UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: FYS3120 Classical Mechanics and Electrodynamics
Day of exam: 9 June 2017
Exam hours: 14:30 (4 hours)
This examination paper consists of 3 (three) pages.
Permitted materials: Approved calculator
Øgrim and Lian: "Fysiske størrelser og enheter"
Angell and Lian: "Fysiske størrelser og enheter"
Rottman: "Matematisk formelsamling".
Collection of formulae for FYS3120
(English, bokmål or nynorsk versions)

Make sure that your copy of this examination paper is complete before answering.

You may answer the questions either in English or Norwegian.

Question 1 Lagrangian mechanics
A particle of mass m moves without friction on a parabola-shaped stiff string under the effects of gravity. The string rotates about the z-axis with constant angular velocity ω, and its shape is given by the equation $z = ar^2$, where a is a constant and r is the distance from the z-axis.

a) Sketch a figure of the system. [2 points]

b) How many degrees of freedom does this system have? [4 points]

c) Find the kinetic energy K of the mass m in terms of r. [5 points]

d) Show that the Lagrangian of the system can be written

$$L = \frac{1}{2} m \left[(1 + 4a^2 r^2)\dot{r}^2 + r^2(\omega^2 - 2ga) \right], \tag{1}$$

where g is the acceleration due to gravity (in the negative z-direction). [3 points]

e) Is the angular momentum around the z-axis a constant of motion / conserved quantity? [2 points]

f) Show that the equation of motion for m is

$$(1 + 4a^2 r^2)\ddot{r} + 4a^2 r\dot{r}^2 - (\omega^2 - 2ga)r = 0. \tag{2}$$

[5 points]

g) Show that there is a solution with constant r, and find the angular velocity ω for this solution. What is the physical interpretation of the solution? [5 points]

Question 2 Relativistic mechanics
First we warm up a little by doing the following:

a) Explain what we mean by a Lorentz vector. (Sometimes sloppily just called a four-vector in the course.) [2 points]

b) Explain why $p^\mu \equiv mU^\mu$ is a Lorentz vector. Here $U^\mu \equiv \frac{dx^\mu}{d\tau}$, m is the mass of the particle, and τ is the proper time. [3 points]

c) Find $p^2 = p^\mu p_\mu$. [3 points]

We will now use this to look at the so-called **pair-production** reaction where a photon γ produces an electron–positron pair e^+e^- in an interaction with a charged particle N:

$$\gamma + N \to N + e^+ + e^-. \tag{3}$$

d) Assume that the mass of the charged particle N is M and that the masses of the electron and positron are both m. Find the smallest photon energy E_γ where the reaction is possible in the laboratory reference frame where N is at rest. Discuss the physics of this threshold in the two cases: i) $M \gg m$ and ii) $M/m \to 0$. *Hint:* It can be useful to consider an invariant in two different rest frames for the two sides of the reaction. [10 points]

e) The centre-of-mass (CM) system, or reference frame, is defined as the reference frame where the sum of the incomming (and outgoing) particle momenta are zero. Find the velocity of the CM system v_{CM} with respect to the laboratory reference frame as a function of the photon energy E_γ. *Hint:* You do not need the answer from the previous question. [7 points]

Question 3 Electromagnetism

Assume that we have a monochromatic plane wave of the form

$$\vec{E}(\vec{r},t) = E_0 \cos(kz - \omega t)\hat{e}_x. \tag{4}$$

a) Find Poynting's vector $\vec{S}$ for this wave in terms of $\vec{E}$. [5 points]

b) What is the physical interpretation of Poynting's vector? [2 points]

We will now look at the scattering of such as wave on a free electron of mass m and charge e. We assume that the electron is initially at rest.

c) Explain why we can ignore the force from the magnetic field as long as $v \ll c$, where v is the velocity of the electron. [4 points]

d) Find the power P radiated by the electron per unit time. Express your answer in terms of the **classical electron radius**

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2}. \tag{5}$$

[4 points]

e) Find the **cross section** σ of the electron for the electromagnetic scattering, given as $\sigma = P/S$, where $S = |\vec{S}|$. Comment on the units of the cross section. [4 points]

Question 4 Ehrenfest's paradox

Imagine that you observing a rigid disc of radius R rotating with a large (relativistic) angular velocity ω. We will use the laboratory rest frame as the reference frame of the problem, with the origin in the centre of the disc. What will you observe for the diameter and the circumference of the disc, and their ratio, using special relativity? [5 points]