

FYS4160

Problem Sheet 1

Discussion on **30th Jan** and **6th Feb 2015**

1. *The strength of gravity compared to the Coulomb force*

- a) Determine the difference in strength between the Newtonian gravitational attraction and the Coulomb force of the interaction of the proton and the electron in a hydrogen atom.
- (b) What is the gravitational force of attraction of two objects of 1kg at a separation of 1m . Compare with the corresponding electrostatic force of two charges of 1C at the same distance.
- (c) Compute the gravitational force between the Earth and the Sun. If the attractive force was not gravitational but caused by opposite electric charges, then what would the charges be?

2. *Falling objects in the gravitational field of the Earth*

- (a) Two test particles are in free fall towards the center of the Earth. They both start from rest at a height of 3 Earth radii and with a horizontal separation of 1m . How far have the particles fallen when the distance between them is reduced to 0.5m ?
- b) Two new test particles are dropped from the same height with a time separation of 1s . The first particle is dropped from rest. The second particle is given an initial velocity equal to the instantaneous velocity of the first particle, and it follows after the first one in the same trajectory. How far and how long have the particles fallen when the distance between them is doubled?

3. *A tidal force pendulum*

- a) Find the expression of the period of a mathematical pendulum with a length l in an area where the acceleration of gravity equals g . How long is the period of such a mathematical pendulum at the Earth's surface when the length of the pendulum is 0.25m ?
- b) A tidal force pendulum consists of two equally massive points that are connected through a stiff, massless rod. The length of the rod is $2l$. The pendulum oscillates with respect to the center of the rod which again is fixed in a constant distance from the center of the Earth. The oscillations takes place in a firm (i.e. non-rotating) vertical plane. The Earth is considered as a spherically symmetric mass distribution. The respective forces on the two massive points are shown in fig 1. Find the two equilibrium positions of the pendulum and perform a stability analysis with respect to these two points. Then use the definitions of torque to find an expression of the period of the harmonic motion of the rod. Find the period at the surface of the Earth. Is it possible to have a similar harmonic motion in a homogeneous gravitational field?

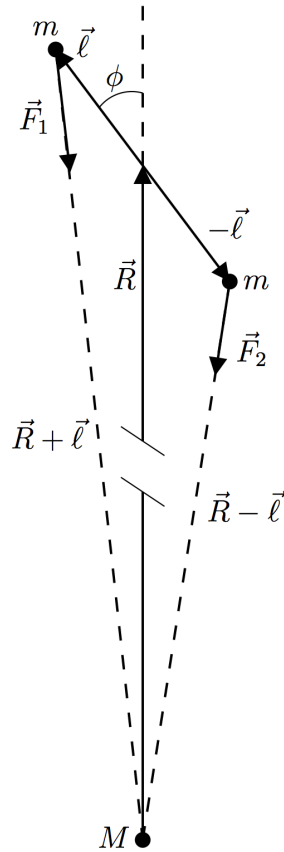


Figure 1: A tidal force pendulum

4. Newtonian potentials for spherically symmetric bodies

a) Calculate the Newtonian potential $\phi(r)$ for a spherical shell of matter. Assume that the thickness of the shell is negligible, and the mass per unit area, σ , is constant on the spherical shell. Find the potential both inside and outside the shell.

b) Let R and M be the radius and the mass of the Earth. Find the potential $\phi(r)$ for $r < R$ and $r > R$. The mass-density is assumed to be constant for $r < R$. Calculate the gravitational acceleration on the surface of the Earth. Compare with the actual value of $g = 9.81 \text{ m/s}^2$ ($M = 6.0 \cdot 10^{24} \text{ kg}$ and $R = 6.4 \cdot 10^6 \text{ m}$).

c) Assume that a hollow tube has been drilled right through the center of the Earth. A small solid ball is then dropped into the tube from the surface of the Earth. Find the position of the ball as a function of time. What is the period of the oscillations of the ball?

d) We now assume that the tube is not passing through the center of the Earth, but at a closest distance s from the center. Find how the period of the oscillations vary as a function of s . Assume for simplicity that the ball is sliding without friction (i.e. no rotation) in the tube.

5. The Earth-Moon system

a) Assume that the Earth and the Moon are point objects and isolated from the rest of the Solar system. Put down the equations of motion for the Earth-Moon system. Show that there

is a solution where the Earth and Moon are moving in perfect circular orbits around their common center of mass. What is the radii of the orbits when we know the mass of the Earth and the Moon, and the orbital period of the Moon?

b) Find the Newtonian potential along the line connecting the two bodies. Draw the result in a plot, and find the point on the line where the gravitational interactions from the bodies exactly cancel each other.

c) The Moon acts with a different force on a 1 kilogram measure on the surface of Earth, depending on whether it is closest to or farthest from the Moon. Find the difference in these forces.

6. *The Roche-limit*

a) A spherical moon with a mass m and radius R is orbiting a planet with mass M . Show that if the moon is closer to its parent planet's center than, $r = \left(\frac{2M}{m}\right)^{1/3} R$, then loose rocks on the surface of the moon will be elevated due to tidal effects.

b) The comet Shoemaker-Levy 9, that in July 1994 collided with Jupiter, was ripped apart already in 1992 after having passed near Jupiter. Assuming the comet had a mass of $m = 2.0 \cdot 10^{12}kg$ and that the closest passing in 1992 was at a distance of $96000km$ from the center of Jupiter, it is possible to estimate the size of the comet. Use that the mass of Jupiter is $M = 1.9 \cdot 10^{27}kg$.