Chapter 8: Classes part 2 - special methods

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0.1 Special methods

- The class constructor has a special name: `__init__`
- The name is recognized by Python, and ensures this method is called when a new class instance is created; e.g. `y = MyClass(4)`
- The constructor is an example of a *special method*
- Special methods have names with leading and trailing double underscores
- Special methods are recognized by Python, and automatically called when we perform various operations on the class instances

0.2 The call special method; motivation

Recall the class for representing a function:

```
class Y:
    def __init__(self, v0):
        self.v0 = v0
        self.g = 9.81

    def value(self, t):
        return self.v0*t - 0.5*self.g*t**2

y = Y(3)
v = y.value(0.1)
```

But it would be more natural to use the class like this:

```
y = Y(3)
v = y(0.1)
```

0.3 The call special method; implementation

Simply replace the `value` method by a *call* special method:

```
class Y:
    def __init__(self, v0):
        self.v0 = v0
        self.g = 9.81

    def __call__(self, t):
        return self.v0*t - 0.5*self.g*t**2
```

Now we can write

```
y = Y(3)
v = y(0.1) # same as v = y.__call__(0.1) or Y.__call__(y, 0.1)
```

Note:

- The instance `y` now behaves and looks as a function!
- The `value(t)` method does the same, but `__call__` allows nicer syntax for computing function values

0.4 Special method for printing

- In Python, we can usually print an object `a` by `print(a)`, works for built-in types (strings, lists, floats, ...)
- Python does not know how to print objects of a user-defined class, but if the class defines a method `__str__`, Python will use this method to convert an object to a string

Example:

```
class Y:
    ...
    def __call__(self, t):
        return self.v0*t - 0.5*self.g*t**2

    def __str__(self):
        return f'v0*t - 0.5*g*t**2; v0={self.v0}'
```

Demo:

```
>>> y = Y(1.5)
>>> y(0.2)
0.1038
>>> print(y)
v0*t - 0.5*g*t**2; v0=1.5
```

0.5 Class Y revisited with print method

```python
class Y:
    """Class for function y(t; v0, g) = v0*t - 0.5*g*t**2."""

    def __init__(self, v0):
        """Store parameters."""
        self.v0 = v0
        self.g = 9.81

    def __call__(self, t):
        """Evaluate function."""
        return self.v0*t - 0.5*self.g*t**2

    def __str__(self):
        """Pretty print."""
        return f'v0*t - 0.5*g*t**2; v0={self.v0}'
```

0.6 Special methods for arithmetic operations

```python
c = a + b    #  c = a.__add__(b)

c = a - b    #  c = a.__sub__(b)

c = a*b      #  c = a.__mul__(b)

c = a/b      #  c = a.__div__(b)

c = a**e     #  c = a.__pow__(e)
```

0.7 Special methods for comparisons

```python
a == b       #  a.__eq__(b)

a != b       #  a.__ne__(b)

a < b        #  a.__lt__(b)

a <= b       #  a.__le__(b)

a > b        #  a.__gt__(b)

a >= b       #  a.__ge__(b)
```

0.8 The programmer is in charge of defining special methods!

How should, for instance, `__add__(self, other)` and `__mul__(self, other)` be defined?

This is completely up to the programmer, depending on what are meaningful results of `object1 + object2` and `object1 * object2`.

0.9 Class for vectors in the plane

Mathematical operations for vectors in the plane:

$$(a, b) + (c, d) = (a + c, b + d)$$
$$(a, b) - (c, d) = (a - c, b - d)$$
$$(a, b) \cdot (c, d) = ac + bd$$
$$(a, b) = (c, d) \text{ if } a = c \text{ and } b = d$$

Desired application code:

```
>>> u = Vec2D(0,1)
>>> v = Vec2D(1,0)
>>> print(u + v)
(1, 1)
>>> a = u + v
>>> w = Vec2D(1,1)
>>> a == w
True
>>> print(u - v)
(-1, 1)
>>> print(u*v)
0
```

0.10 Class for vectors; implementation

```
class Vec2D:
    def __init__(self, x, y):
        self.x = x;  self.y = y

    def __add__(self, other):
        return Vec2D(self.x+other.x, self.y+other.y)

    def __sub__(self, other):
        return Vec2D(self.x-other.x, self.y-other.y)

    def __mul__(self, other):
        return self.x*other.x + self.y*other.y

    def __abs__(self):
        return math.sqrt(self.x**2 + self.y**2)

    def __eq__(self, other):
        return self.x == other.x and self.y == other.y

    def __str__(self):
        return f'({self.x}, {self.y})'
```

0.11 Class for polynomials; functionality

A polynomial can be specified by a list of its coefficients. For example, $1-x^2+2x^3$ is

$$1 + 0 \cdot x - 1 \cdot x^2 + 2 \cdot x^3$$

and the coefficients can be stored as `[1, 0, -1, 2]`

Desired application code:

```
>>> p1 = Polynomial([1, -1])
>>> print(p1)
1 - x
>>> print(p1(x=0.5))
0.5
>>> p2 = Polynomial([0, 1, 0, 0, -6, -1])
>>> p3 =(p1 + p2)
>>> print(p3.coeff)
[1, 0, 0, 0, -6, -1]
>>> print(p3)
1 - 6*x^4 - x^5
>>> p2.differentiate()
>>> print(p2)
1 - 24*x^3 - 5*x^4
```

How can we make class `Polynomial`?

0.12 Class Polynomial; basic code

```python
class Polynomial:
    def __init__(self, coefficients):
        self.coeff = coefficients

    def __call__(self, x):
        s = 0
        for i in range(len(self.coeff)):
            s += self.coeff[i]*x**i
        return s
```

0.13 Class Polynomial; addition

```python
class Polynomial:
    ...

    def __add__(self, other):
        # return self + other

        # start with the longest list and add in the other:
        if len(self.coeff) > len(other.coeff):
            coeffsum = self.coeff[:]  # copy!
            for i in range(len(other.coeff)):
                coeffsum[i] += other.coeff[i]
        else:
            coeffsum = other.coeff[:] # copy!
            for i in range(len(self.coeff)):
                coeffsum[i] += self.coeff[i]
        return Polynomial(coeffsum)
```

0.14 Class Polynomial; multiplication

Mathematics: Multiplication of two general polynomials:

$$\left(\sum_{i=0}^{M} c_i x^i\right)\left(\sum_{j=0}^{N} d_j x^j\right) = \sum_{i=0}^{M}\sum_{j=0}^{N} c_i d_j x^{i+j}$$

The coeff. corresponding to power $i+j$ is $c_i \cdot d_j$. The list `r` of coefficients of the result: `r[i+j] = c[i]*d[j]` (`i` and `j` running from 0 to M and N, resp.)

Implementation:

```
class Polynomial:
    ...
    def __mul__(self, other):
        M = len(self.coeff) - 1
        N = len(other.coeff) - 1
        coeff = [0]*(M+N+1)  # or zeros(M+N+1)
        for i in range(0, M+1):
            for j in range(0, N+1):
                coeff[i+j] += self.coeff[i]*other.coeff[j]
        return Polynomial(coeff)
```

0.15 Class Polynomial; differentation

Mathematics: Rule for differentiating a general polynomial:

$$\frac{d}{dx}\sum_{i=0}^{n} c_i x^i = \sum_{i=1}^{n} i c_i x^{i-1}$$

If `c` is the list of coefficients, the derivative has a list of coefficients, `dc`, where `dc[i-1] = i*c[i]` for `i` running from 1 to the largest index in `c`. Note that `dc` has one element less than `c`.

Implementation:

```
class Polynomial:
    ...
    def differentiate(self):    # change self
        for i in range(1, len(self.coeff)):
            self.coeff[i-1] = i*self.coeff[i]
        del self.coeff[-1]

    def derivative(self):       # return new polynomial
        dpdx = Polynomial(self.coeff[:])  # copy
        dpdx.differentiate()
        return dpdx
```

0.16 Class Polynomial; pretty print

```
class Polynomial:
    ...
    def __str__(self):
        s = ''
        for i in range(0, len(self.coeff)):
            if self.coeff[i] != 0:
                s += f' + {self.coeff[i]}*x^{i}'
```

```
# fix layout (lots of special cases):
s = s.replace('+ -', '- ')
s = s.replace(' 1*', ' ')
s = s.replace('x^0', '1')
s = s.replace('x^1 ', 'x ')
s = s.replace('x^1', 'x')
if s[0:3] == ' + ':  # remove initial +
    s = s[3:]
if s[0:3] == ' - ':  # fix spaces for initial -
    s = '-' + s[3:]
return s
```

0.17 Class for polynomials; usage

Consider

$$p_1(x) = 1 - x, \quad p_2(x) = x - 6x^4 - x^5$$

and their sum

$$p_3(x) = p_1(x) + p_2(x) = 1 - 6x^4 - x^5$$

```
>>> p1 = Polynomial([1, -1])
>>> print(p1)
1 - x
>>> p2 = Polynomial([0, 1, 0, 0, -6, -1])
>>> p3 =(p1 + p2)
>>> print p3.coeff
[1, 0, 0, 0, -6, -1]
>>> p2.differentiate()
>>> print(p2)
1 - 24*x^3 - 5*x^4
```

0.18 Example; "automatic" differentiation

Given some mathematical function in Python, say

```
def f(x):
    return x**3
```

can we make a class `Derivative` and write

```
dfdx = Derivative(f)
```

so that `dfdx` behaves as a function that computes the derivative of `f(x)`?

```
print(dfdx(2))    # computes 3*x**2 for x=2
```

0.19 Automagic differentiation; solution

Method. We use numerical differentiation "behind the curtain":

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

for a small (yet moderate) h, say $h = 10^{-5}$

Implementation.

```
class Derivative:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)

    def __call__(self, x):
        f, h = self.f, self.h      # make short forms
        return (f(x+h) - f(x))/h
```

0.20 Automagic differentiation; demo

```
>>> from math import *
>>> df = Derivative(sin)
>>> x = pi
>>> df(x)
-1.000000082740371
>>> cos(x)  # exact
-1.0
>>> def g(t):
...     return t**3
...
>>> dg = Derivative(g)
>>> t = 1
>>> dg(t)  # compare with 3 (exact)
3.000000248221113
```

0.21 Automagic differentiation; useful in Newton's method

Newton's method solves nonlinear equations $f(x) = 0$, but the method requires $f'(x)$

```
def Newton(f, xstart, dfdx, epsilon=1E-6):
    ...
    return x, no_of_iterations, f(x)
```

Suppose $f'(x)$ requires boring/lengthy derivation, then class `Derivative` is handy:

```
>>> def f(x):
...     return 100000*(x - 0.9)**2 * (x - 1.1)**3
...
>>> df = Derivative(f)
>>> xstart = 1.01
>>> Newton(f, xstart, df, epsilon=1E-5)
(1.0987610068093443, 8, -7.5139644257961411e-06)
```

0.22 Class introduction - summary

- Classes pack together data and functions that naturally belong together
- We define a class, and then create *instances* (or objects) of that class
  - Different instances will have different data, but they all have the same functions operating on that data
- In IN1900 codes, classes are never really necessary, but sometimes convenient
- In "real-world" programs, with tens of 1000s of lines, the extra organization offered by classes may be the difference between a code that works and one that doesn't

0.23 Summary of special methods

- `c = a + b` implies `c = a.__add__(b)`
- There are special methods for `a+b`, `a-b`, `a*b`, `a/b`, `a**b`, `-a`, `if a:`, `len(a)`, `str(a)` (pretty print), etc.
- With special methods we can create new mathematical objects like vectors, polynomials and complex numbers and write "mathematical code" (arithmetics)
- The call special method is particularly handy: `v = c(5)` means `v = c.__call__(5)`
- Functions with parameters should be represented by a class with the parameters as attributes and with a call special method for evaluating the function