# Ch.6: Array computing and curve plotting

**Joakim Sundnes**[1,2]

[1]Simula Research Laboratory
[2]University of Oslo, Dept. of Informatics

Sep 20, 2021

## 0.1 Plan for week 38

Monday 20 september

- Quick recap of last week
- Magic functions 'eval' and 'exec' (left from last week)
- Intro to NumPy arrays and plotting
- Live programming of ex 4.6, 5.7, 5.9, (5.10, 5.11)

Thursday 23 september

- Live programming of ex 5.10, 5.11, 5.13
- Making movies and animations from plots

## 0.2 Recap from last week - user input

**Alternative 1:**

The function `input` makes the program stop and wait for user input:

```
var = input('Please provide some input data:')
```

Simple and intuitive to use, slow and annoying in the long run

**Alternative 2:**

Use `sys.argv` to access *command line arguments*:

```
import sys
var = sys.argv[1]
```

Run the program from the terminal:

```
python myprog.py 2.05
```

Or in iPython/Spyder:

```
run myprog.py 2.05
```

## 0.3 Recap from last week - file read/write

**Reading from a file:**

```
with open('myfile.txt','r') as infile:
    l = infile.readline() #read a single line

    for line in infile:
        words = line.split()
        var = float(words[-1])  # etc
```

**Write to a file:**

```
data = [...]
with open('myfile.txt','w') as outfile:
    for myvar in data:
        outfile.write(myvar)
        outfile.write('\n')  #linebreak
```

## 0.4 Recap from last week - error handling

**Handle wrong input with a try/except block:**

```
import sys
try:
    h = float(sys.argv[1])
except IndexError:
    print('No command line argument for h!')
    sys.exit(1)  # abort execution
except ValueError:
    print(f'h must be a pure number, not {sys.argv[1]}')
    exit()
```

## 0.5 Recap from last week - data conversion

- All input data is *text* (type `str`)
- We usually want numbers, and need to convert with `var = float(var)`
- What if we want to input something else; a list, a mathematical formula, etc.?

## 0.6 Alternative conversion with the magic `eval` function

- `eval(s)` evaluates a string object `s` as if the string had been written directly into the program
- Gives a more flexible alternative to converting with `float(s)`

```
>>> s = '1+2'
>>> r = eval(s)
>>> r
3
>>> type(r)
<type 'int'>

>>> r = eval('[1, 6, 7.5] + [1, 2]')
>>> r
[1, 6, 7.5, 1, 2]
>>> type(r)
<type 'list'>
```

## 0.7 With eval, a little program can do much

**Program `input_adder.py`:**

```
i1 = eval(input('Give input: '))
i2 = eval(input('Give input: '))
r = i1 + i2
print (f'{type(i1)} + {type(i2)} becomes {type(r)} \nwith value {r}')
```

## 0.8 This great flexibility also quickly breaks programs...

```
Terminal> python input_adder.py
operand 1: (1,2)
operand 2: [3,4]
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: can only concatenate tuple (not "list") to tuple

Terminal> python input_adder.py
operand 1: one
Traceback (most recent call last):
  File "add_input.py", line 1, in <module>
    i1 = eval(raw_input('operand 1: '))
  File "<string>", line 1, in <module>
NameError: name 'one' is not defined

Terminal> python input_adder.py
operand 1: 4
operand 2: 'Hello, World!'
Traceback (most recent call last):
  File "add_input.py", line 3, in <module>
    r = i1 + i2
TypeError: unsupported operand type(s) for +: 'int' and 'str'
```

## 0.9 A similar magic function: exec

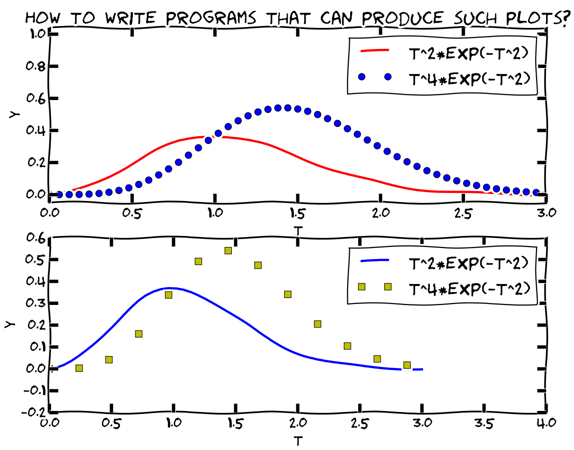
- `eval(s)` evaluates an *expression* `s`
- `eval('r = 1+1')` is illegal because this is a statement, not only an expression
- ...but we can use `exec` to turn one or more complete statements into live code:

```
statement = 'r = 1+1'    # store statement in a string
exec(statement)
print(r)                  # prints 2
```

For longer code we can use multi-line strings:

```
somecode = '''
def f(t):
    term1 = exp(-a*t)*sin(w1*x)
    term2 = 2*sin(w2*x)
    return term1 + term2
'''
exec(somecode)  # execute the string as Python code
```

## 0.10 Goal: learn to visualize functions




## 0.11 We need to learn about a new object: array

- Curves $y = f(x)$ are visualized by drawing straight lines between points along the curve

- Need to store the coordinates of the points along the curve in lists or *arrays* `x` and `y`
- Arrays $\approx$ lists, but computationally much more efficient
- To compute the `y` coordinates (in an array) we need to learn about *array computations* or *vectorization*
- Array computations are useful for much more than plotting curves!

## 0.12 The minimal need-to-know about vectors

- Vectors are known from high school mathematics, e.g., point $(x, y)$ in the plane, point $(x, y, z)$ in space
- In general, a vector $v$ is an $n$-tuple of numbers: $v = (v_0, \ldots, v_{n-1})$
- Vectors can be represented by lists: $v_i$ is stored as `v[i]`, but we shall use arrays instead

## 0.13 Arrays can have more than one index

Just as nested lists, arrays can have multiple indices: $A_{i,j}$, $A_{i,j,k}$
Example: table of numbers, one index for the row, one for the column

$$\left[\begin{array}{cccc} 0 & 12 & -1 & 5 \\ -1 & -1 & -1 & 0 \\ 11 & 5 & 5 & -2 \end{array}\right] \qquad A = \left[\begin{array}{ccc} A_{0,0} & \cdots & A_{0,n-1} \\ \vdots & \ddots & \vdots \\ A_{m-1,0} & \cdots & A_{m-1,n-1} \end{array}\right]$$

- The no of indices in an array is the *rank* or *number of dimensions*
- Vector = one-dimensional array, or rank 1 array
- In Python code, we use Numerical Python arrays instead of nested lists to represent mathematical arrays (because this is computationally more efficient)

## 0.14 Storing (x,y) points on a curve in lists

**Collect points on a function curve $y = f(x)$ in lists:**

```
def f(x):
    return x**3

n = 5                   # no of points
dx = 1.0/(n-1)          # x spacing in [0,1]

for i in range(n):
    x.append(i*dx)
    y.append(f(x))

#turn lists into NumPy arrays
import numpy as np          # module for arrays
x = np.array(xlist)      # turn list xlist into array
y = np.array(ylist)
```

## 0.15 Make arrays directly (instead of lists)

**Or drop the lists and make NumPy arrays directly:**

```
>>> n = 5                      # number of points
>>> x = np.linspace(0, 1, n) # n points in [0, 1]
>>> y = np.zeros(n)          # n zeros (float data type)
>>> for i in range(n):
...     y[i] = f(x[i])
...
```

## 0.16 Arrays are not as flexible as list, but computationally much more efficient

- List elements can be *any* Python objects
- Array elements can only be of *one object type*
- Arrays are very efficient to store in memory and compute with if the element type is `float`, `int`, or `complex`
- Rule: use arrays for sequences of numbers!

## 0.17 We can work with entire arrays at once - instead of one element at a time

Compute the sine of an array:

```
from math import sin

for i in range(len(x)):
    y[i] = sin(x[i])
```

However, if `x` is array, `y` can be computed by

```
import numpy as np
y = np.sin(x)                    # x: array, y: array
```

The loop is now inside `np.sin` and implemented in very efficient C code.

**Vectorization gives:**

- shorter, more readable code, closer to the mathematics
- much faster code

## 0.18 A function `f(x)` written for a number `x` usually works for array `x` too

```
from numpy import sin, exp, linspace

def f(x):
    return x**3 + sin(x)*exp(-3*x)

x = 1.2                     # float object
y = f(x)                    # y is float

x = linspace(0, 3, 10001)   # 10000 intervals in [0,3]
y = f(x)                    # y is array
```

## 0.19 NOTE: `math` is for numbers and `numpy` for arrays

```
>>> import math, numpy
>>> x = numpy.linspace(0, 1, 11)
>>> math.sin(x[3])
0.2955202066613396
>>> math.sin(x)
...
TypeError: only length-1 arrays can be converted to Python scalars
>>> numpy.sin(x)
array([ 0.      ,  0.09983,  0.19866,  0.29552,  0.38941,
        0.47942,  0.56464,  0.64421,  0.71735,  0.78332,
        0.84147])
```

## 0.20 Very important application: vectorized code for computing points along a curve

$$f(x) = x^2 e^{-\frac{1}{2}x} \sin(x - \frac{1}{3}\pi), \quad x \in [0, 4\pi]$$

**Vectorized computation of $n + 1$ points along the curve.**

```
import numpy as np

n = 100
x = np.linspace(0, 4*pi, n+1)
y = 2.5 + x**2*np.exp(-0.5*x)*np.sin(x-pi/3)
```

## 0.21 New term: vectorization

- *Scalar*: a number
- *Vector* or *array*: sequence of numbers (vector in mathematics)
- We speak about scalar computations (one number at a time) versus vectorized computations (operations on entire arrays, no Python loops)

- *Vectorized functions* can operate on arrays (vectors)
- *Vectorization* is the process of turning a non-vectorized algorithm with (Python) loops into a vectorized version without (Python) loops
- Mathematical functions in Python without `if` tests automatically work for both scalar and vector (array) arguments (i.e., no vectorization is needed by the programmer)

## 0.22 Small quiz:

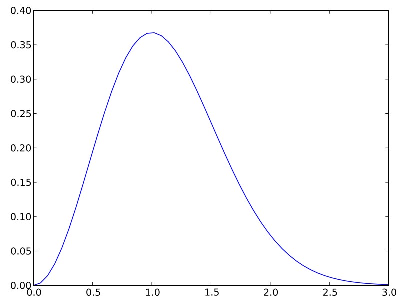
What is output from the following code? Why?

```
import numpy as np

l = [0,0.25,0.5,0.75,1]
a = np.array(l)

print(l*2)
print(a*2)
```

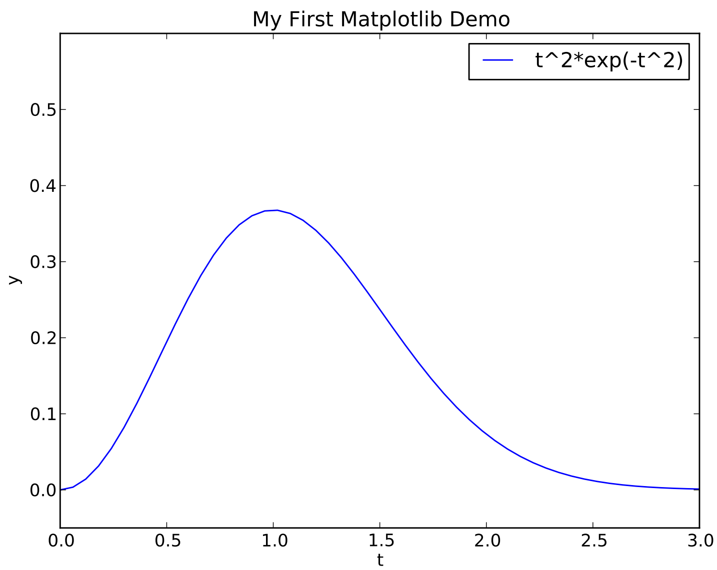
## 0.23 Plotting the curve of a function: the very basics

**Plot the curve of $y(t) = t^2 e^{-t^2}$:**

```
import matplotlib.pyplot as plt  # import and plotting
import numpy as np

# Make points along the curve
t = np.linspace(0, 3, 51)      # 50 intervals in [0, 3]
y = t**2*np.exp(-t**2)         # vectorized expression

plt.plot(t, y)            # make plot on the screen
plt.savefig('fig.pdf')    # make PDF image for reports
plt.savefig('fig.png')    # make PNG image for web pages
plt.show()
```




## 0.24 A plot should have labels on axis and a title




## 0.25 The code that makes the last plot

```
import matplotlib.pyplot as plt
import numpy as np

def f(t):
    return t**2*np.exp(-t**2)
```

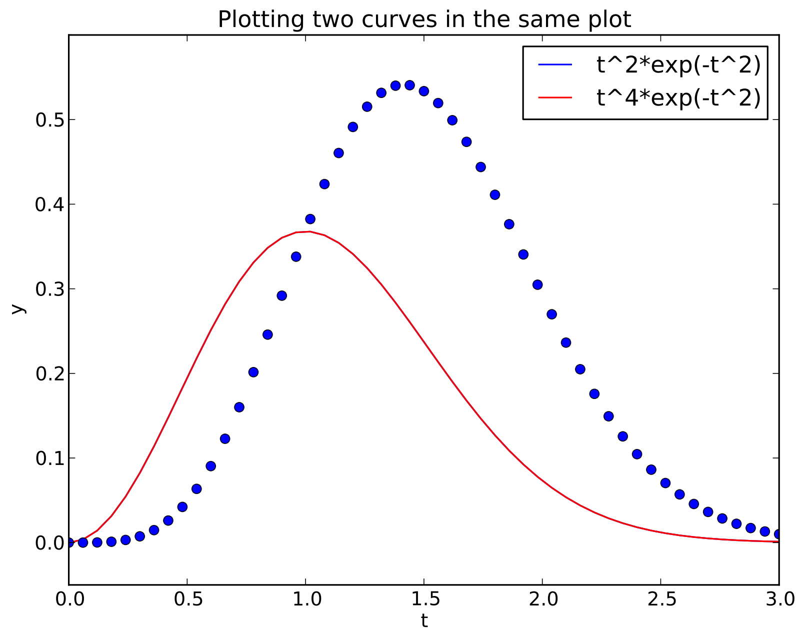
```
t = np.linspace(0, 3, 51)    # t coordinates
y = f(t)                     # corresponding y values

plt.plot(t, y,label="t^2*exp(-t^2)")

plt.xlabel('t')               # label on the x axis
plt.ylabel('y')               # label on the y axix
plt.legend()                  # mark the curve
plt.axis([0, 3, -0.05, 0.6])  # [tmin, tmax, ymin, ymax]
plt.title('My First Matplotlib Demo')
plt.show()
```

## 0.26 Plotting several curves in one plot

**Plot $t^2e^{-t^2}$ and $t^4e^{-t^2}$ in the same plot:**

```
import matplotlib.pyplot as plt
import numpy as np

def f1(t):
    return t**2*np.exp(-t**2)

def f2(t):
    return t**2*f1(t)

t = np.linspace(0, 3, 51)
y1 = f1(t)
y2 = f2(t)

plt.plot(t, y1,  'r-', label = 't^2*exp(-t^2)')
plt.plot(t, y2, 'bo', label = 't^4*exp(-t^2)')

plt.xlabel('t')
plt.ylabel('y')
plt.legend()
plt.title('Plotting two curves in the same plot')
plt.savefig('tmp2.png')
plt.show()
```

## 0.27 The resulting plot with two curves




## 0.28 Controlling line styles

When plotting multiple curves in the same plot, the different lines (normally) look different. We can control the line type and color, if desired:

```
plot(t, y1, 'r-')   # red (r) line (-)
plot(t, y2, 'bo')   # blue (b) circles (o)

# or
plot(t, y1, 'r-', t, y2, 'bo')
```

Documentation of colors and line styles, see the online Matplotlib documentation or

```
Unix> pydoc matplotlib.pyplot
```

## 0.29 Quick plotting with minimal typing

### A lazy pro would do this:

```
t = np.linspace(0, 3, 51)
plt.plot(t, t**2*exp(-t**2), t, t**4*exp(-t**2))
```

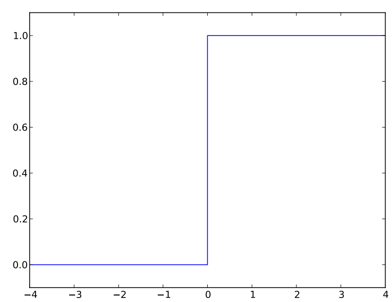
## 0.30 Example: plot a discontinuous function

The Heaviside function is frequently used in science and engineering:

$$H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

Python implementation:

```
def H(x):
    if x < 0:
        return 0
    else:
        return 1
```




## 0.31 Plotting the Heaviside function: first try

**Standard approach:**

```
x = np.linspace(-10, 10, 5)  # few points (simple curve)
y = H(x)
plt.plot(x, y)
```

First problem: `ValueError` error in `H(x)` from `if x < 0`
Let us debug in an interactive shell:

```
>>> x = np.linspace(-10,10,5)
>>> x
array([-10.,  -5.,   0.,   5.,  10.])
>>> b = x < 0
>>> b
array([ True,  True, False, False, False], dtype=bool)
>>> bool(b)  # evaluate b in a boolean context
...
ValueError: The truth value of an array with more than
one element is ambiguous. Use a.any() or a.all()
```

## 0.32 `if x < 0` does not work if `x` is array

**Remedy 1: use a loop over `x` values.**

```
def H_loop(x):
    r = zeros(len(x))  # or r = x.copy()
    for i in range(len(x)):
        r[i] = H(x[i])
    return r

n = 5
x = np.linspace(-5, 5, n+1)
y = H_loop(x)

#or loop over x and call the original function
y = np.zeros_like(x)
for i in range(len(x)):
    y[i] = H(x[i])
```

Downside: much to write, slow code if `n` is large

## 0.33 `if x < 0` does not work if `x` is array

**Remedy 2: use `numpy.vectorize`.**

```
# Automatic vectorization of function H
Hv = np.vectorize(H)
# Hv(x) works with array x
```

Downside: The resulting function is as slow as Remedy 1

## 0.34 `if x < 0` does not work if `x` is array

**Remedy 3: code the `if` test differently.**

```
def Hv(x):
    return np.where(x < 0, 0.0, 1.0)
```

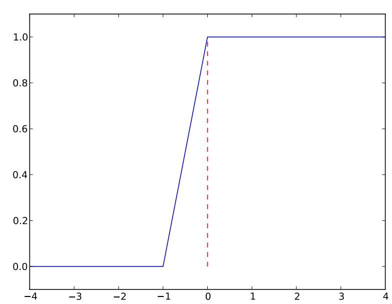
**More generally:**

```
def f(x):
    if condition:
        x = <expression1>
    else:
        x = <expression2>
    return x

def f_vectorized(x):
    x1 = <expression1>
    x2 = <expression2>
    r = np.where(condition, x1, x2)
    return r
```

## 0.35 Back to plotting the Heaviside function

With a vectorized `Hv(x)` function we can plot in the standard way

```
x = linspace(-10, 10, 5)    # linspace(-10, 10, 50)
y = Hv(x)
plot(x, y, axis=[x[0], x[-1], -0.1, 1.1])
```




## 0.36 How to make the function look discontinuous in the plot?

We could use a lot of $x$ points to make the curve look steeper, but it does still not really look like a discontinuous function.

**Question.** How can we make the plot look like a proper discontinuous function?

## 0.37 Example: Plot function given on the command line

**Task: plot function given on the command line.**

```
Terminal> python plotf.py expression xmin xmax
Terminal> python plotf.py "exp(-0.2*x)*sin(2*pi*x)" 0 4*pi
```

Should plot $e^{-0.2x}\sin(2\pi x)$, $x \in [0, 4\pi]$. `plotf.py` should work for "any" mathematical expression.

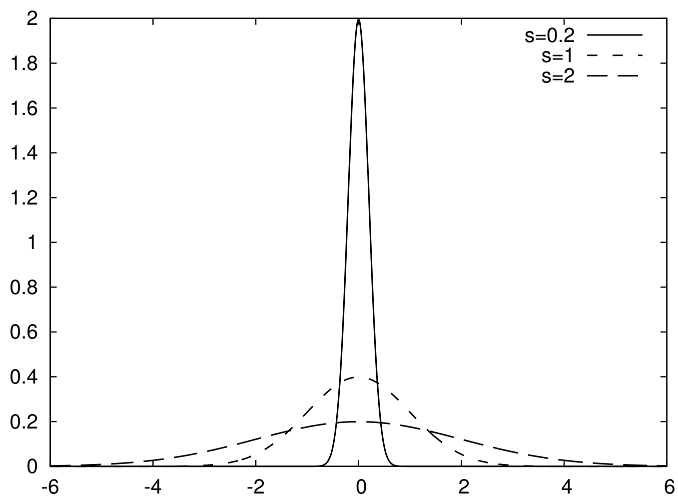
## 0.38 Solution

**Complete program:**

```
from numpy import *
import matplotlib.pyplot as plt
import sys

formula = sys.argv[1]
xmin = eval(sys.argv[2])
```

```
xmax = eval(sys.argv[3])

x = linspace(xmin, xmax, 101)
y = eval(formula)
plt.plot(x, y)
plt.title(formula)
plt.show()
```

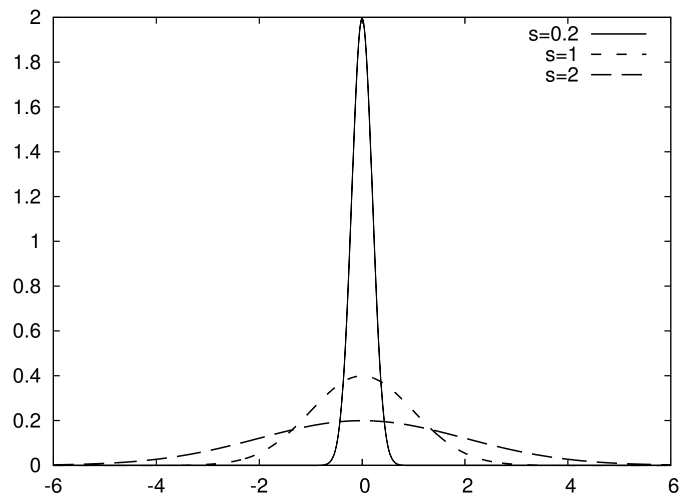
## 0.39 Let's make a movie/animation




## 0.40 The Gaussian/bell function

$$f(x; m, s) = \frac{1}{\sqrt{2\pi}} \frac{1}{s} \exp\left[ -\frac{1}{2} \left( \frac{x-m}{s} \right)^2 \right]$$

- $m$ is the location of the peak
- $s$ is a measure of the width of the function
- Make a movie (animation) of how $f(x; m, s)$ changes shape as $s$ goes from 2 to 0.2




## 0.41 Movies are made from a (large) set of individual plots

- Goal: make a movie showing how $f(x)$ varies in shape as $s$ decreases
- Idea: put many plots (for different $s$ values) together (exactly as a cartoon movie)
- Very important: fix the $y$ axis! Otherwise, the $y$ axis always adapts to the peak of the function and the visual impression gets completely wrong

## 0.42 Three alternative recipes

1. Let the animation run *live*, without saving any files
   - Not possible to pause, slow down etc
2. Loop over all data values, plot and make a hardcopy (file) for each value, combine all hardcopies to a movie
   - Requires separate software (for instance *ImageMagick*) to see the animation
3. Use a 'FuncAnimation' object from 'matplotlib'
   - Plays the animation *live*
   - Relies on external software to save a movie file

## 0.43 Alt. 1: General idea

- Fix the axes!
- Use a 'for'-loop to loop over $s$-values
- Compute new $y$-values and update the plot for each run through the loop

## 0.44 Alt. 1: Complete code

```
import matplotlib.pyplot as plt
import numpy as np

def f(x, m, s):
    return (1.0/(np.sqrt(2*np.pi)*s))*np.exp(-0.5*((x-m)/s)**2)

m = 0;  s_start = 2;  s_stop = 0.2
s_values = np.linspace(s_start, s_stop, 30)

x = np.linspace(m -3*s_start, m + 3*s_start, 1000)
# f is max for x=m (smaller s gives larger max value)
max_f = f(m, m, s_stop)

y = f(x,m,s_stop)
lines = plt.plot(x,y)  #Returns a list of line objects!

plt.axis([x[0], x[-1], -0.1, max_f])
plt.xlabel('x')
plt.ylabel('f')

for s in s_values:
    y = f(x, m, s)
    lines[0].set_ydata(y) #update plot data and redraw
    plt.draw()
    plt.pause(0.1)
```

## 0.45 Alt. 2: General idea

- Same 'for'-loop as alternative 1
- Use 'printf'-formatting to generate a unique file name for each plot
- Save file

## 0.46 Alt. 2: Complete code

```
import matplotlib.pyplot as plt
import numpy as np

def f(x, m, s):
    return (1.0/(np.sqrt(2*np.pi)*s))*np.exp(-0.5*((x-m)/s)**2)

m = 0;  s_start = 2;  s_stop = 0.2
s_values = np.linspace(s_start, s_stop, 30)

x = np.linspace(m -3*s_start, m + 3*s_start, 1000)
# f is max for x=m (smaller s gives larger max value)
max_f = f(m, m, s_stop)

y = f(x,m,s_stop)
lines = plt.plot(x,y)

plt.axis([x[0], x[-1], -0.1, max_f])
plt.xlabel('x')
plt.ylabel('f')

frame_counter = 0
for s in s_values:
    y = f(x, m, s)
    lines[0].set_ydata(y) #update plot data and redraw
    plt.draw()
    plt.savefig(f'tmp_{frame_counter:04d}.png') #unique filename
    frame_counter += 1
```

## 0.47 How to combine plot files to a movie (video file)

We now have a lot of files:

`tmp_0000.png  tmp_0001.png  tmp_0002.png  ...`

We use some program to combine these files to a video file:

- `convert` for animated GIF format (if just a few plot files)
- `ffmpeg` (or `avconv`) for MP4, WebM, Ogg, and Flash formats

## 0.48 Make and play animated GIF file

Tool: `convert` from the ImageMagick software suite.
Unix command:

```
Terminal> convert -delay 20 tmp_*.png movie.gif
```

Delay: 30/100 s, i.e., 0.5 s between each frame.
Play animated GIF file with `animate` from ImageMagick:

```
Terminal> animate movie.gif
```

or open the file in a browser.

## 0.49 Alt. 3: General idea

- Make a function to update the plot:
  - Updates the plot by calculating values and calling `set_ydata`
  - (Optional function to initialize the plot)
- Make a list or array of the argument that changes (here $s$)
- Pass the function and the list as arguments to create a `FuncAnimation` object
- Use functions in that object to animate, save a movie file etc.

## 0.50 Alt. 3: Complete code

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation

def f(x, m, s):
    return (1.0/(np.sqrt(2*np.pi)*s))*np.exp(-0.5*((x-m)/s)**2)

m = 0; s_start = 2; s_stop = 0.2
s_values = np.linspace(s_start,s_stop,30)

x = np.linspace(-3*s_start,3*s_start, 1000)

max_f = f(m,m,s_stop)

plt.axis([x[0],x[-1],0,max_f])
plt.xlabel('x')
plt.ylabel('y')

y = f(x,m,s_start)
lines = plt.plot(x,y) #initial plot to create the lines object

def next_frame(frame):
    y = f(x, m, frame)
    lines[0].set_ydata(y)
    return lines

ani = FuncAnimation(plt.gcf(), next_frame, frames=s_values, interval=100)
ani.save('movie.mp4',fps=20)
plt.show()
```

## 0.51 Notes on making movies

- Making actual movie files require external software such as `ImageMagick` or `ffmpeg`

- The software may be tricky to install (simple recipes exist, but don't always work)
- For the animation assignments in this course, you do not have to make movie files. You either:
  - Use Alt 1 or Alt 3 to make the animation run *live*
  - Use Alt 2 to create a lot of image files
- If you can also make the movie files this is great, but it will not be required