UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120/FYS 4120 Classical mechanics and electrodynamics
Day of exam: Thursday June 8, 2006
Exam hours: 3 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Øgrim og Lian or Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection FYS 3120/4120

This paper is available also in Norwegian (Bokmål or Nynorsk) language.

Make sure that your copy of this examination paper is complete before you begin.

PROBLEM 1
Composite system
A composite mechanical system, shown in Fig. 1, consists of two parts. Part A is a cylinder with

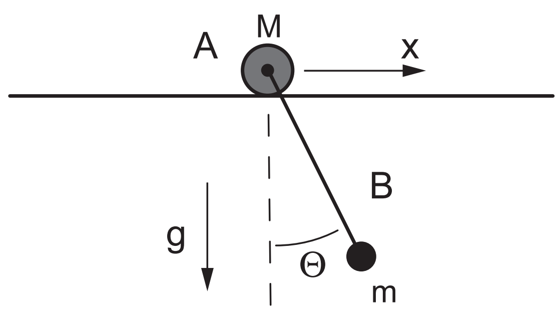

Figure 1:

mass M, radius R and moment of inertia about the symmetry axis $I = \frac{1}{2}MR^2$. The cylinder rolls without sliding on a horizontal plane. Part B is a pendulum with the pendulum rod attached to the symmetry axis of the cylinder. It oscillates without friction about the point of attachment in the plane orthogonal to the symmetry axis of the cylinder. The pendulum rod has length L and we consider it as massless. The mass of the pendulum bob is $m = M/2$.

Use in the following the horizontal displacement x of the cylinder and the pendulum angle θ as generalized coordinates for the composite system.

a) Find the Lagrangian of the system expressed as a function of the generalized coordinates and their time derivatives.

b) Examine if there are cyclic coordinates, and find constants of motion.

c) Formulate Lagrange's equations for the system and assume the following initial conditions: $x = 0, \dot{x} = 0, \theta = \theta_0, \dot{\theta} = 0$ at time $t = 0$. Assume $\theta_0 << 1$ and simplify the equations by using a small angle approximation. Show that the system in this case will perform small oscillations of the form $\theta(t) = \theta_0 \cos \omega t$ and determine the oscillation frequency ω. What is the corresponding expression for $x(t)$?

PROBLEM 2
Accelerated charge

An electron, with charge e, moves in a constant electric field $\mathbf{E}$. The motion is determined by the relativistic Newton's equation

$$\frac{d}{dt}\mathbf{p} = e\mathbf{E} \tag{1}$$

where $\mathbf{p}$ denotes the relativistic momentum $\mathbf{p} = m_e \gamma \mathbf{v}$, with m_e as the electron rest mass, $\mathbf{v}$ as the velocity and $\gamma = 1/\sqrt{1-(v/c)^2}$ as the relativistic gamma factor. We assume the electron to move along the field lines, that is, there is no velocity component orthogonal to $\mathbf{E}$.

a) Show that the electron has a constant proper acceleration $\mathbf{a}_0 = e\mathbf{E}/m_e$, which is the acceleration in an instantaneous rest frame of the electron.

b) Show that if $v = 0$ at time $t = 0$, then γ depends on time t as

$$\gamma = \sqrt{1 + \kappa^2 t^2} \tag{2}$$

and find κ expressed in terms of a_0.

c) Show that if we write $\gamma = \cosh \kappa\tau$ then τ is the proper time of the electron.

As a reminder we give the following functional relations:

$$\cosh^2 x - \sinh^2 x = 1\,, \quad \frac{d}{dx}\cosh x = \sinh x\,, \quad \frac{d}{dx}\sinh x = \cosh x \tag{3}$$

PROBLEM 3
Rotating dipole

A thin rigid rod of length ℓ rotates in a horizontal plane (the x,y-plane) as shown in Fig. 2. At the two end points there are fixed charges of opposite sign, $+q$ and $-q$. The rod is rotating with constant angular frequency ω. This gives rise to a time dependent electric dipole moment

$$\mathbf{p}(t) = q\ell(\cos\omega t\,\mathbf{i} + \sin\omega t\,\mathbf{j}) \tag{4}$$

a) Use the general expression for the radiation fields of an electric dipole (see the formula collection of the course) to show that the magnetic field in the present case can be written as

$$\mathbf{B}(\mathbf{r},t) = B_0(r)(\cos\theta\sin(\omega(t-\frac{r}{c}))\,\mathbf{i} - \cos\theta\cos(\omega(t-\frac{r}{c}))\mathbf{j} - \sin\theta\sin(\omega(t-\frac{r}{c})-\phi)\,\mathbf{k} \tag{5}$$

with (r,θ,ϕ) as the polar coordinates of $\mathbf{r}$. Find the expression for $B_0(r)$.

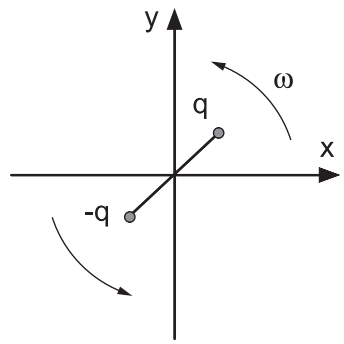

Figure 2:

What is the general relation between the electric field $\mathbf{E}(\mathbf{r}, t)$ and the magnetic field $\mathbf{B}(\mathbf{r}, t)$ in the radiation zone? (A detailed expression for $\mathbf{E}(\mathbf{r}, t)$ is not needed.)

b) Show that radiation in the x-direction is linearly polarized. What is the polarization of the radiation in the z-direction?

c) Find the time-averaged expression for the energy density of the radiation. In what direction has the radiated energy its maximum?