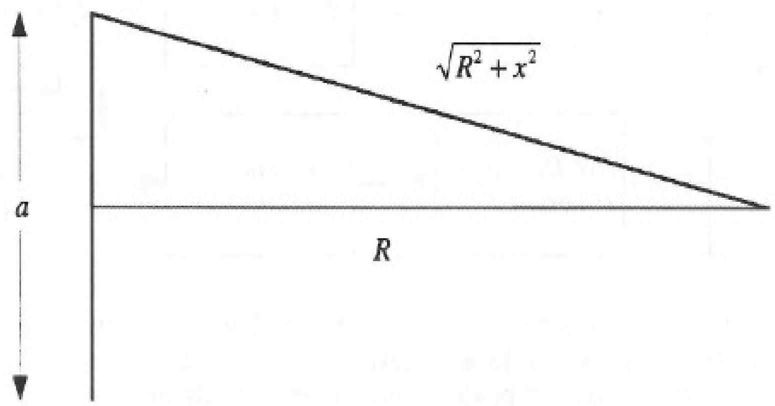
INF-GEO 4310 (Imaging)

Problems in Acoustic Imaging

Problem 1

The Near Field/Far Field Crossover

Consider a uniform linear aperture of extent *a* excited with a monochromatic signal having wavelength λ. Consider a point target at a distance *R* from the aperture. The radiation from each incremental part of the aperture arrives at the target according to the propagation path length associated with the incremental aperture position and the target location. We can analyze this with the following figure.



a) Compute the differential path length Δ associated with a point x on the aperture and a range R. You should be able to simplify the expression by using the relation $\sqrt{1+y} \approx 1 + y/2$.

This differential error across the aperture is thus essentially quadratic, and can be reduced arbitrarily be increasing *R*. That is, in the far field the radiation from each point on the aperture arrives (essentially) coherently, adding constructively. As we move the point target closer to the aperture, the delay error increases inversely with *R* until, at some crossover range R_C between the near field and far field, it becomes non-negligible. We define this range $R = R_C$ (rather arbitrarily) as that for which the maximum error is

$$\Delta = \lambda/8$$

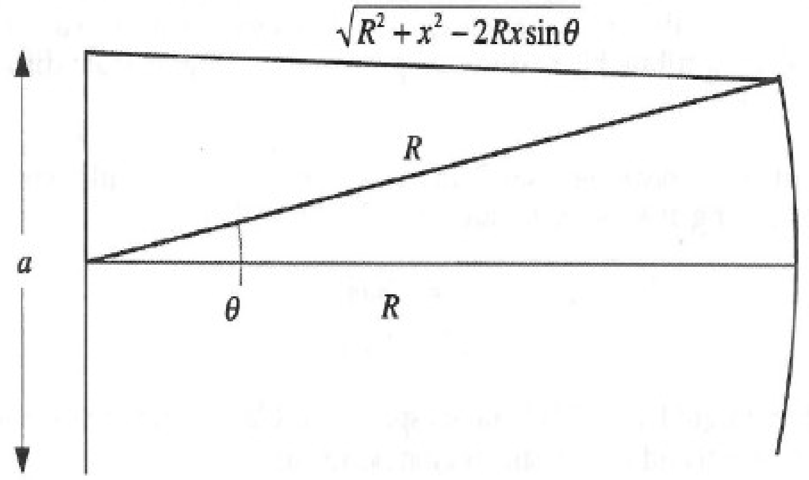
b) The maximum error will always be associated with the ends of the aperture. Substitute $x = a/2$ in the expression found in a) and compute the crossover range between near field and far field. Have you seen this expression before?

c) In ultrasound imaging, we often use an aperture operating at a center frequency of 3.5MHz. Let $a = 22mm$ and the speed of sound, $c = 1540m/s$. When imaging with this aperture, at what range do we not have to compensate for near field effects?

Problem 2

Azimuthal Resolution and the Rayleigh Criterion of a Linear Aperture

Consider again a uniform linear aperture of extent a excited with a monochromatic signal having wavelength λ. Consider a point target in the far field, moved around a circular arc of constant range. Because we are in the far field, there is negligible phase error when the angle of the target is zero (i.e., the radiation from each aperture point arrives in phase). But as we traverse the circular arc, the phase error increases until some parts of the aperture start contributing destructively.



a) Compute the differential path length Δ between an aperture point x and the center, as the point target moves along the circular arc. Use the same simplification as i a) and that $\sin\theta \approx \theta$ for $\theta << 1$.
By expanding the expression in terms of x/R and θ and ignoring quadratic term on the basis of far field operation, you should be able to establish a linear relation between Δ and x.

b) Destructive interference occurs when the magnitude of any differential path length error exceeds $\lambda/4$. Calculate the angular extent for which $-\lambda/4 \leq \Delta \leq \lambda/4$.

c) Substitute $x = \pm a/2$ into the relation found in b) (since the maximum error always will be associated with the ends of the aperture). What do we call this relation?