Intensity interferometry for identical particles

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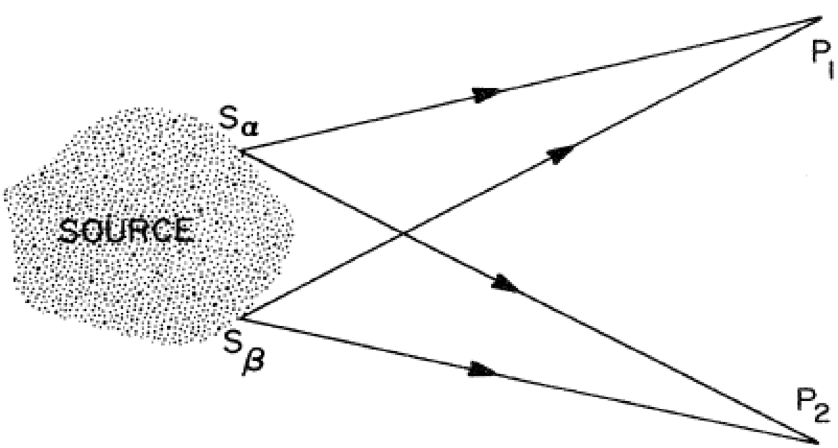
Introduction

- The technique of intensity interferometry has its origins in astrophysics, but it has seen significant theoretical theoretical development and widespread application in subatomic physics.
- Intensity interferometry was developed by Hanbury-Brown and Twiss in the 1950s, as a means of determining the dimension of distant astronomical objects.
- The method involves the construction of a two-particle correlation function from the distribution of particles radiated from a hot, spatially localized source.
- In the original astrophysics applications of the technique, the source was a distant radio-wave emitter. In applications involving the collision of nuclei or particles, the source is the reaction region.

Introduction



- A finite size source emits indistinguishable particles (e.g., from positions S_α and S_β) and the particles are later observed at positions P_1 and P_2 . Both emission points contribute to both observation points.
- The HBT correlation function is proportional to the intensity of the particles at P_1 and P_2 .In a classic amplitude interferometry, the two detecting points, can be used as 2 slits to produce interference patterns which lead to the measurement of relative phase of the 2 particles.

Theoretical basics

- To understand the origin of the correlations, we need a description of the state of the particles (e.g. pions) at the moment of their production at the source points, and of their propagation from the source points to the detection points.
- $$x' - x \approx (k/k^0)(t' - t)$$
- The amplitude for a particle to go from point x to point x', using Feynman path integral method leads to:
- $$A[(x,t) \rightarrow (x',t')] \equiv \psi(k : x \rightarrow x') = \sum_{all\ paths} e^{iS(path)}$$
- where S(path) is the action of the pion particles.

$$\psi(k : x \rightarrow x') \simeq e^{iS(classical\ path,\ k : x \rightarrow x')} \approx \mathrm{e}^{ik(x-x')},\ S(classical\ path,\ k : x \rightarrow x') \approx k(x - x')$$

Theory basics

- The production probability amplitude for producing a pion with momentum k at x by a magnitude A(k,x) and a phase Φ(x). A(k,x) can be taken to be real and nonnegative.
- The behaviour of the production phase Φ(x) at different source points describes the degree of coherence or chaoticity of the pion production process.
- The complete probability amplitude for a pion of momentum k to be produced from the source point x, to propagate along the classical trajectory, and to arrive at x is:

$$\Psi(k : x \rightarrow x') = A(k, x)\, e^{i\phi(x)}\, \psi(k : x \rightarrow x')$$

Theory basics

- The total amplitude for the detection of a pion at x' is the sum of the probability amplitudes from all source points:
- $$\Psi(k:\{all\ x\ points\}\rightarrow x') = \sum_x A(k,x)\mathrm{e}^{i\phi(x)}\psi(k:x\rightarrow x') = \sum_x A(k,x)e^{i\phi(x)}e^{ik(x-x')}$$
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- The single particle momentum distribution, P(k), is the absolute square of the total probability amplitude

$$P(k) = |\Psi(k:\{all\ x\ points\}\rightarrow x')|^2 = |\sum_x A(k,x)e^{i\phi(x)}e^{ik(x-x')}|^2 = |\sum_x A(k,x)e^{i\phi(x)}e^{ikx}|^2$$

Theory basics

- Expanding the momentum distribution like
- $$P(k)=\sum_{x} A^2(k,x)+\sum_{x\neq y} A(k,x)A(k,y)e^{i\phi(x)}e^{-i\phi(y)}e^{ik(x-y)}$$
-
- and if we take into account the randomness of the phases of the source at different points, we are left with:

$$P(k)=\sum_x A^2(k,x)$$

$$P(k)=\int dx\,\rho(x)A^2(k,x)$$

Theory basics

- Let's find out now the 2-particle distribution function. The probability amplitude for particle 1 with momentum k_1 to go from x_1 to x_1', in coincidence with particle 2 (k_2) going from x_2 to x_2' is the following:
- $$\psi(k_1 : x_1 \rightarrow x_1')\psi(k_2 : x_2 \rightarrow x_2')$$
- Using the same formalism as for 1 particle distribution, the amplitude becomes

$$\psi(k_1 : x_1 \rightarrow x_1')\psi(k_2 : x_2 \rightarrow x_2') = e^{ik_1(x_1 - x_1')} e^{ik_2(x_2 - x_2')}$$

Theory basics

- The probability amplitude for the two pions to be produced at the source points, to propagate from the source points, and to arrive at the detection point is
- $$A(k_1, x_1) e^{i\phi(x_1)} A(k_2, x_2) e^{i\phi(x_2)} \psi(k_1 : x_1 \rightarrow x_1') \psi(k_2 : x_2 \rightarrow x_2')$$
- $$A(k_1, x_1) e^{i\phi(x_1)} A(k_2, x_2) e^{i\phi(x_2)} e^{ik_1(x_1 - x_1')} e^{ik_2(x_2 - x_2')}$$
- Because of the indistinguishability of the pions and the Bose-Einstein statistics of identical bosons, the probability amplitude must be symmetrical with respect to the interchange of the labels of the pions. So, we get an additional cross term.

$$\frac{1}{\sqrt{2}} \{ A(k_1, x_1) e^{i\phi(x_1)} A(k_2, x_2) e^{i\phi(x_2)} e^{ik_1(x_1 - x_1')} e^{ik_2(x_2 - x_2')} \}$$

$$\frac{+1}{\sqrt{2}} \{ A(k_1, x_2) e^{i\phi(x_2)} A(k_2, x_1) e^{i\phi(x_1)} e^{ik_1(x_2 - x_1')} e^{ik_2(x_1 - x_2')} \} \equiv e^{i\phi(x_1)} e^{i\phi(x_2)} \Phi(k_1 k_2 : x_1 x_2 \rightarrow x_1' x_2')$$

Theory basics

- Summing over all points
- $$\Psi(k_1 k_2 : \{all\, x_1 x_2\, points\} \rightarrow x_1' x_2') = \sum_{x_1, x_2} e^{i\phi(x_1)} e^{i\phi(x_2)} \Phi(k_1 k_2 : x_1 x_2 \rightarrow x_1' x_2')$$
- $$P(k_1, k_2) = \frac{1}{2!} |\Psi(k_1 k_2 : \{all\, x_1 x_2\, points\} \rightarrow x_1' x_2')|^2$$
- For a chaotic source, we again make use of the random nature of the phases and expand the above formula:

$$P(k_1, k_2) = \sum_{x_1, x_2} |\Phi(k_1 k_2 : x_1 x_2 \rightarrow x_1' x_2')|^2$$

$$P(k_1, k_2) = \int dx_1\, dx_2\, \rho(x_1)\, \rho(x_2) |\Phi(k_1 k_2 : x_1 x_2 \rightarrow x_1' x_2')|^2$$

$$P(k_1, k_2) = \int dx_1 \rho(x_1) A^2(k_1, x_1) \int dx_2 \rho(x_2) A^2(k_2, x_2) + \int dx_1 \rho(x_1) A(k_1, x_1) A(k_2, x_1) e^{i(k_1 - k_2) x_1} \times \int dx_2 \rho(x_2) A(k_2, x_2) A(k_1, x_2) e^{i(k_2 - k_1) x_2}$$

$$P(k_1, k_2) = P(k_1) P(k_2) + |\int dx\, e^{i(k_1 - k_2) x} \rho(x) A(k_1, x) A(k_2, x)|^2$$

Theory basics

- Now we can use an effective density like this:
- $$P(k_1, k_2) = P(k_1)P(k_2)(1 + |\int dx\, e^{i(k_1 - k_2)x} \rho_{eff}(x; k_1, k_2)|^2)$$
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- $$\rho_{eff}(x; k_1, k_2) = \rho(x) A(k_1, x) A(k_2, x) / \sqrt{P(k_1) P(k_2)}$$
- $$\tilde{\rho}_{eff}(q; k_1 k_2) = \int dx\, e^{iqx} \rho_{eff}(x; k_1 k_2)$$
- The distribution function becomes

$$P(k_1, k_2) = P(k_1)P(k_2)(1 + \left|\tilde{\rho}_{eff}(q; k_1 k_2)^2\right|)$$

Theory basics

- Now we can define our correlation function like
- $$C_2(k_1, k_2) = \frac{P(k_1, k_2)}{P(k_1)P(k_2)}$$
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- Thus, for an extended chaotic source, the two pion correlation function is directly related to the Fourier transform of the effective density.

$$C_2(k_1, k_2) = 1 + |\tilde{\rho}_{eff}(q; k_1 k_2)|^2$$

Model sources for particle emission

Static Gaussian source

Is the most extensively used parametrization and corresponds to particle emission from a Gaussian source that may move with respect to the laboratory frame but does not otherwise evolve with time.

$$\rho(r) = \frac{1}{\sqrt{a_x^2 a_y^2 a_z^2 \pi^3 R^6}} \exp -[(x/a_x)^2 + (y/a_y)^2 + (z/a_z)^2]/R^2,$$

where a_i 's are dimensionless constants that allow for non-spherical sources.

If the two body wave functions are plane waves, the correlation function is given by the square of the Fourier transform of the source distribution:

$$C_2(k_1, k_2) = 1 + \exp\{-2[(q_x a_x)^2 + (q_y a_y)^2 + (q_z a_z)^2]R^2\}$$

Two gaussian source

This type of source is a refinement of the static single-gaussian source model

$$\rho(r)=\frac{\mu_1}{(piR_1^2)^{3/2}}\exp(-r^2/R_1^2)+\frac{\mu_2}{(piR_2^2)^{3/2}}\exp(-r^2/R_2^2)$$

where $\mu_1+\mu_2=1$. One of the length scales could refer to direct pion production, while the second could refer to pions produced through resonance decays. The corresponding correlation function is

$$C_2(k_1,k_2)=\mu_1\exp[-(k_1-k_2)^2 R_1^2/2]+\mu_2\exp[-(k_1-k_2)^2 R_2^2/2]$$

Finite source lifetime Gaussian-source model

This is a further refinement of the Gaussian-source model.

$$\rho(r,t)=\frac{1}{\pi^2 r_0^3 \tau}\exp\{-r^2/r_0^2-t^2/\tau^2\}$$

The corresponding correlation function is, in the plane wave limit

$$C_2(k_1,k_2)=1+\exp\{-[(k_1-k_2)^2 r_0^2/2]-[(E_1-E_2)^2\tau^2/2]\}$$

Lorentz invariant Gaussian-source model

A further time dependence of the source distribution is the translation of the source in the laboratory frame. The effect of this motion lead to the following Lorentz invariant form of the Gaussian source:

$$\rho(x^\mu)=\frac{1}{\pi^2 r_0^3 \tau}\exp[-B_1(x_\mu S^\mu)^2+B_2 x_\mu x^\mu]$$

where B1 and B2 are source parameters and S_μ is the total four-momentum of the source system. For a choice of the parameters

$$B_1=(r_0^{-2}+\tau^{-2})/s,\ B_2=r_0^{-2},\ where\ s=S_\mu S^\mu$$

we get the following correlation function:

$$C_2(k_1,k_2)=1+\exp[-2B_1 r_0^2\tau^2(q_\mu S^\mu)^2+2q_\mu q^\mu/B_2],\ where\ q^\mu=(k_1^\mu-k_2^\mu)/2$$

Kopylov-Podgoretskii model

Kopylov and Podgoretskii propose a more complicated model for the pion emitting source. Basically, they consider a two step approach to pion production. In the first step, an oscillator is excited. In the second step it deexcites by emitting a pion. The decay is assumed to take place statistically. Two models are considered for the spatial region where the oscillators are produced. In the first model, the oscillators are uniformly distributed inside an ellipsoidal region, while in the second model, the oscillators are distributed only on the surface of an ellipsoid.

In their model, the probability of detecting two particles is

$$P_{12} \approx 1 + \frac{\exp(i\Delta)}{2(\xi_\alpha - i)(\xi_\beta + i)} + \frac{\exp(-i\Delta)}{2(\xi_\alpha + i)(\xi_\beta - i)} \approx 1 + \frac{(1+\xi_\alpha \xi_\beta)\cos\Delta + (\xi_\alpha - \xi_\beta)\sin\Delta}{(1+\xi_\alpha^2)(1+\xi_\beta^2)},$$

where

$$\Delta = k_1 r_{\alpha 1} + k_2 r_{\beta 2} - k_1 r_{\beta 1} - k_2 r_{\alpha 2} - (E_1 - E_2)(t_\alpha - t_\beta)$$

$$\xi_{\alpha(\beta)} = [E_1 - E_2 - v_{\alpha(\beta)}(k_1 - k_2)] / \Gamma_{\alpha(\beta)}$$

Kopylov-Podgoretskii model

The first source region is a uniform ellipsoid and the correlation function is:

$$C_2(k_1, k_2) = 1 + [3\, j_1(\kappa)/\kappa]^2/(1+\xi^2),$$

where $j_1(\kappa)$ is a spherical Bessel function and

$$\kappa = 2[(q_x A_x)^2 + (q_y A_y)^2 + (q_z A_z)^2]^{1/2}$$

The A_i parameters are the semiaxes of the ellipsoid.

As a second source distribution, Kopylov and Podgoretskii consider emission from an ellipsoidal surface. This gives the result:

$$C_2(k_1, k_2) = 1 + [J_1(\kappa)/\kappa]^2/(1+\xi^2),$$

where $J_1(\kappa)$ is a cylindrical Bessel function.

Time evolving sources

Pratt (1984) has developed a correlation function formalism based on Wigner functions, which he applies to particle emission from a spherically expanding shell. Pion emission is treated from a source whose Wigner function has the form

$$g(x,k)=\delta(r-R_0)\exp(-t^2/\tau^2)\exp[-E'(k,r)/T],$$

where the pion is emitted at space-time coordinates $(R_0 n, t)$ from a spherical shell characterized by a radius R0, lifetime parameter τ , and temperature T. The unit vector normal to the sphere surface is denoted by n. In a frame co-moving with the shell at velocity vn, the pions energy is $E'(k,r)=(E_p - v\,\boldsymbol{nk})(1-v^2)^{-1/2}$

The correlation function has a form in which the apparent source size depends on the total momenta of the pion pair.

$$R_S(\boldsymbol{K})=R_0[(z\tanh z)^{-1}-\sinh^{-2} z]^{1/2},$$

where $\boldsymbol{K}=\boldsymbol{k}_1+\boldsymbol{k}_2$ and $z=\boldsymbol{K}\gamma v/(2T)$

Final state interactions

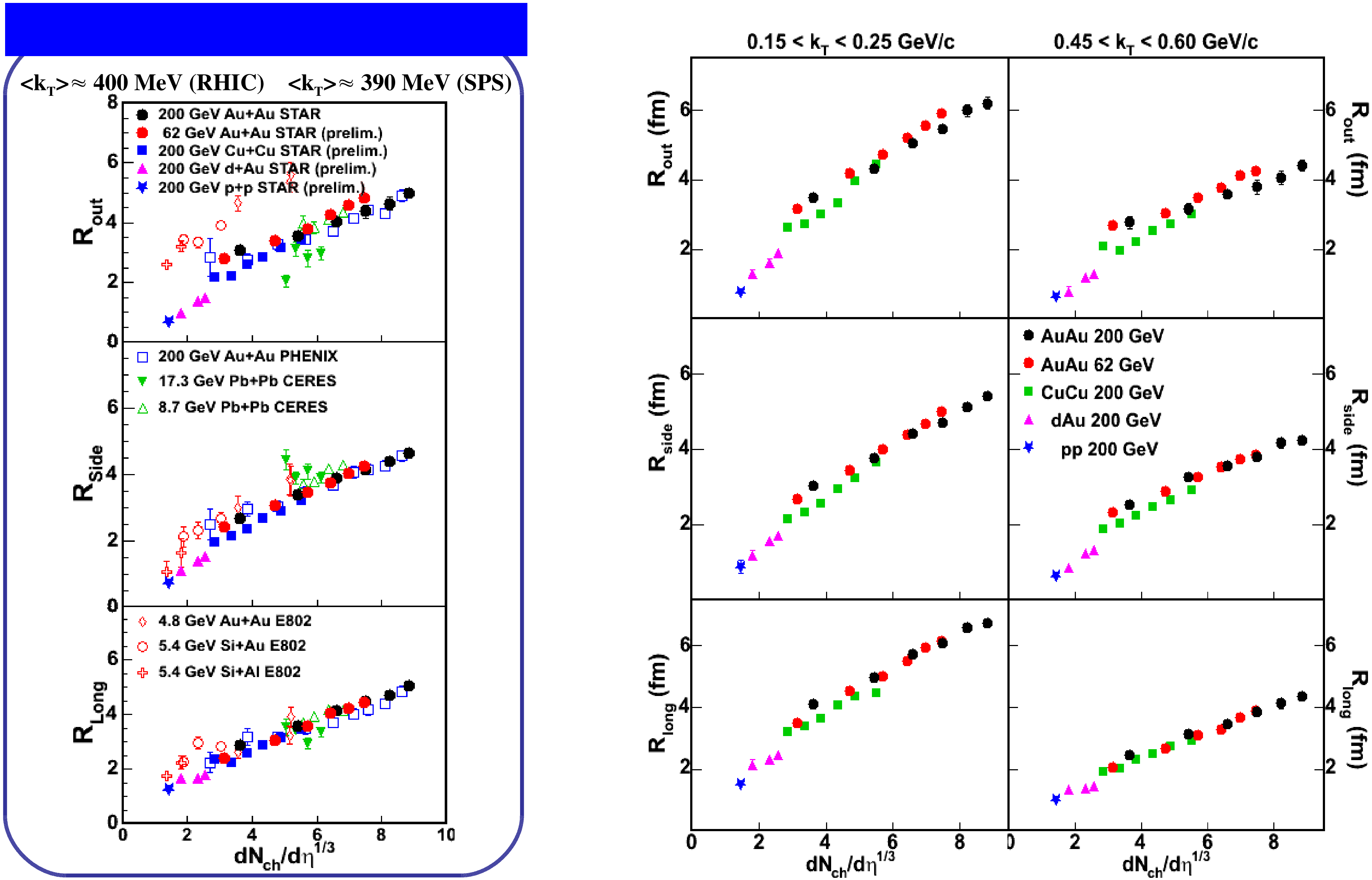
Coulomb interaction is corrected by using the Gamow factor.

$$[C_2(q)]_{theory+Coulomb} = W(p_1, p_2) C_2(q)$$

$$W(p_1, p_2) = 2\pi\eta / [\exp(2\pi\eta) - 1], \eta = \alpha m_\pi / |p_1 - p_2|$$

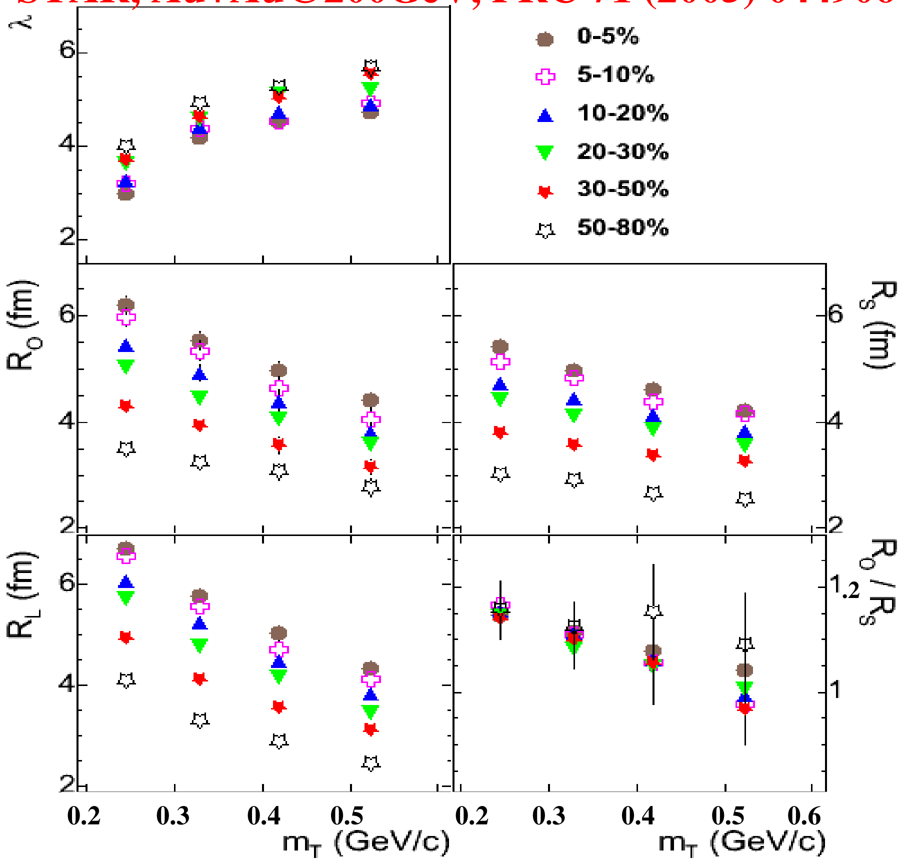
The strong interaction is believed to distort the correlation factor as well. Unfortunately, there is considerable uncertainty in the measured phase shifts introduced by the strong interaction, so that the magnitude of the strong interaction effects is not well determined. Different authors agree that strong interactions may suppress the correlation function at q=0 by as much as 20%.

Recent experimental results

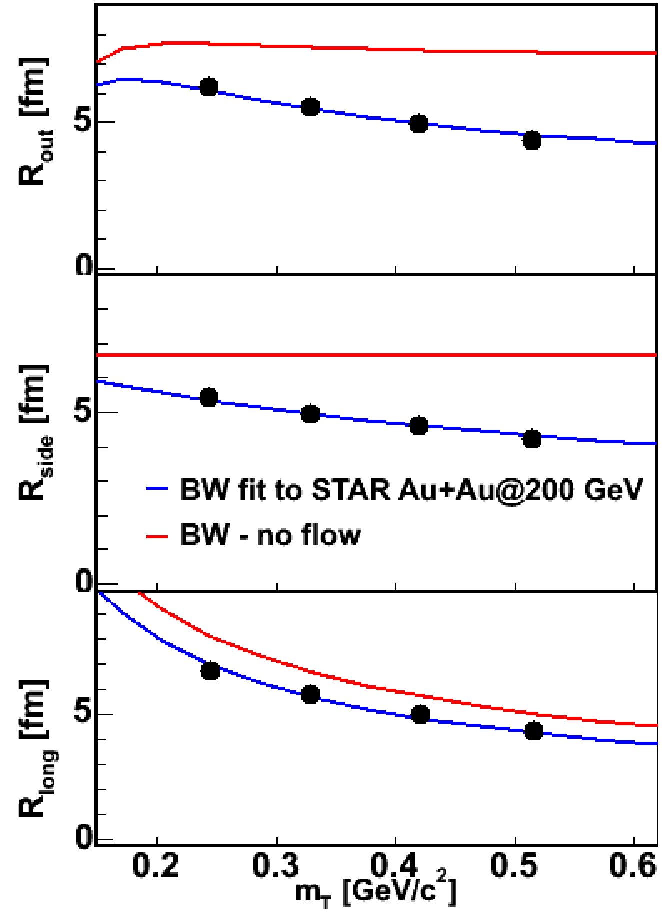


Apparent source size measured by STAR as a function of multiplicity. The data is compared with results at lower energies also. The scaling between source size, fitted from the experimental correlation function, and multiplicity can be observed.

STAR, Au+Au@200GeV, PRC 71 (2005) 044906

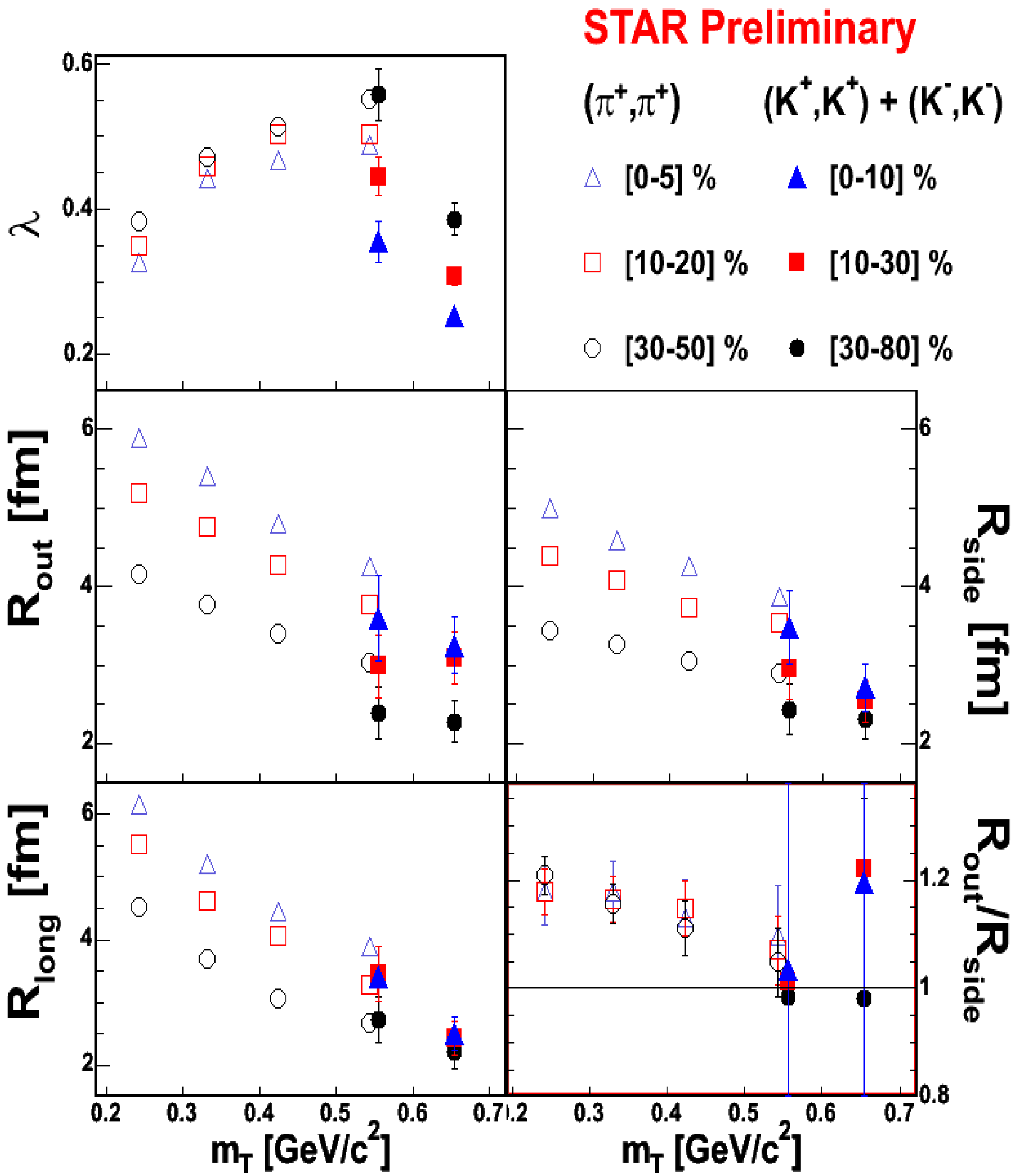


In Au+Au p_T (m_T) dependence attributed to collective expansion of the source



Calc. with Blast-Wave - Retiere, Lisa, PRC 70 (2004) 044907

The chaoticity parameter increases with increasing transverse mass of the pair, meaning that lower pt particles come from a more coherent source. The observed radius size is dropping with increasing transverse mass leading to the conclusion that high pt particles are created in a central hot core, while low pt particles come from a more extended source.



Comparison between pion-pion correlations and kaon-kaon correlation results from STAR in AuAu at 62.4 GeV collisions. Within statistical errors, at the same transverse mass, the kaon correlations give about the same source size as the pion correlations.