UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Exam in: FYS 3120 Classical Mechanics and Electrodynamics
Day of exam: Tuesday June 2, 2015
Exam hours: 4 hours, beginning at 14:30
This examination paper consists of 3 pages
Permitted materials: Calculator
Angell og Lian: Størrelser og enheter i fysikken
Rottmann: Matematisk formelsamling
Formula Collection/Formelsamling FYS 3120
Language: This paper is available also in Norwegian

Make sure that your copy of this examination paper is complete before answering.

PROBLEM 1
Cylinder with pendulum
A composite mechanical system, shown in Fig. 1, consists of two parts. Part A is a cylinder with

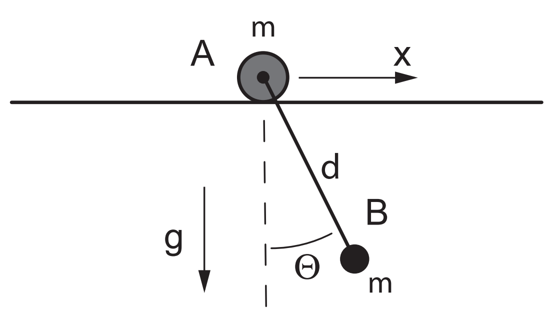

Figure 1:

mass m, radius R and moment of inertia about the symmetry axis $I = \frac{1}{2}mR^2$. The cylinder rolls without sliding on a horizontal plane. Part B is a pendulum with the pendulum rod attached to the symmetry axis of the cylinder. It oscillates without friction under influence of gravitation in the plane orthogonal to the symmetry axis of the cylinder. The pendulum rod, which we consider as massless, has length d. The mass of the pendulum bob is m, the same as the mass of the cylinder.

Use in the following the horizontal displacement x of the cylinder and the pendulum angle θ as generalized coordinates for the composite system. Assume the initial conditions: $x = 0$, $\dot{x} = 0$, $\theta = \theta_0 \neq 0$, $\dot{\theta} = 0$ at time $t = 0$.

a) Find the Lagrangian of the system expressed as a function of the generalized coordinates and their time derivatives. Identify constants of motion.

b) Find Lagrange's equations for the system, and show that by eliminating the variable x that the equation of motion takes the form

$$(1 - \frac{2}{5}\cos^2\theta)\ddot{\theta} + \frac{2}{5}\cos\theta\sin\theta\dot{\theta}^2 + \frac{g}{d}\sin\theta = 0 \qquad (1)$$

c) Assume $\theta_0 << 1$ and simplify the equation by using the small angle approximation. Show that the system in this case will perform oscillations of the form $\theta(t) = \theta_0 \cos\omega t$ and determine the oscillation frequency ω. What is the corresponding expression for $x(t)$?

PROBLEM 2
τ^- decay
We consider a process where a negatively charged tau-lepton (τ^-) decays into a K^--meson and a neutrino. The mass of the tau-lepton is $m_\tau = 1777 MeV/c^2$, the mass of the K-meson is $m_K = 494 MeV/c^2$, and the neutrino we consider as massless. In the figure the decay process

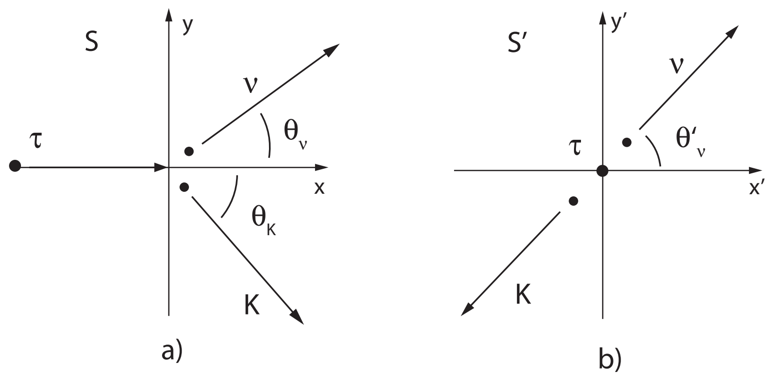

Figure 2:

is shown both in the laboratory frame S, and in the rest frame S' of the tau-lepton. The relative velocity of the two reference frames is $v = 0.9c$, with direction along the x-axis (in both frames).

a) We study first the process in the rest frame S'. Use the relativistic conservation laws for energy and momentum to determine the energies E'_ν and E'_K, of the neutrino and the K-meson, respectively. Determine also the absolute value of the (three-) momenta of the two particles in this reference frame.

b) Assume the direction of the neutrino in reference frame S' to be orthogonal to the relative velocity of the two frames, $\theta'_\nu = \pi/2$. Write the transformation formula for relativistic 4-momenta, and use it to determine the energies E_ν and E_K of the two particles in the lab frame S. Determine also the angles θ_ν and θ_K of the two particles.

c) With no knowledge of the direction of the neutrino, explain why the probability is 50% for the neutrino to have energy larger than the value found in b) and also 50% probability to find it within a smaller angle θ_ν than the value evaluated in b).

PROBLEM 3
Oscillating electron
An electromagnetic plane wave has electric field components

$$E_x = E_0 \cos(kz - \omega t) \,, \quad E_y = E_z = 0 \tag{2}$$

An electron, with charge e and mass m, performs oscillations in the field.

a) Give the expressions for the components of the magnetic field $\mathbf{B}$ and determine Poynting's vector $\mathbf{S}$ of the plane wave.

b) Assume the motion of the electron, within a good approximation, is given by

$$\dot{x} = -\frac{eE_0}{m\omega} \sin(kz - \omega t) \,, \quad \dot{y} = \dot{z} = 0 \tag{3}$$

with (x, y, z) as the electron coordinates. Specify the conditions for this approximation to be valid.

c) The interaction cross section σ between the wave and the electron is defined as the power of the radiated energy from the electron, divided by the energy current density of the incoming wave, both averaged over time. Determine the cross section σ, with the electron motion given by the above approximation.

d) Determine the time averaged, differential power $\frac{dP}{d\Omega}$ of the radiation from the electron. Specify what are directions with maximal and minimal radiation. Characterize also the polarization of the radiation.

We give the following (non-relativistic) expressions for the the radiated power (Larmor's formula) and the radiation fields from the accelerated charge e,

$$P = \frac{\mu_0 e^2}{6\pi c} \mathbf{a}^2 \,, \quad \mathbf{B}_{rad}(\mathbf{r}, t) = \frac{\mu_0 e}{4\pi c} [\frac{\mathbf{a} \times \mathbf{n}}{R}]_{ret} \,, \quad \mathbf{E}_{rad}(\mathbf{r}, t) = c\mathbf{B}_{rad}(\mathbf{r}, t) \times \mathbf{n}_{ret} \tag{4}$$

Here $\mathbf{a}$ is the acceleration of electron, $\mathbf{n} = \mathbf{R}/R$, with $\mathbf{R} = \mathbf{r} - \mathbf{r}(t)$, as the the vector between the point $\mathbf{r}$ where the radiation fields are measured, and the the position $\mathbf{r}(t)$ of the electron. Far from the charge we may use the approximation $\mathbf{R} = \mathbf{r}$.