INF-GEO 4310 (Imaging)

Problems in Sonar and Medical Ultrasound Imaging

Problem 1 Doppler shift in sonar

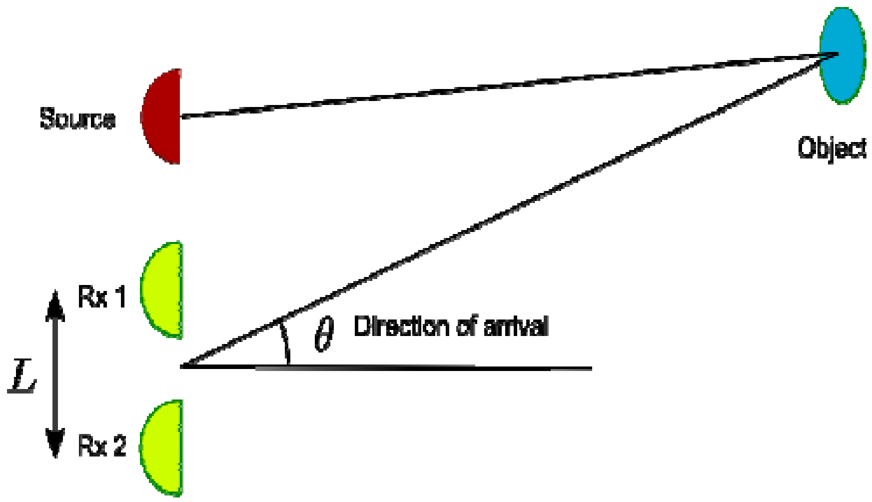
Assume that you are on a ship during war. Your sonar detects an underwater object which seems to be moving straight towards your ship. The raw data from the sonar are:

- Two-way travel time: 2 seconds
- Sonar centre frequency: 20 kHz
- Return signal's centre frequency: 20400 Hz

How much time do you have to react before something will hit your vessel?

Problem 2 Sonar Imaging

Assume a sonar source and two receivers (Rx 1 and Rx2) forming a plane as illustrated below. The receivers are displaced L apart. The source transmits a pulse with wavelength λ. The transmitted signal is reflected by an object. The reflected signal propagates to the receiver array with velocity c. Rx 2 observes the reflected signal δt delayed compared to Rx 1.

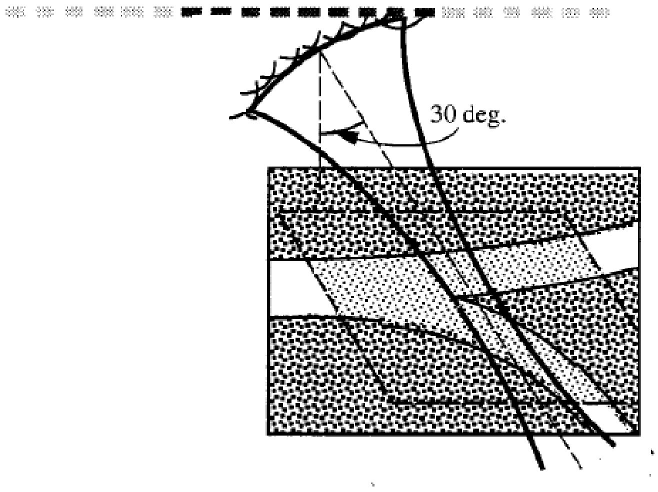


a) Assume that the reflector is in the far field. What is the direction of arrival of the reflected signal expressed by the terms given in the exercise?

b) Assume that the smallest time difference possible to detect is equivalent to 1/8 period (time accuracy). What is then the angular accuracy of the system?

Problem 3 Doppler shift and vascular imaging

Assume that a beam is used for estimating blood velocity in the carotid artery. The beam is sent at an angle of 30 degrees as in the image. Although the image below also shows the bifurcation in the neck where the external and internal carotid split off, assume that the blood flow is to be estimated before the division, i.e. to the very left in the image.



1. The Doppler signal has a centre frequency of 5 MHz and the measured back scattered peak frequency is 635 Hz lower than the transmitted frequency. What is the peak velocity of the blood flow?
2. Assume that there is an uncertainty in the angle measurement of +/- 5 degrees. What is the corresponding uncertainty in the estimate of the peak velocity?
3. It is desirable to have as little sensitivity to uncertainty in angle as possible. Should the steering angle be made larger or smaller than 30 degrees in order to achieve this? Justify your answer.

Problem 4

You are using a single transducer to examine a heart valve. Assume that in a given heart cycle the range of the valve is given by $z(t) = 16 + 0.5e^{-t/\tau}u(t)$cm where $\tau = 10$ ms, $u(t)$ is the unit step function, and the speed of sound is 1540 m/s.

(a) Sketch $z(t)$ over a couple of heart cycles (assume that the heart rate is 1 Hz).

(b) Assume that your transducer is fired at $t = 0$ and it has a typical transmit waveform. Carefully sketch the A-mode signal (as a function of time) that you would observe on an oscilloscope.

(c) Now assume that you repeatedly fire your transducer every 1 ms. Sketch the M-mode image that would be generated, being careful to label the axes.

(d) Suppose you wanted to image the motion of this valve by making a B-mode image of it. If it can be "covered" by 10 scan lines (given the beam size at 16 cm range), describe what steps you would take to make this image in real time. Do you think it is possible?

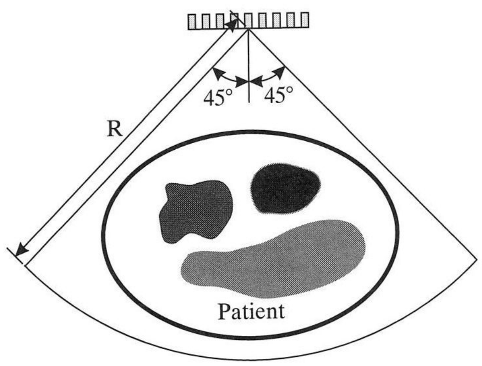
Hint: ***u(t) = 1 for t>0 and 0 otherwise.***
Here it just means that you should start the plot at t=0

Problem 5

11.6 Consider the linear array in Figure 11.15. Suppose we want to image a cross section of a patient as shown in Figure P11.2, acquiring a scan line every $\Delta\theta = 1^\circ$. Assume there are 101 transducers operating at a frequency of 2 MHz and $c = 1540$ m/s and $d = 0.6$ mm.

(a) Find an expression for the time delay of the ith transducer as a function of angle of the transmitted plane wave.

(b) How long does it take to scan the entire FOV assuming $R = 20$ cm?



FOV = Field of View

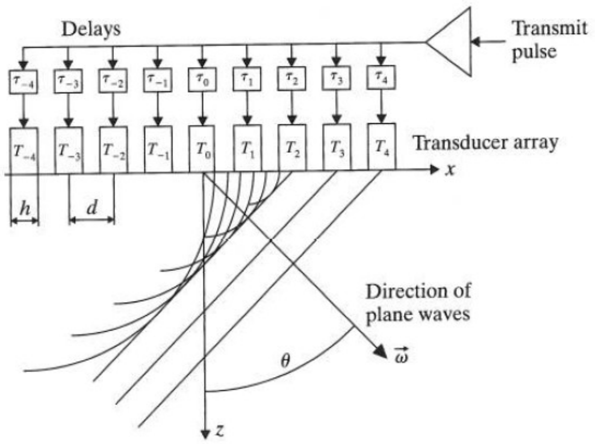


Figure 11.15
Geometry for steering a phased array.

(Problems 4 and 5 are from Prince & Links, Medical Imaging, Signals and Systems, Pearson, 2006)