2]	(2)	$(\forall y)(\forall x)(Gyx \supset Fxy)$	P
[1,3]	(3)	$(\forall y)Gay$	(1)a EII
[1,3]	(4)	Gab	3, UI
[2]	(5)	$(\forall x)(Gax \supset Fxa)$	2, UI
[2]	(6)	$Gab \supset Fba$	5, UI
[1,2,3]	(7)	Fba	4,6 TF
[1,2,3]	(8)	$(\exists x)Fbx$	7, EG
[1,2,3]	(9)	$(\forall y)(\exists x)Fyx$	8, UG
[1,2]	(10)	$(\forall y)(\exists x)Fyx$	[3](9) EI ϵ

Oppgave 3

Mange muligheter, deriblant:

a) UD: {a,b}

F: {a}

G: {b}

H: {a,b}

b) UD: {a,b,c}

G: {a}

F: {b}

H: {b}

Viktig her er at de som er G ikke må være F eller H

c) UD: {a,b,c}

F: {a}

G: { $\langle a, b \rangle, \langle c, a \rangle$ }