Strangeness production in Heavy Ion Collisions

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Strangeness content at thermal and chemical equilibrium

- In nucleon-nucleon collisions we have production of quark-antiquark pairs.
- Produced strange quarks and antiquarks will then combine with neighboring quarks and antiquarks to form strange particles
- In the Schwinger model of particle production we get the following formula which gives the ratio of strange to nonstrange pairs as 0.1

$$\frac{\Delta N}{\Delta t \, \Delta x \, \Delta y \, \Delta z} = \frac{\kappa^2}{8\pi^3} \exp\left\{ \frac{-\pi m_q^2}{\kappa} \right\}$$

Strangeness content at thermal and chemical equilibrium

- Counting the valence quarks we can find the ratio of strange to nonstrange quarks related to the K^+/π^+ ratio
- In p-Be collisions at 14.6 GeV the K^+/π^+ ratio has been measured to about 0.08. This gives a strange to nonstrange quark ratio in the order of 0.05

$$\frac{s+\bar{s}}{u+\bar{u}+d+\bar{d}}=\frac{K^{+1}/\pi^{+1}}{1.5+K^{+1}/\pi^{+1}}$$

Hadronic gas in equilibrium

- In nucleus-nucleus collisions large numbers of hadrons are produced.
- What happens to the ratio of strange to nonstrange particles if the hadronic gas is allowed to reach equilibrium?
- We consider an electrically neutral boson gas of pions and kaons in thermal and chemical equilibrium.
- We then have zero chemical potential for both charged and neutral mesons.
- The density of a particle is then given by:

$$n_i = \frac{1}{(2\pi)^3} \int_0^\infty \frac{4\pi \boldsymbol{p}^2 d|\boldsymbol{p}|}{e^{\sqrt{\boldsymbol{p}^2+m_i^2}/T} - 1} = \frac{T m_i^2}{2\pi^2} \sum_{k=1}^\infty \frac{1}{k} K_2\left(\frac{k m_i}{T}\right)$$

Hadronic gas in equilibrium

- The integral can be written as a sum over the modified Bessel function of order 2.

- Using tabulated values of the Bessel function we can calculate particle densities at T=200 MeV

$$\frac{n_{K^{+1}}}{n_{\pi^{+1}}}=0.3792$$

$$\frac{n_s+n_{\bar{s}}}{n_u+n_{\bar{u}}+n_d+n_{\bar{d}}}=\frac{n_{K^{+1}}/n_{\pi^{+1}}}{1.5+n_{K^{+1}}/n_{\pi^{+1}}}=0.2018$$

Hadronic gas in equilibrium

- We see that both the K^+/π^+ ratio of 0.38 and strange to nonstrange ratio of 0.2 have been enhanced over the respective ratios of 0.08 and 0.05 in p-Be collisions
- An important question is whether the reaction rate is fast enough for the hadron gas to reach equilibrium within the timeframe of the collision process.
- The biggest problem is strangeness production which has a large energy threshold compared to the temperature of the hadron gas T=200MeV

Strangeness content in a quark-gluon plasma

- The strangeness content of the QGP is governed by the dynamical state of the plasma.
- The plasma is in thermal equilibrium when the momentum distributions of the particles do not change.
- Similarly the plasma is in chemical equilibrium when the particle densities do not change.
- Processes changing the momentum distribution or chemical equilibrium are negated by inverse or other reactions
- The state of the plasma is characterized by the temperature T and the chemical potentials μ_i of the various particles.

Strangeness content in a quark-gluon plasma

- The quark density is given as an integral over the Fermi-Dirac distribution.
- We consider the case when $\mu_u = \mu_d = \mu_s = 0$, and the temperature is of the same order as the strange quark mass. The densities of all quarks and antiquarks are then nearly the same. Fig 18.2
- This is why an enhancement of the number of strange quarks is a suggested signal for the production of quark-gluon plasma

$$n_q(\mu_q) = \frac{N_c N_s}{(2\pi)^3} \int_0^\infty \frac{4\pi \boldsymbol{p}^2 d|\boldsymbol{p}|}{1 + e^{(\sqrt{\boldsymbol{p}^2 + m_q^2} - \mu_q^2)/T}},$$

$$n_{\bar{q}}(\mu_q) = n_q(-\mu_q)$$

Strangeness content in a quark-gluon plasma

- Enhancement of the number of strange quarks leads to an enhancement of the number of strange hadrons.
- With nearly equal number of quarks and antiquarks we have an equal probability of creating hyperons and antihyperons.
- The enhancement of the production of strange antihyperons can be used as a signal of production of quark-gluon plasma with $\mu_u = \mu_d = 0$.
- In nucleon-nucleon collisions, production of antihyperons is greatly suppressed by the Schwinger factor.

The heavy-ion stopping regime

- In heavy ion collisions we have nonzero chemical potentials μ_u and μ_d. This is because of the valence quarks in the baryons participating in the collision.
- With no strange valence quarks in the colliding baryons the strange chemical potential μ_s remains zero.
- The light quark densities are now greater than strange quark and antiquark densities which again are greater than light antiquark densities.
- This means that strange antiquarks are most likely to combine with a light quark to form a K^+ or K^o while strange quarks are more likely to combine with light quarks to form a hyperon.

$$n_u, n_d > n_s, n_{\bar{s}} > n_{\bar{u}}, n_{\bar{d}}$$

Rate of approach to chemical equilibrium in a quark-gluon plasma

- It is important to know the time scale for the strangeness content of a QPG to reach equilibrium
- We consider a thermalized QGP consisting of light quarks, antiquarks and gluons, with negligible strangeness content.
- Strange quarks can be produced during the following processes

$$u+\bar{u}\rightarrow s+\bar{s}$$
$$d+\bar{d}\rightarrow s+\bar{s}$$
$$g+g\rightarrow s+\bar{s}$$

The cross sections for these reactions are:

$$\sigma_{q\overline{q}}(M) = \frac{8\pi\alpha_s^2}{27M^2}\left(1+\frac{\eta}{2}\right)\sqrt{1-\eta}$$
$$\sigma_{gg}(M) = \frac{\pi\alpha_s^2}{3M^2}\left[\left(1+\eta+\frac{1}{16}\eta^2\right)\ln\left(\frac{1+\sqrt{1-\eta}}{1-\sqrt{1-\eta}}\right)\left(\frac{7}{4}+\frac{31}{16}\eta\right)\sqrt{1-\eta}\right]$$
$$\eta = \frac{4m_s^2}{M^2}$$

The reaction cannot proceed unless the energy M is larger than the threshold energy $2m_s$. In the limit $M \approx 2m_s$ we have:

$$\sigma_{q\overline{q}} \sim \frac{\pi\alpha_s^2}{3M^2}\frac{4}{3}\sqrt{1-\eta}$$
$$\sigma_{gg} \sim \frac{\pi\alpha_s^2}{3M^2}\frac{7}{16}\sqrt{1-\eta}$$

In the limit of very high energies $M >> m_s$ we have:

$$\sigma_{q\overline{q}} \sim \frac{\pi\alpha_s^2}{3M^2}\frac{8}{9}$$
$$\sigma_{gg} \sim \frac{\pi\alpha_s^2}{3M^2}\left[(1+\eta)\ln\left(\frac{M^2}{m_s^2}\right)-\left(\frac{7}{4}+\frac{17\eta}{16}\right)\right]$$

Strangeness production in a quark-gluon plasma

- We see that the cross-section for production from gluons dominate at high energies. At low energies production from light quarks dominate. Fig 18.4

- Using the cross sections we can determine the rate of strangeness production in the plasma.

- We start by looking at production from light quarks, then we use the same method on the gluons.

The number of quarks with one flavor and momentum $\vec{p}_1$, in the volume element d^3x and the momentum elemtent d^3p_1 is:

$$dN_q = N_c N_s \frac{d^3x d^3p_1}{(2\pi)^3} f_q(E_1)$$

where $f_q(E_1)$ is the Fermi-Dirac distribution:

$$f_q(E_1) = \frac{1}{e^{(E_1-\mu)/T} + 1}$$

The volume swept by the cross section $\sigma_{q\overline{q}}(M)$ per unit time due to the relative motion of the quark and antiquark is $\sigma_{q\overline{q}}(M)v_{12}$. The number of antiquarks available for a quark for strangeness productions is then:

$$\sigma_{q\overline{q}}(M) v_{12} N_c N_s f_{\overline{q}}(E_2) \frac{d^3p_2}{(2\pi)^3}$$

where $f(E_2)$ is the Fermi-Dirac distribution for an antiparticle:

$$f_{\overline{q}}(E_2) = \frac{1}{e^{(E_2+\mu)/T} + 1}$$

The number of $s\overline{s}$ pairs produced per unit time from light quarks is given as the integral:

$$\frac{dN_{s\overline{s}}}{dt}(q\overline{q} \to s\overline{s}) = N_c^2 N_s^2 \sum_{f=1}^{N_f=2} \int \frac{d^3x d^3p_1 d^3p_2}{(2\pi)^6} f_q(E_1) f_{\overline{q}}(E_2) \sigma_{q\overline{q}}(M) v_{12}$$

Strangeness production per unit time per spatial volume is then:

$$\frac{dN_{s\bar{s}}}{dtd^3x}(q\bar{q} \to s\bar{s}) = N_c^2 N_s^2 \sum_{f=1}^{N_f=2} \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} f_q(E_1) f_{\bar{q}}(E_2) \sigma_{q\bar{q}}(M) v_{12}$$

By the same method we can find the rate of strangeness production from the process $gg \to s\bar{s}$:

$$\frac{dN_{s\bar{s}}}{dtd^3x}(gg \to s\bar{s}) = N_g^2 N_\epsilon^2 \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} f_g(E_1) f_g(E_2) \sigma_{gg}(M) v_{12}$$

where $f_g(E)$ is the Bose-Einstein distribution function:

$$f_g(E) = \frac{1}{e^{E/T} - 1}$$

By looking at the degrees of freedom we find:

$$N_{q\bar{q}} = N_c^2 N_s^2 N_f = 72$$
$$N_{gg} = N_g^2 N_\epsilon^2 = 256$$

The general expression for the production rate is:

$$\frac{dN_{s\bar{s}}}{dtd^3x} = N\int \frac{d^3p_1 d^3p_2}{(2\pi)^6} f_1(E_1)f_2(E_2)\sigma(M)v_{12}$$

This integral can be solved with the saddle point method. By assuming the light quarks to be massless we can write the equation as:

$$\frac{dN_{s\bar{s}}}{dtd^3x} = N\frac{\sigma(M)M^2}{2(2\pi)^4} f_1(\epsilon(M))F_2\left(\frac{M^2}{4\epsilon(M)}\right)\sqrt{\frac{2\pi}{w(\epsilon)}}$$

where

$$F_2(E) = -\int_\infty^E f_2(E')dE'$$

and ϵ is the root of the extremum condition

$$\left\{\frac{d}{dE}\left[\ln f_1(E) + \ln F_2\left(\frac{M^2}{4E}\right)\right]\right\}_{E=\epsilon} = 0$$

and $w(\epsilon)$ is given by the second derivatives

$$w(\epsilon) = -\left\{\frac{d^2}{dE^2}\left[\ln f_1(E) + \ln F_2\left(\frac{M^2}{4E}\right)\right]\right\}_{E=\epsilon}$$

We find for the case $\mu = 0, T << 2m_s$ the following expression

$$\frac{dn_s}{dM^2dt} \simeq N\frac{\sigma(M)M^2}{2(2\pi)^4}T^2\frac{\pm\ln(1\pm e^{-M/2T})}{e^{M/2T}\pm 1}\sqrt{\frac{\pi M}{2T}}$$

Which gives the following rate of strange quark density change

$$\frac{dn_s}{dt} \simeq N_i\int dM^2\frac{\sigma_i(M)M^2}{2(2\pi)^4}T^2\frac{\pm\ln(1\pm e^{-M/2T})}{e^{M/2T}\pm 1}\sqrt{\frac{\pi M}{2T}}$$

For the case $\mu \neq 0$ and T not small ϵ and w must be evaluated numerically. We can now use our results at chemical equilibrium to find the equilibration time τ:

$$\tau = \frac{n_s}{dn_s/dt}$$

Equilibration time

- The equilibration time gives the order of magnitude of the time scale for a quark-gluon plasma to reach equilibrium from an initial state void of strangeness
- We have not taken into account the effects of reverse processes and Pauli exclusion. But it still gives an estimate of the time needed to form a QGP
- For a temperature of T=200 MeV we have of τ=10 fm/c.
- The collision process in a heavy ion collision takes place over 5-10 fm/c.
- Equilibration of strangeness will not be completed for temperatures of 200 MeV, but may be close to completion at T=400 MeV

Experimental results

- Following are recent experimental results regarding strange particles.

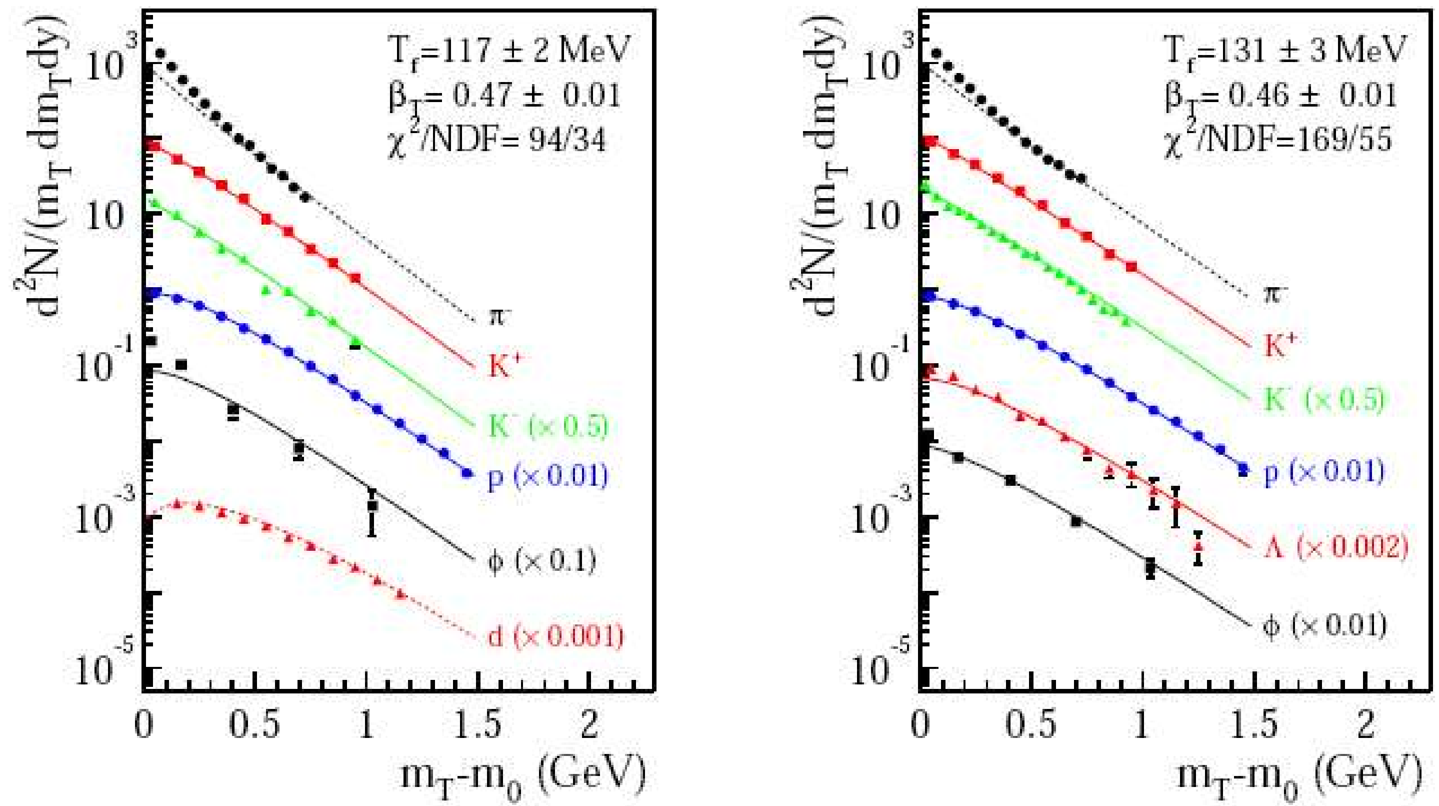


FIG. 1. Transverse mass spectra of hadrons produced in central Pb+Pb collisions at 20 (left) and 30 (right) AGeV. The solid lines indicate fits of a blast wave parametrization [9].

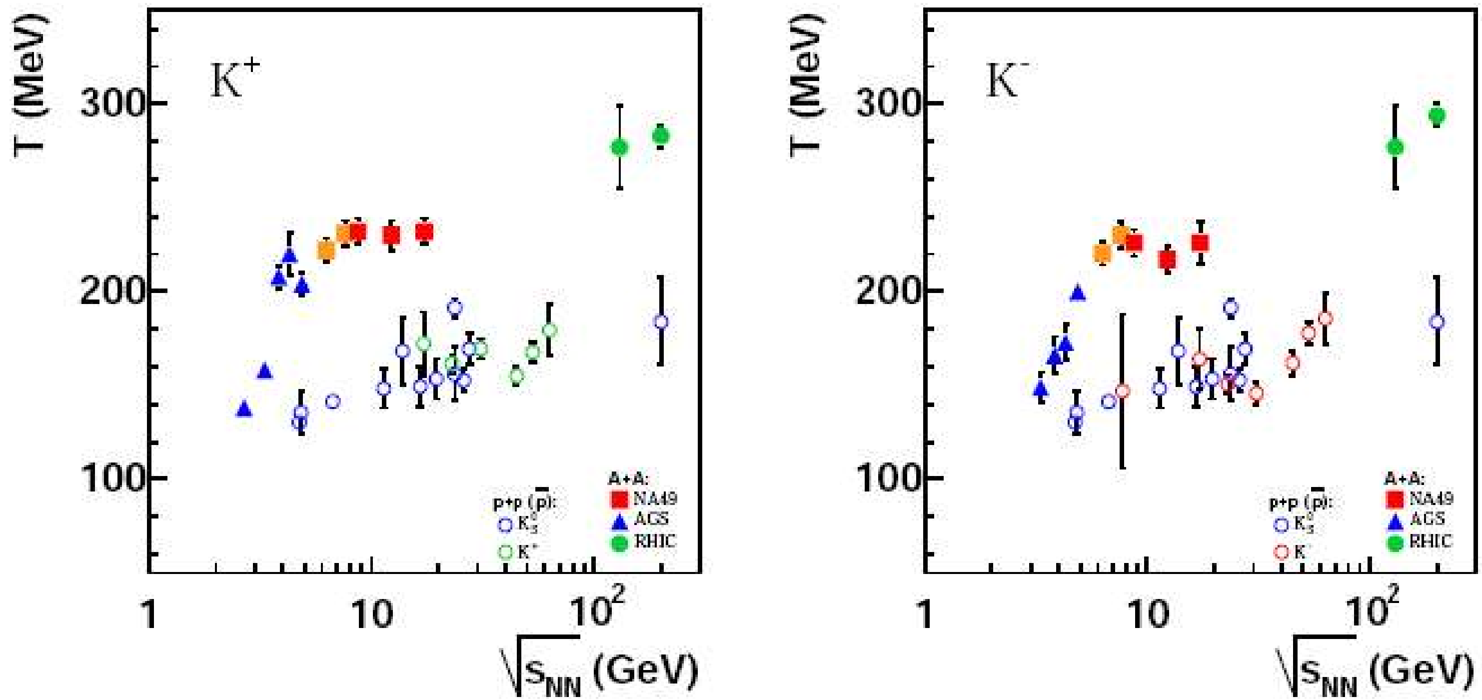


FIG. 2. Energy dependence of the inverse slope parameter, T, of the transverse mass spectra of K^+ (left) and K^- (right) produced in central Pb+Pb (Au+Au) collisions (solid symbols) and p+p interactions (open symbols). $\sqrt{s_{NN}}$ is the c.m.s. energy per nucleon-nucleon pair.

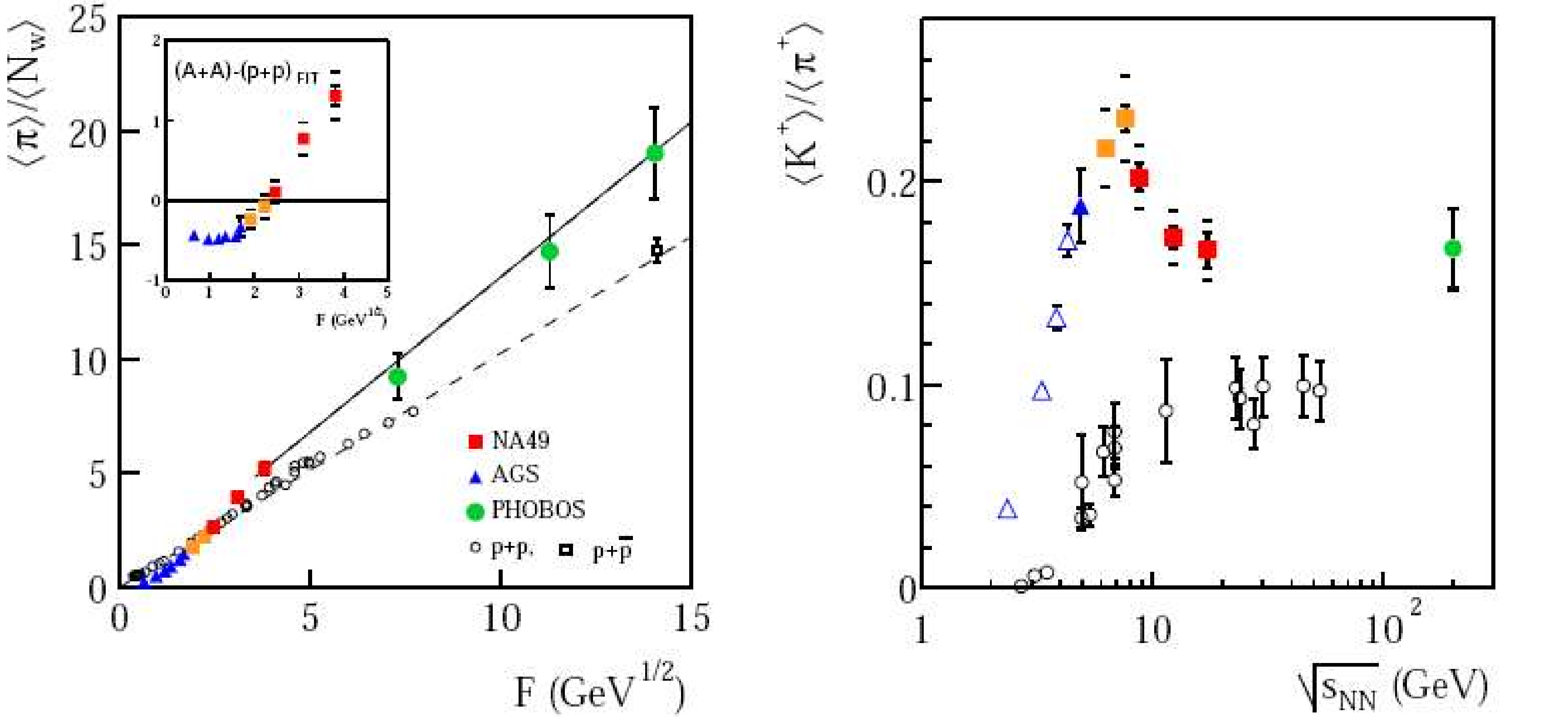


FIG. 4. Left: The dependence of total pion multiplicity per wounded nucleon on Fermi's energy measure F ($F \equiv (\sqrt{s_{NN}} - 2m_N)^{3/4}/\sqrt{s_{NN}}^{1/4}$, where $\sqrt{s_{NN}}$ is the c.m.s. energy per nucleon–nucleon pair and m_N the rest mass of the nucleon) for central Pb+Pb (Au+Au) collisions (closed symbols) and inelastic p+p($\overline{\text{p}}$) interactions (open symbols). Right: The dependence of the $\langle K^+ \rangle / \langle \pi^+ \rangle$ ratio on the collision energy for central Pb+Pb (Au+Au) collisions (closed symbols) and inelastic p+p interactions (open symbols).

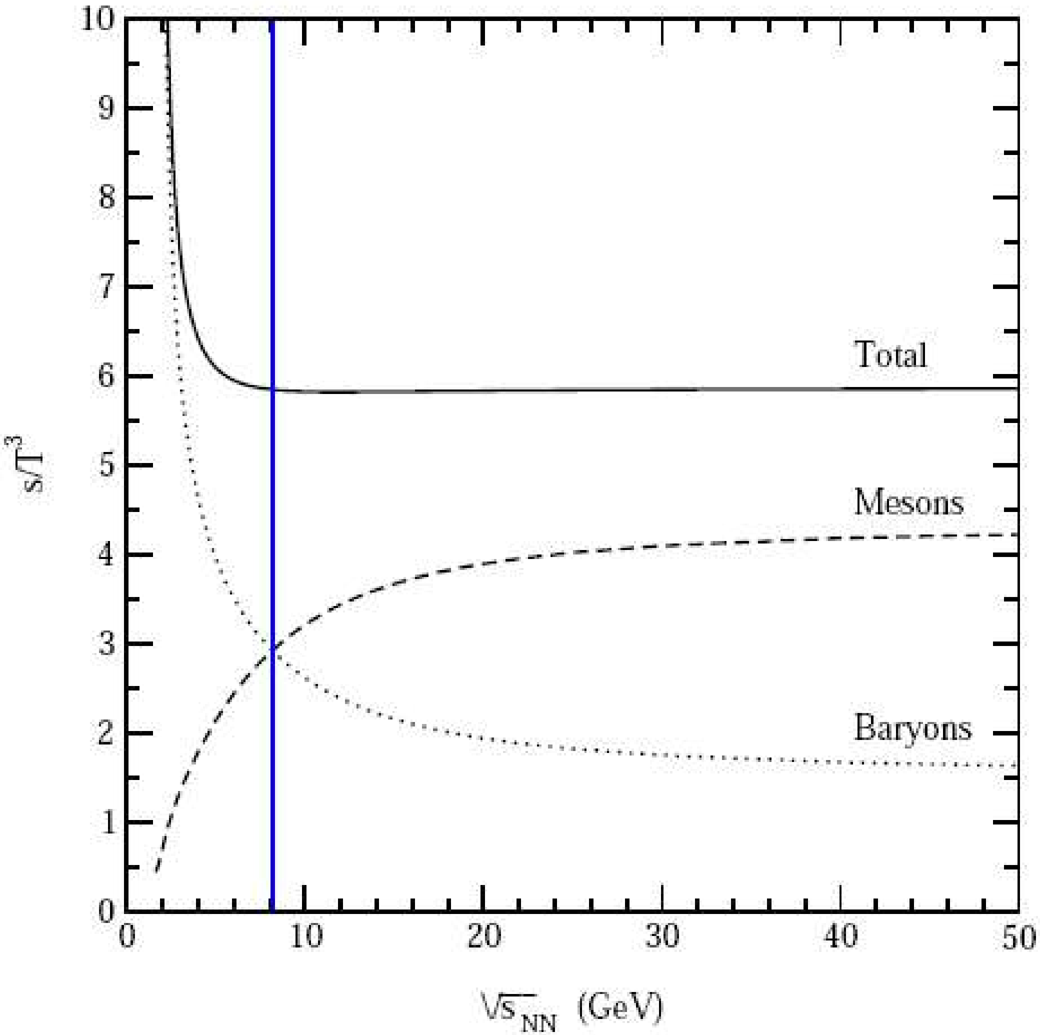


Fig. 2. The entropy density normalised to T^3 as a function of the beam energy as calculated in the statistical model using THERMUS [21].

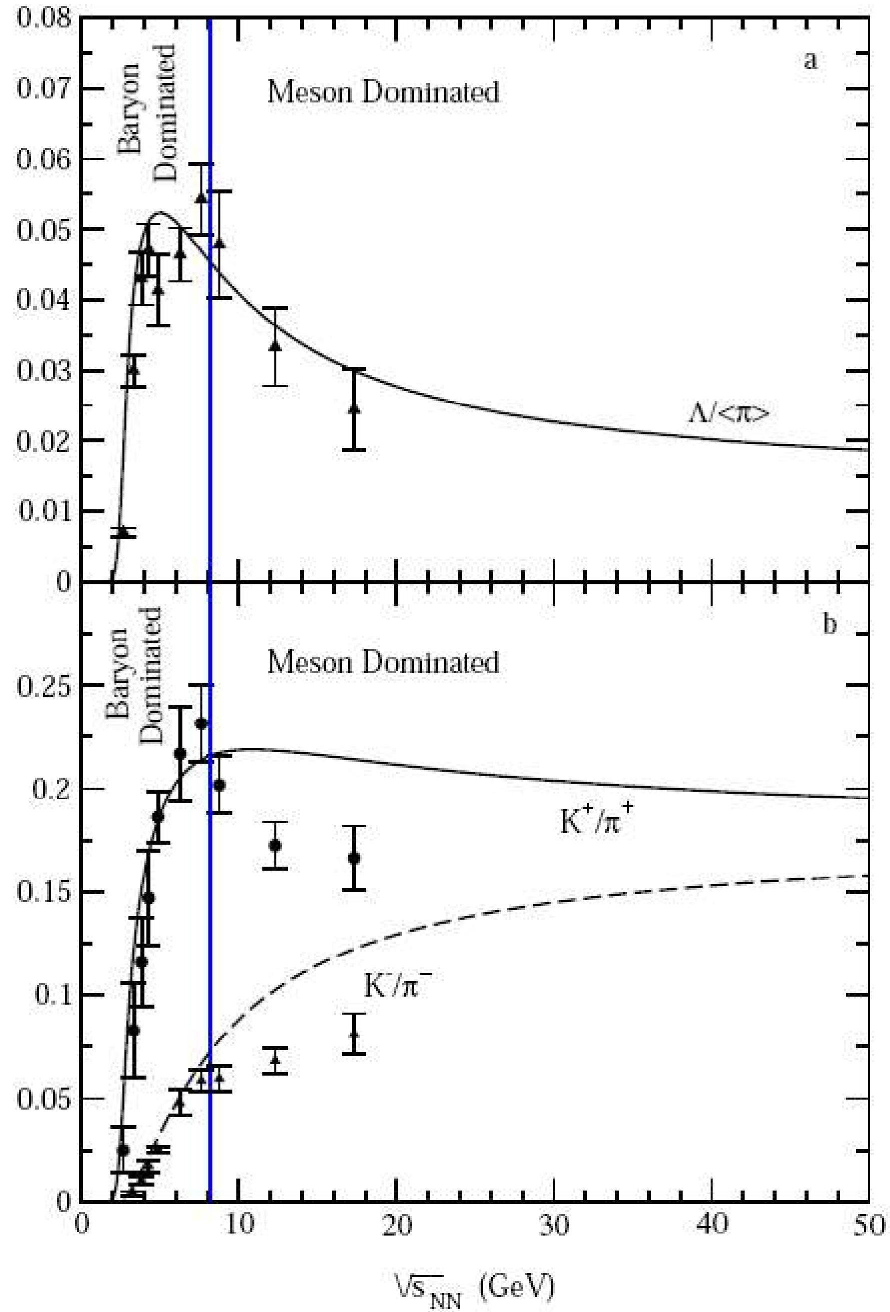
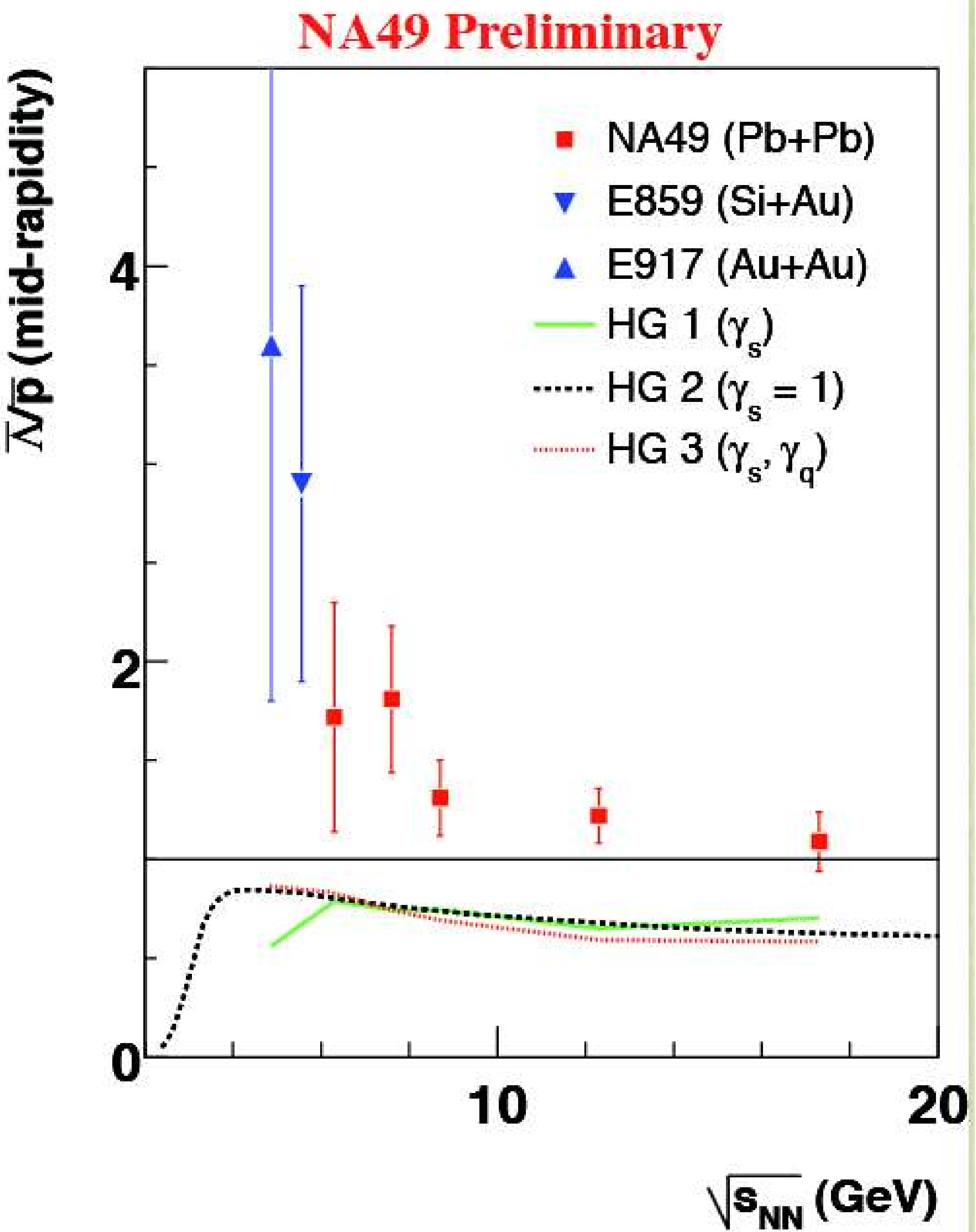


Fig. 4. a) The $\Lambda/\langle\pi\rangle$ ratio as a function of beam energy. b) The K^+/π^+ and K^-/π^- ratios as a function of energy. The solid and dashed lines are the predictions of the statistical model calculated using THERMUS [21]. The data points are from Ref. [1,2,3,4,7,9].



NA49 Preliminary
$\overline{\Lambda}/\overline{p}$ (mid-rapidity)
NA49 (Pb+Pb)
E859 (Si+Au)
E917 (Au+Au)
HG 1 (γ_s)
HG 2 (γ_s = 1)
HG 3 (γ_s, γ_q)
4
2
0
10
20
$\sqrt{s_{NN}}$ (GeV)

Summary

- **Comprehensive results on strange hadron production from NA49.**
- **Transverse mass spectra**
  - kinetic freeze-out at $T \approx 100 - 120$ MeV, $\beta_T \approx 0.8$
  - hyperons are consistent with the step structure observed in $\langle m_t \rangle - m_0$ for kaons.
- **Energy dependence**
  - midrapidity yields increase with energy
  - the strangeness to pion ratio shows a maximum at low SPS energies
  - energy dependence of $\bar{B}/B$ ratio gets weaker with increasing strangeness content.
  - $\bar{\Lambda}/\bar{p}$ ratio increases with decreasing energy
  - stronger Ξ enhancement than Λ with centrality
  - differences between NA49 and NA57 hyperon yields at midrapidity
- **High p_t yield ratios**
  - R_{CP} is different between mesons and baryons
  - R_{CP} at pt $\approx$ 3 GeV/c increases with decreasing energy
- **Λ flow**
  - Substantial Λ elliptic flow observed increasing with p_t
  - Mass ordering of v_2