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Interaction Between Ionizing Radiation And Matter, Problems Charged particles

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Problem 8.1 (see fig 8.9)

Solution: Photons:

$$p(2\overline{x}) = e^{-\mu 2\overline{x}} = e^{-2} = 0.14$$

$$\overline{x} = 1/\mu$$

Charged particels:

$$p(2\overline{x}) \cong 0$$

$$Heavy\ Charged\ \ particles:\ p(\overline{x}) \cong 1$$

Problem 8.2 (Chap. 8.II.A.B)

Solution:

Soft: - b>>a: particle passes an atom in a large distance
- Small energy transitions to the atom
- The result is excitations (dominant) and ionization; amount energy transferred range from E_{min} to a certain energy H

Hard: - b<<a: particle passes trough the atom
- Large (but few) energy transactions to single electron
- Amount energy transferred range from H to E_{max}
- Can be seen as an elastic collision between free particles (bonding energy nelectable)

Problem 8.5 (Eq. 8.4 and 8.11)

Solution: Classic: $E_{max} = \frac{1}{2} m_2 v_{2,max}^2 = 4 \frac{m_1 m_2}{(m_1 + m_2)^2} T_0$

Relativistic: $T'_{max} = 2 m_e c^2 \left(\frac{\beta^2}{1-\beta^2} \right)$

$\beta^2 = 1 - \left(\left(T / M_0 c^2 \right) + 1 \right)^{-2}$

$\Rightarrow T'_{max} = 4 T_0 \frac{m_e}{M_0} \left[1 + \frac{T_0}{2 M_0 c^2} \right]$

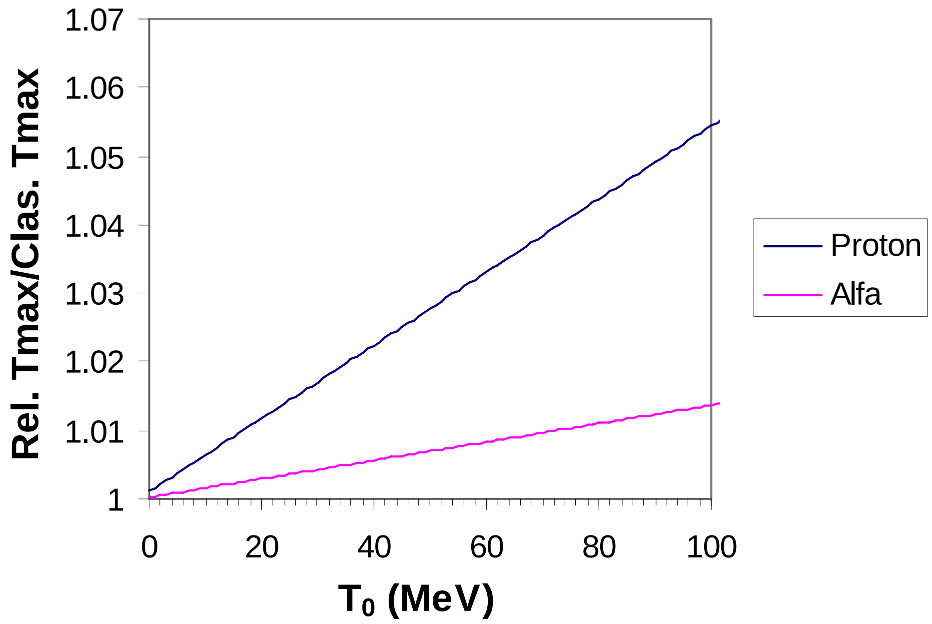
T_0=25 MeV

electron: $T_{max}=T_o/2=12.5$ MeV

positron: $T_{max}=T_o=25$ MeV

proton: $T_{max}=T'_{max}=55.19$ keV

α-particle: $T_{max}=T'_{max}=13.75$ keV



Problem 8.6 (Eq. 8.11)

Solution:

$$v_{electron} = v_{proton}$$
$$\Downarrow$$
$$T_{0electron} = T_{0proton} \frac{m_e}{m_p}$$

$T_{0proton}$=25 MeV

electron: T_0=13.62 keV→ T_{max}=6.81 keV
positron: T_0=13.62 keV→ T_{max}=13.62 keV
proton: T_0=25.00MeV→ T_{max}=55.19 keV
α-particle: T_0=99.33MeV→ T_{max}=55.19 keV

Problem 8.7 (Eq. 8.4, 8.6, 8.7, 8.8 and 8.10)

Solution: $z=+1$, $M_0c^2=3*938.26\text{MeV}=2815\text{MeV}$, $T=800\text{MeV}$

$\rightarrow \beta=0.6274$, $T_{max}=663.4\text{ keV}$

Copper: $H=100\text{eV}$, $I=322\text{eV}$, $Z/A=0.4564$

Soft:

$$\left(\frac{dT_s}{\rho dx}\right)_C = 0.150\frac{Z}{A}\,\text{cm}^2\!/\text{g}\,\frac{2m_ec^2z^2}{\beta^2}\left[\ln\left(\frac{2m_ec^2\beta^2H}{I^2\left(1-\beta^2\right)}\right)-\beta^2\right]$$

$$= 0.1778\,\text{MeVcm}^2\!/\text{g}\left[\ln(639.8)-0.3936\right] = \underline{\underline{1.079\,\text{MeVcm}^2\!/\text{g}}}$$

Hard:

$$\left(\frac{dT_h}{\rho dx}\right)_C = 0.150\frac{Z}{A}\,\text{cm}^2\!/\text{g}\,\frac{2m_ec^2z^2}{\beta^2}\left[\ln\left(\frac{T_{max}}{H}\right)-\beta^2\right]$$

$$= 0.1778\,\text{MeVcm}^2\!/\text{g}\left[\ln(6634)-0.3936\right] = \underline{\underline{1.495\,\text{MeVcm}^2\!/\text{g}}}$$

Total:

$$\left(\frac{dT}{\rho dx}\right)_C = 2\cdot 0.150\frac{Z}{A}\,\text{cm}^2\!/\text{g}\,\frac{2m_ec^2z^2}{\beta^2}\left[\ln\left(\frac{2m_ec^2\beta^2}{I\left(1-\beta^2\right)}\right)-\beta^2\right]$$

$$= 0.3555\,\text{MeVcm}^2\!/\text{g}\left[\ln(2060)-0.3936\right] = \underline{\underline{2.573\,\text{MeVcm}^2\!/\text{g}}}$$

Problem 8.8 (Eq. 8.11 and 8.10)

Solution: a) $$1-\left(\left(T_T / M_T c^2\right)+1\right)^{-2} = 1-\left(\left(T_\alpha / M_\alpha c^2\right)+1\right)^{-2} = \beta^2$$

$$\left(T_T / M_T c^2\right) = \left(T_\alpha / M_\alpha c^2\right)$$

$$T_\alpha = T_T \frac{M_\alpha}{M_T}$$

$$T_\alpha \approx T_T \frac{4}{3} = 800 \cdot \frac{4}{3} \text{MeV} = 1067 \text{MeV}$$

$$T_\alpha = T_T \frac{4.0026}{3.0160} = \underline{\underline{1062 \text{MeV}}}$$

b) $$\left(\frac{dT}{\rho dx}\right)_{C,\alpha} = \left(\frac{dT}{\rho dx}\right)_{C,T} \cdot \frac{z_\alpha^2}{z_T^2} = 4 \cdot 2.573 \, \text{MeVcm}^2/\text{g} = \underline{\underline{10.29 \, \text{MeVcm}^2/\text{g}}}$$

Problem 8.9 (next lection and Eq. 8.11 and 8.10)

Solution: Cyclotron: $|F| = zvB = ma = \frac{mv^2}{r} \Rightarrow v = \frac{zBr}{m} \Rightarrow T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{z^2}{m}(Br)^2$

a) Deuterons: $T_D = T_p \frac{{z_D}^2}{{z_p}^2}\frac{m_p}{m_D} \approx T_p \frac{1}{1}\frac{1}{2} = \frac{1}{2}T_p, \quad T_D = T_p \frac{1}{1}\frac{1.008}{2.014} = \underline{\underline{50.05\text{MeV}}}$

α-particle: $T_\alpha = T_p \frac{{z_\alpha}^2}{{z_p}^2}\frac{m_p}{m_\alpha} \approx T_p \frac{4}{1}\frac{1}{4} = T_p, \quad T_\alpha = T_p \frac{4}{1}\frac{1.008}{4.003} = \underline{\underline{100.7\text{MeV}}}$

b) Water: I=75eV, z=2, Z/A=(8+2*1)/(16+2*1)=10/18

$$\beta^2 = 1 - \left(\frac{1}{(100.7/3753)+1}\right)^2 = 0.05158, \quad m_\alpha c^2 \approx 4 \cdot m_p c^2 = 4 \cdot 938.26\text{MeV} = 3753.0\text{MeV}$$

$$\left(\frac{dT}{\rho dx}\right)_C = 2 \cdot 0.150 \frac{Z}{A} \,\text{cm}^2/\text{g} \,\frac{2m_e c^2 z^2}{\beta^2}\left[\ln\left(\frac{2m_e c^2 \beta^2}{I(1-\beta^2)}\right) - \beta^2\right]$$

$$= 13.21 \,\text{MeVcm}^2/\text{g}\left[\ln(741.1) - 0.05158\right] = \underline{\underline{86.61 \,\text{MeVcm}^2/\text{g}}}$$

Problem 8.10 (Eq. 8.11 and 8.10)

Solution: Proton: m_pc^2=938.26MeV, z=1, T=20MeV, β^2=0.04131

Lead: I=823eV, $\frac{Z}{A} = \frac{82}{207.2} = 0.3958$

$$\left(\frac{dT}{\rho dx}\right)_C = 2\cdot 0.150\frac{Z}{A}\,\mathrm{cm^2/g}\,\frac{2m_ec^2z^2}{\beta^2}\left[\ln\left(\frac{2m_ec^2\beta^2}{I\left(1-\beta^2\right)}\right)-\beta^2\right]$$

$$= 2.938\,\mathrm{MeVcm^2/g}\left[\ln\left(53.51\right)-0.04131\right] = \underline{\underline{11.57\,\mathrm{MeVcm^2/g}}}$$

Problem 8.11 (Eq. 8.13 and 8.13ab)

Solution: Electrons: z=1, T=50MeV, β^2 =0.9999≈1, $\tau = T/m_e c^2 = 50/0.511 = 97.85$

Aluminium: I=166eV, δ=5.068, $\frac{Z}{A} = \frac{13}{26.98} = 0.4818$

a) electrons: $F^-(\tau) = 1 - \beta^2 + \frac{\tau^2/8 - (2\tau+1)\ln 2}{(\tau+1)^2} = \frac{1197\text{-}136.3}{9771} = \underline{0.1086}$

$$\left(\frac{dT}{\rho dx}\right)_C = 0.1535\,{}^{\text{MeVcm}^2}\!/_{\text{g}}\,\frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F^-(\tau) - \delta\right]$$

$$= 0.07396\,{}^{\text{MeVcm}^2}\!/_{\text{g}}\,[29.14 + 0.1086 - 5.068] = \underline{\underline{1.788\,{}^{\text{MeVcm}^2}\!/_{\text{g}}}}$$

b) positrons: $F^+(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left\{23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right\} = 1.386 - 1.917\text{-}0.01168\text{-}0\text{-}0 = \text{-}\underline{0.5427}$

$$\left(\frac{dT}{\rho dx}\right)_C = 0.1535\,{}^{\text{MeVcm}^2}\!/_{\text{g}}\,\frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F^+(\tau) - \delta\right]$$

$$= 0.07396\,{}^{\text{MeVcm}^2}\!/_{\text{g}}\,[29.14\text{-}0.5427 - 5.068] = \underline{\underline{1.740\,{}^{\text{MeVcm}^2}\!/_{\text{g}}}}$$

Problem 8.12 (Eq. 8.15 and 8.13(b))

Solution: $\left(\frac{dT}{\rho dx}\right)_R = \left(\frac{dT}{\rho dx}\right)_C \frac{TZ}{n}$

Electrons: T=50MeV, n=750MeV

a) Al, Z=13: $\left(\frac{dT}{\rho dx}\right)_R = \frac{50 \cdot 13}{750} 1.788 \, \mathrm{MeVcm^2}/\mathrm{g} = \underline{\underline{1.550 \, \mathrm{MeVcm^2}/\mathrm{g}}}$

b) Lead, Pb: Z=82, I=823eV, Z/A=0.3958

$$\left(\frac{dT}{\rho dx}\right)_C = 0.1535 \, \mathrm{MeVcm^2}/\mathrm{g} \, \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{\tau^2(\tau+2)}{2\left(I/m_e c^2\right)^2} \right) + F^-(\tau) - \delta \right]$$

$$= 0.06076 \, \mathrm{MeVcm^2}/\mathrm{g} \, [25.94 + 0.1086 - 3.579] = \underline{1.365 \, \mathrm{MeVcm^2}/\mathrm{g}}$$

$$\left(\frac{dT}{\rho dx}\right)_R = \frac{50 \cdot 82}{750} 1.365 \, \mathrm{MeVcm^2}/\mathrm{g} = \underline{\underline{7.462 \, \mathrm{MeVcm^2}/\mathrm{g}}}$$

Problem 8.13 (Chapter 8.III.G)

Solution: Radiation yield: Y(10MeV)= 0.2265 and Y(7MeV)= 0.1734

Energy of radiation per electron as it slows down to rest is: $E_e = Y(T_0)T_0$

Total radiation energy from 10MeV electrons: $E_{10} = N\,E_e = NY(T_0)T_0$
$= 10^{15}\ 0.2265\ 10\text{MeV}\ 1.602\ 10^{-19}\text{J/eV} = 362.9\text{J}$

Total radiation energy from 7MeV electrons: $E_7 = N\,E_e = NY(T_0)T_0$
$= 10^{15}\ 0.1734\ 7\text{MeV}\ 1.602\ 10^{-19}\text{J/eV} = 194.5\text{J}$

Radiation energy from 10MeV electrons until 7 MeV electrons:
$E_{10} - E_7 = \underline{168.4\text{J}}$

Problem 8.14 (Eq. 8.23 and text below)

Solution: Iron, Z=26

Carbon, Z=6: $\left(\Re_{CSDA}\right)_C \simeq \left(\frac{T_0^{1.77}}{415} + \frac{1}{670}\right) g/cm^2$

At 30 MeV proportional to $Z^{0.3}$: $\frac{\left(\Re_{CSDA}\right)}{\left(\Re_{CSDA}\right)_{Z_2}} \simeq \left(\frac{Z_1}{Z_2}\right)^{0.3}$

$$\left(\Re_{CSDA}\right)_{Fe} \simeq \left(\frac{26}{6}\right)^{0.3} \left(\Re_{CSDA}\right)_C = 1.55\left(\frac{30^{1.77}}{415} + \frac{1}{670}\right) g/cm^2 = \underline{\underline{1.54 g/cm^2}}$$

Problem 8.15 (Argument before and after Fig 8.8.The points a.,b.,c.)

Solution: a) $T_D = T_P m_D / m_P = 2T_P = 60$ MeV

b) $\left(\Re_{CSDA}\right)_{Fe,D} = \left(\Re_{CSDA}\right)_{Fe,p} \dfrac{m_D}{m_p} = \underline{\underline{3.085 g/cm^2}}$