

#### UiO **\* Fysisk institutt**

Det matematisk-naturvitenskapelige fakultet

#### Lecture 4



## This week

- Monday: Symmetries of systems (Section 2.3)
- Wednesday: Energy conservation, velocity dependent potentials. (Sections 2.3 and 2.4)
- **Problem session:** Problem set 2 (main topic: finding the e.o.m. from Lagrange's equation)

### Recap

• The essence of Lagrange-Hamilton formalism is



• The Lagrange equation dealing with part 3 is  $\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0, \quad j = 1, 2, ..., d$ where the Lagrangian *L* is given by  $L(q, \dot{q}, t) = K(q, \dot{q}, t) - V(q, t)$ 

# Plan for today

- Symmetries of systems (Section 2.3)
  - Conserved quantities (constants of motion)
  - Noether's theorem
  - Cyclic coordinates
  - Generalized momentum/conjugate momentum
  - More general symmetries of the Lagrangian
  - A proof of conservation of angular momentum! (If we have time)

# Summary

- Cyclic coordinates are generalized coordinates q<sub>i</sub> that do not appear in the Lagrangian
- The corresponding conjugate momentum p<sub>i</sub> is conserved

$$p_i \equiv \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right)$$

• If Q is the parameter of a transformation leaving the Lagrangian invariant the conserved quantity K is  $K = \sum_{i} \frac{\partial L}{\partial \dot{q}_{i}} Q_{i}$