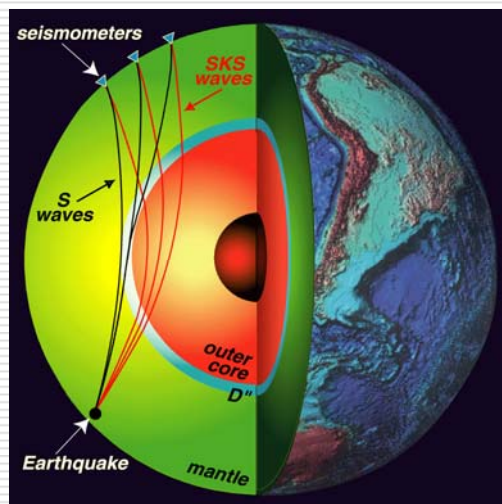# INF-GEO 3310/4310:
Imaging in seismology

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1

The Earth internal structure seen by seismology

What is imaging in seismology?

Most of the imaging studies concern imaging the interior of the structure of the Earth. By analyzing the arrival times of waves, we map the 3-D distribution of the P and S-wave velocities and the locations in discontinuities (interfaces between layers). Variations in wave velocity inside the Earth are primarily related to variations in composition and to variations in temperature: warm material are softer and the waves travel more slowly there. In the mantle, the composition is believed to be rather homogeneous and the dominant effect is the one related to temperature. Here you see the main structure of the Earth: the mantle in green with its lowermost layer in blue. And the iron-rich core with its liquid outer core in light red and solid inner core in dark red. The crust at the surface is a thin layer of 10 to 70km which you do not see at the scale of this figure. Imaging in seismology covers all scales from the scale of seismics (a few km) to the whole Earth.
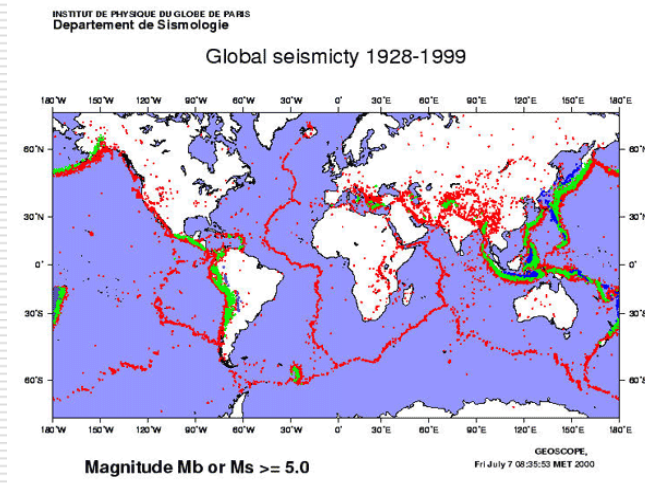
But since the arrival time of the waves at a given seismological station depends on the origin time and location of the earthquake, on which we have no control, we cannot do tomographic studies without locating the earthquakes in space and time. This has therefore to be done first. In addition, if we know the structure well enough, we can use the seismograms to analyze the rupture itself along the fault plane where the earthquake has occurred. These source studies are not as numerous as the structure studies but they are very important because they contribute to our understanding of the physics of earthquake rupture and will hopefully lead to methods for predicting earthquakes.

Differences with seismics:

- Fewer sources and no control on source location.
- Fewer stations.
- Few earthquakes and few stations: no single imaging method based on a transformation: iterative procedure based on improving initial guesses

3

- **What is an earthquake?**
- Imaging: early studies and the mean structure of the Earth
- P-wave tomography
- Normal mode tomography
- Source location and time-reversal

4

Location of earthquakes

INSTITUT DE PHYSIQUE DU GLOBE DE PARIS
Departement de Sismologie

Global seismicty 1928-1999

Magnitude Mb or Ms >= 5.0

GEOSCOPE,
Fri July 7 08:35:53 MET 2000

5

First, for the non-geophysicists, I will explain shortly what earthquakes are.

This shows the distribution of earthquakes of magnitude at least 5 during most of the last century. You see that they are not distributed randomly, but follow basically some lines along the surface of the Earth. In red you have the shallow earthquakes and in green and blue the deep ones. The deepest earthquakes are located at about 700km depth.

The earthquakes occur along the boundaries of the tectonic plates and arise from sudden relaxation of the stresses which occur when two plates are trying to move past each other.
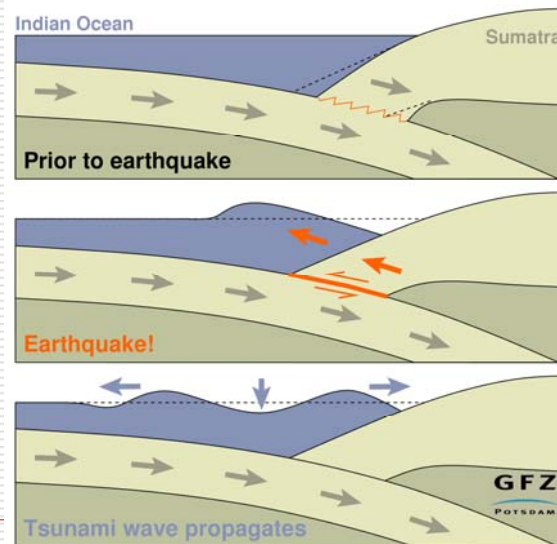
This example illustrates how earthquakes occur at subduction zones. South of
Indonesia, the Indian Ocean plate is moving northwards and diving below the
Eurasian plate, which is relatively fixed at the Earth surface. Due to friction at the
contact between the two plates, the Eurasian plate is gradually bent at its south
end by the movement of the Indian plate. When the forces associated with the
bending get larger than a given threshold, the Eurasian plate suddenly slips on
the Indian plate and gets back to its undeformed position. This occurs over
basically a few seconds, up to a few minutes for the very large earthquakes, and
this movement generates vibrations in the whole Earth, which are then registered
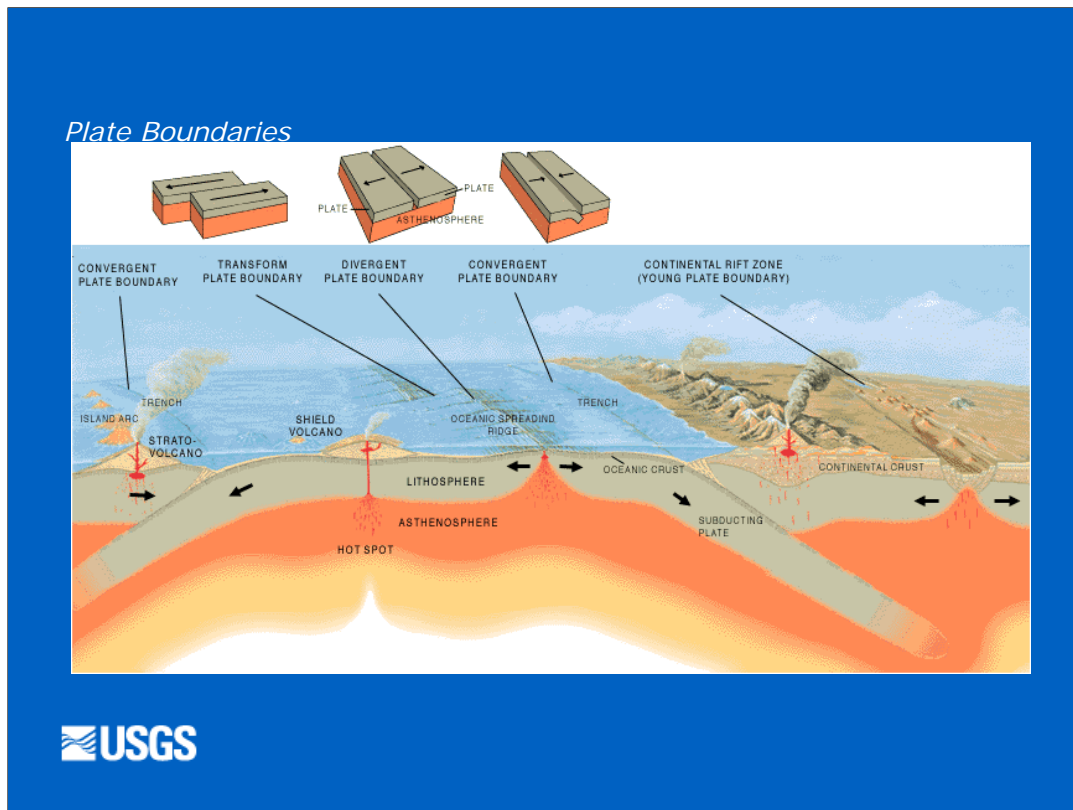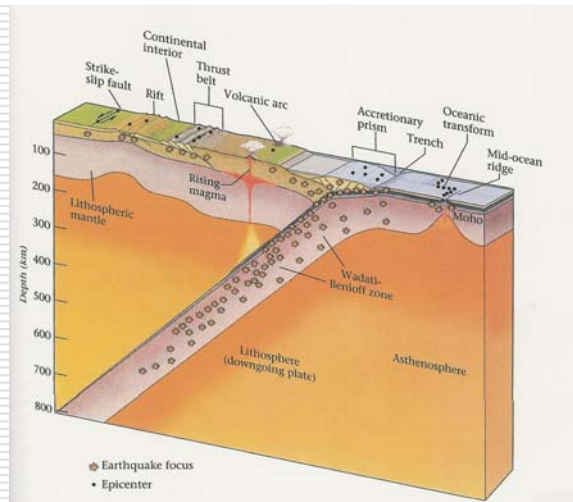by seismometers.
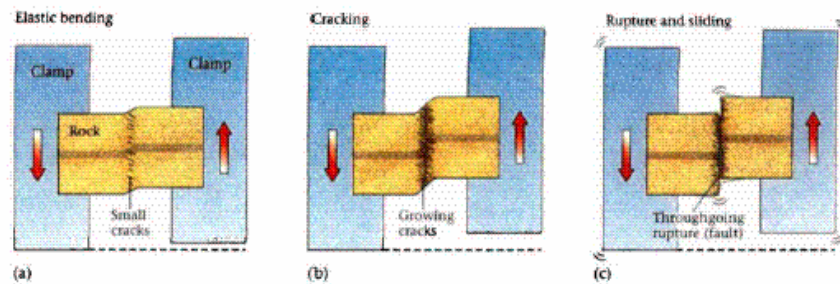
Not only subduction zone earthquakes, but basically all earthquakes occur as the sudden release of accumulated tectonic stresses.

Earthquakes occur therefore mostly where we have tectonic plate
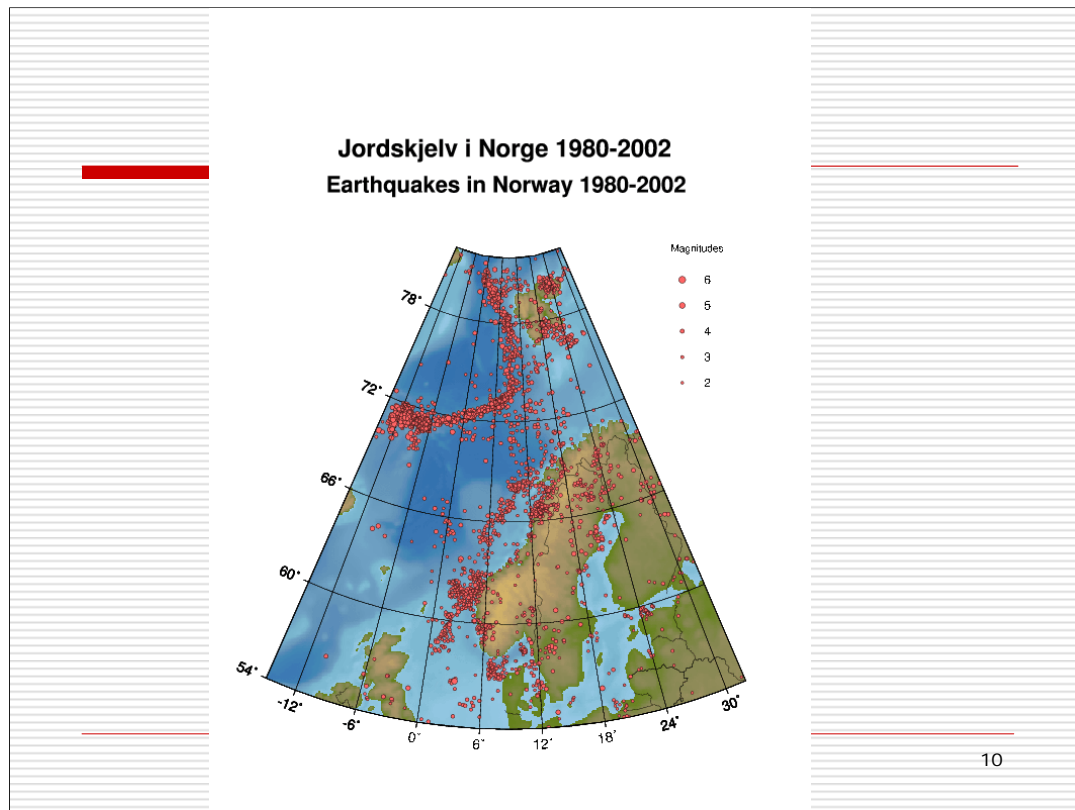boundaries

- Where two plates slide past one another

- Where two plates run into one another - convergent boundary
(subduction or continental collision)

- Where plates move away from one another - divergent boundary

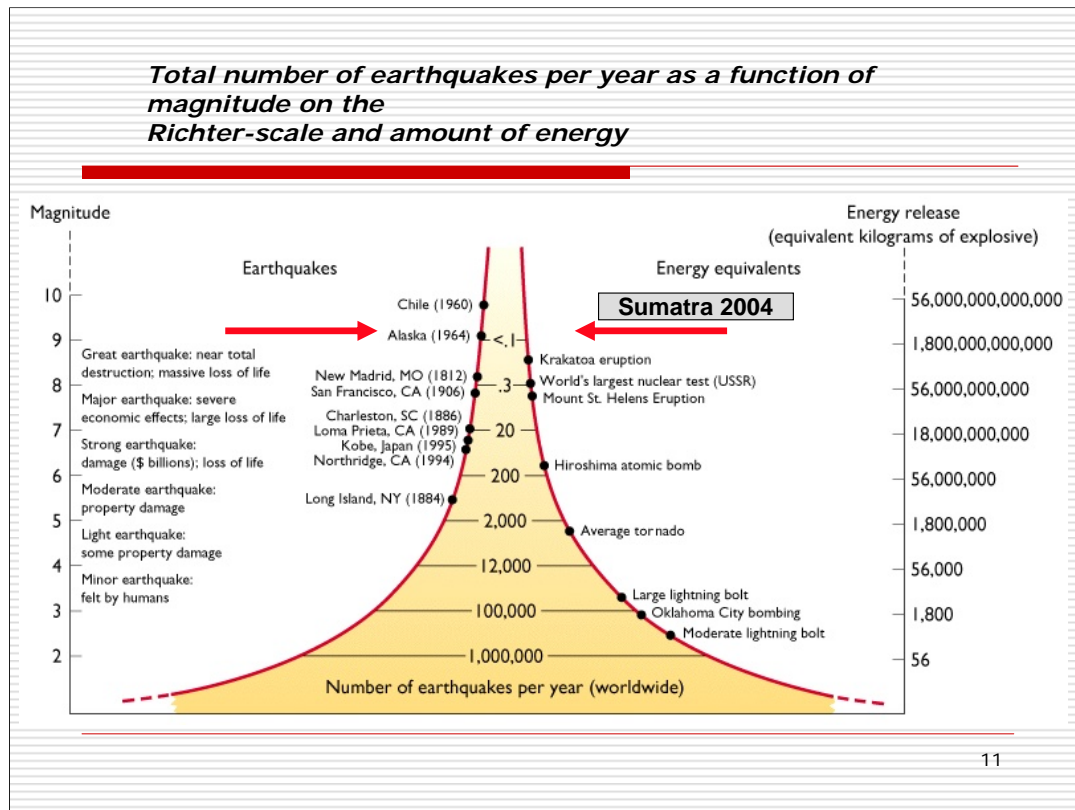In subduction zones, the earthquakes occur down to 700km depth. In other parts
of the Earth, they are confined to the first 100km.

Jordskjelv i Norge 1980-2002
Earthquakes in Norway 1980-2002

Although most large earthquakes occur along plate boundaries, we have also quite a lot of small earthquakes inside the plates. This is because there are also deformations and stresses inside the plate. This is an example showing the distribution of the seismicity in Norway. There are many small earthquakes, but notice that even inside a plate the distribution is not random and that there are regions with earthquakes and regions basically without earthquakes.

Total number of earthquakes per year as a function of magnitude on the Richter-scale and amount of energy

The amount of energy released by earthquakes is quite enormous and they are really very efficient sources of vibrations for imaging the Earth. In addition to earthquakes, it is possible to use nuclear underground explosions as source of vibration for studying the Earth. But they are of course much more seldom than earthquakes!
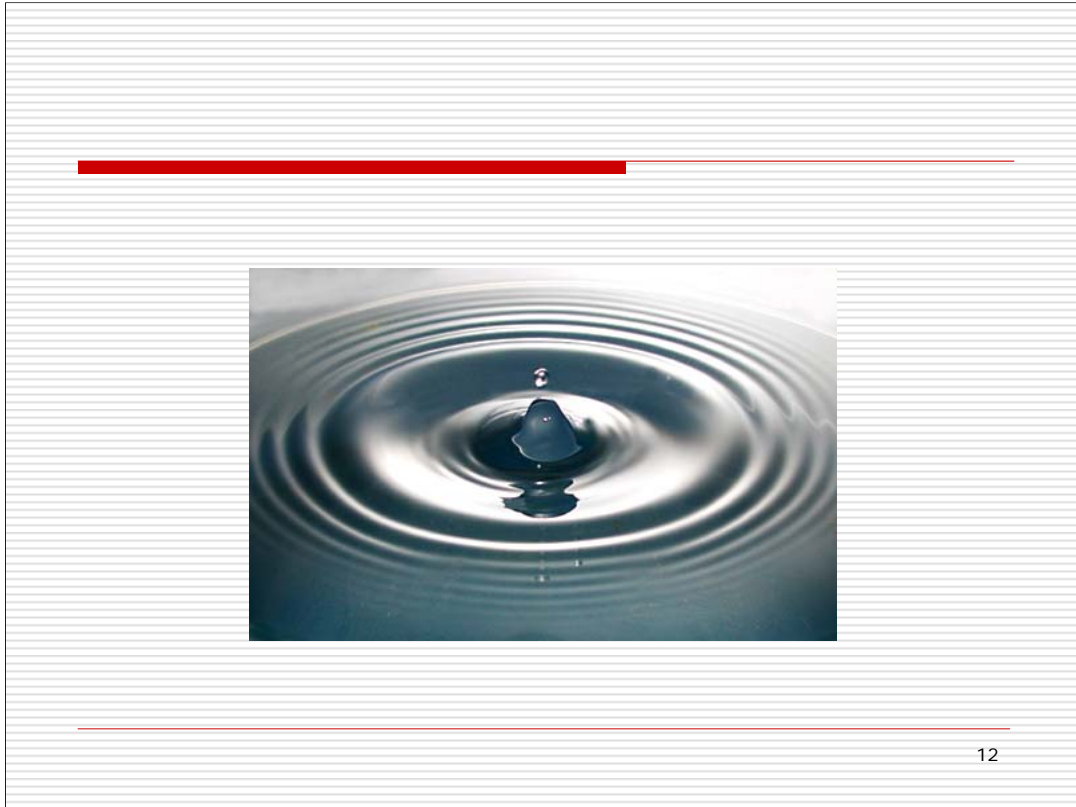
As a stone which you drop in the water perturbs the surface and generates waves, do earthquakes generate seismic waves which propagate in the whole Earth.
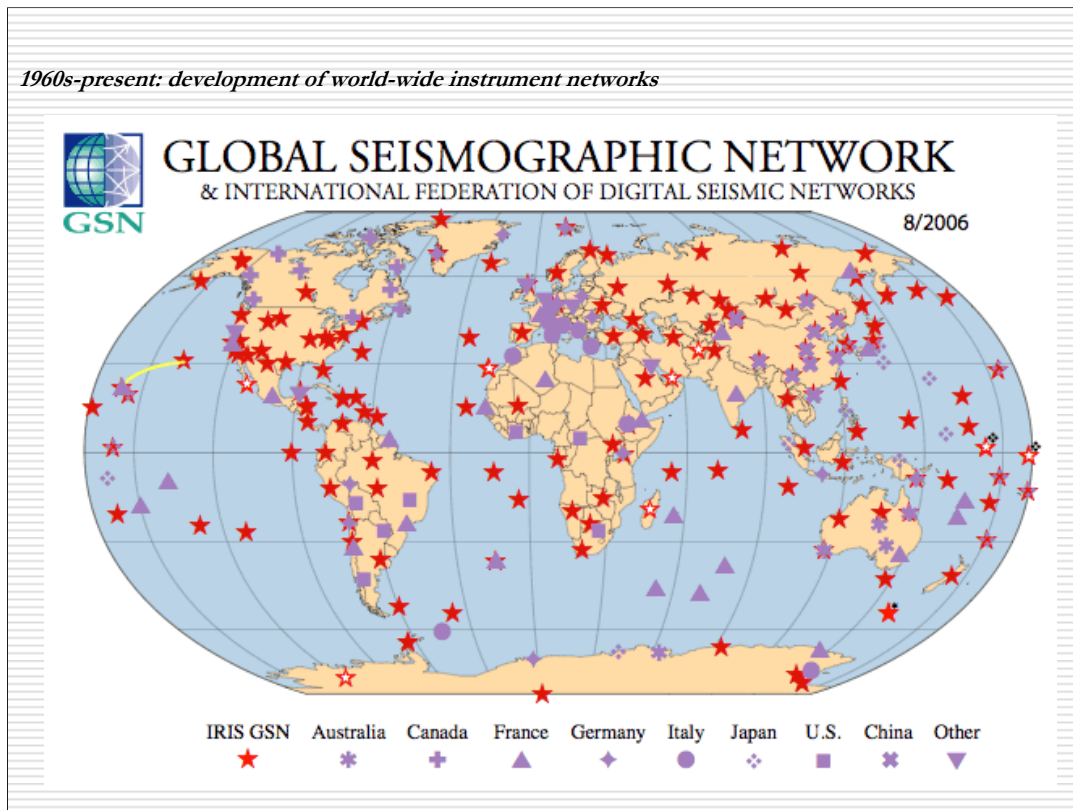
Late 19th century: development of the seismometer

The first signal recognized as coming
from a distant earthquake (Japan to
Potsdam, Germany) was identified
by E. von Rebeur-Paschwitz in 1889.
(From Nature, Vol. 40, 1889.)

1889 APRIL 17. GR.M.T.
POTSDAM.

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This recording is the first seismogram. The instrument that von Reuber-Paschwitz had installed was not meant to record earthquakes but just study the movement of the ground. Von Rebeur-Paschwitz heard that there had been an earthquake in Japan and put that into relation to a strange signal he had seen on his instrument the same day. From there on, seismometers were installed in several places, in particular in Bergen, and seismology started to develop.

1960s-present: development of world-wide instrument networks

GLOBAL SEISMOGRAPHIC NETWORK
& INTERNATIONAL FEDERATION OF DIGITAL SEISMIC NETWORKS
8/2006

IRIS GSN Australia Canada France Germany Italy Japan U.S. China Other

Since then, the number of available seismic records has been increasing
very quickly… This shows the global network of long-period instruments in
2006. In addition, there are many short-period stations and temporary
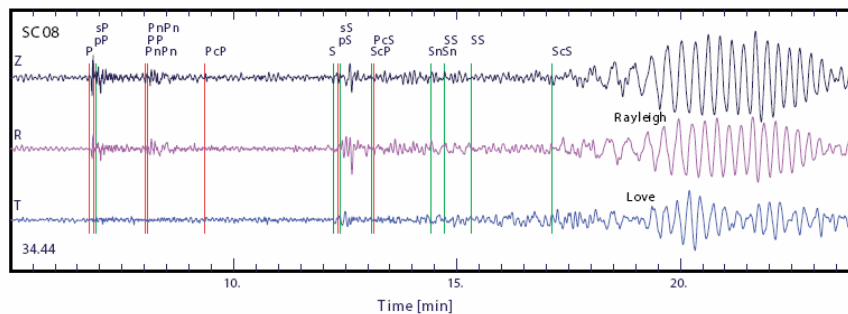broadband instruments which are not shown on this figure.

Figure 1: Example of a 3-component (Z, R, T), broadband seismic record, with particularly prominent surface wave signal; epicentral distance ~ 34°. A few phases have been identified on the seismograms. I took this and some of the following images from http://wwwrses.anu.edu.au/seismology/SHon2002/

This is an example of the ground motion that one records nowadays after an earthquake at a distance of about 4000km from the earthquake. The three plots show the three components of the motion (vertical, and two horizontal directions) as a function of time. The zero time here is the time of the earthquake. The first vibrations arrive about 4 mn after the earthquake and the last ones arrive almost half-an-hour after the earthquake has occurred.

The earthquake itself lasts only a few seconds. The reason for the long duration of the seismogram is the fact that the Earth is not homogeneous and that we have many echoes. It is by studying these echoes that we can infer the structure of the Earth, or do the imaging of the Earth's interior.

The periods of these waves:

from around 0.01s (local
  earthquakes)
to 53 mn (maximum on Earth)

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The period of the waves which are generated by earthquakes are large compared
to the period of the other waves you have seen in this course. We usually
express frequencies in mHz (milliHertz) in seismology. The wave velocities are of
the order of a few km/s. This gives wavelengths varying basically from a few tens
of meters to several thousand km. The resolution you can obtain from such
waves is of course not very large. But you have to consider this relatively to the
size of the Earth.

- What is an earthquake?
- Imaging: early studies and the mean structure of the Earth
- P-wave tomography
- Normal mode tomography
- Source location and time-reversal
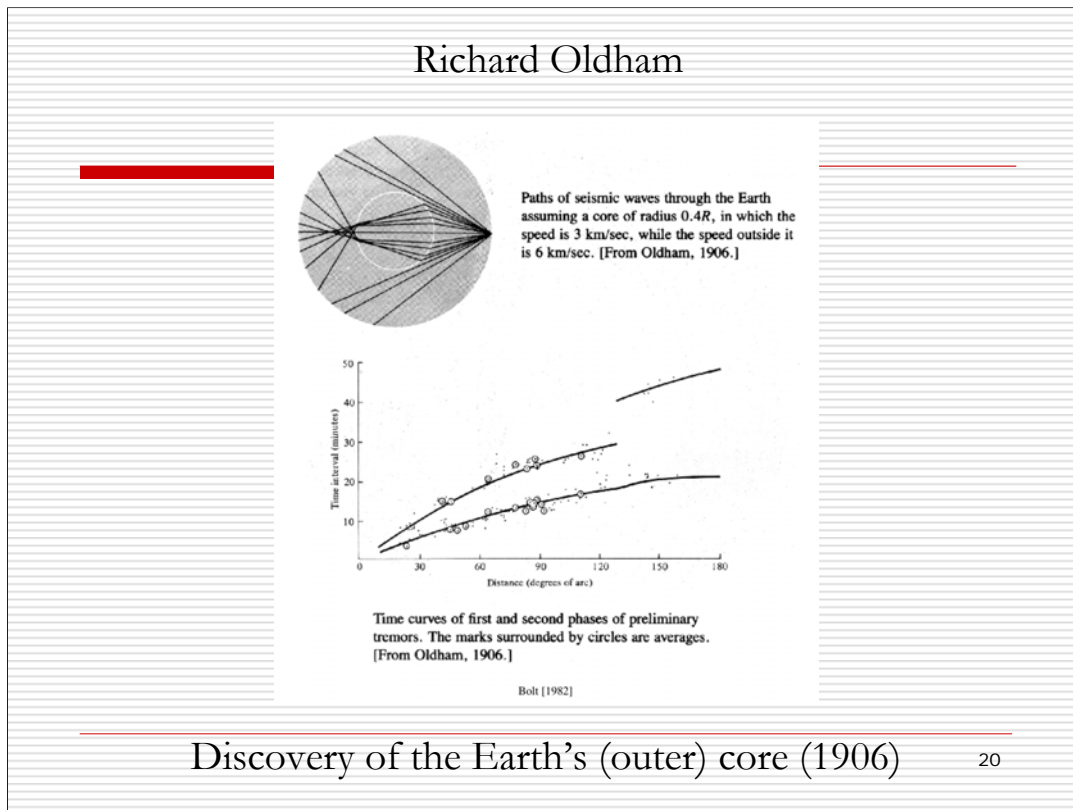
17

Wave propagation for distant earthquakes
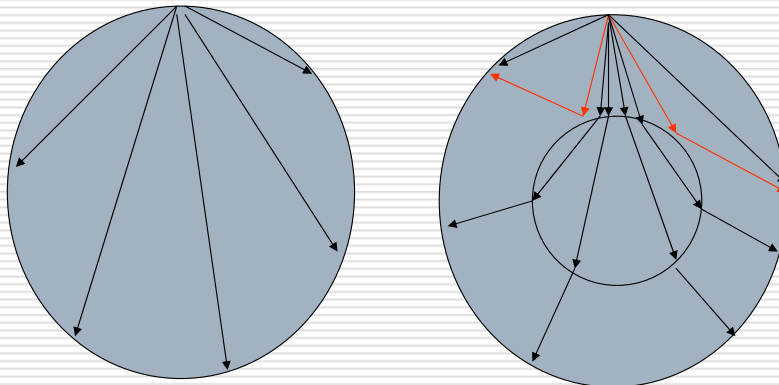
If the Earth had been an homogenous spherical body, as on the left figure, we would have had a simple geometrical relation between the traveltimes of the P and S waves and epicentral distance (the distance along the surface of the Earth between the earthquake and the seismometer). Due to the structure inside the Earth, this relation is more complicated. The traveltime as a function of distance is actually THE major data used in imaging the Earth, both for early studies, as we will see now, and in modern P-wave tomographies as we will see in the second example.

P

v_1=constant

sini1/v1=sini2/v2

S

v_1

P

$v_2 > v_1$

19
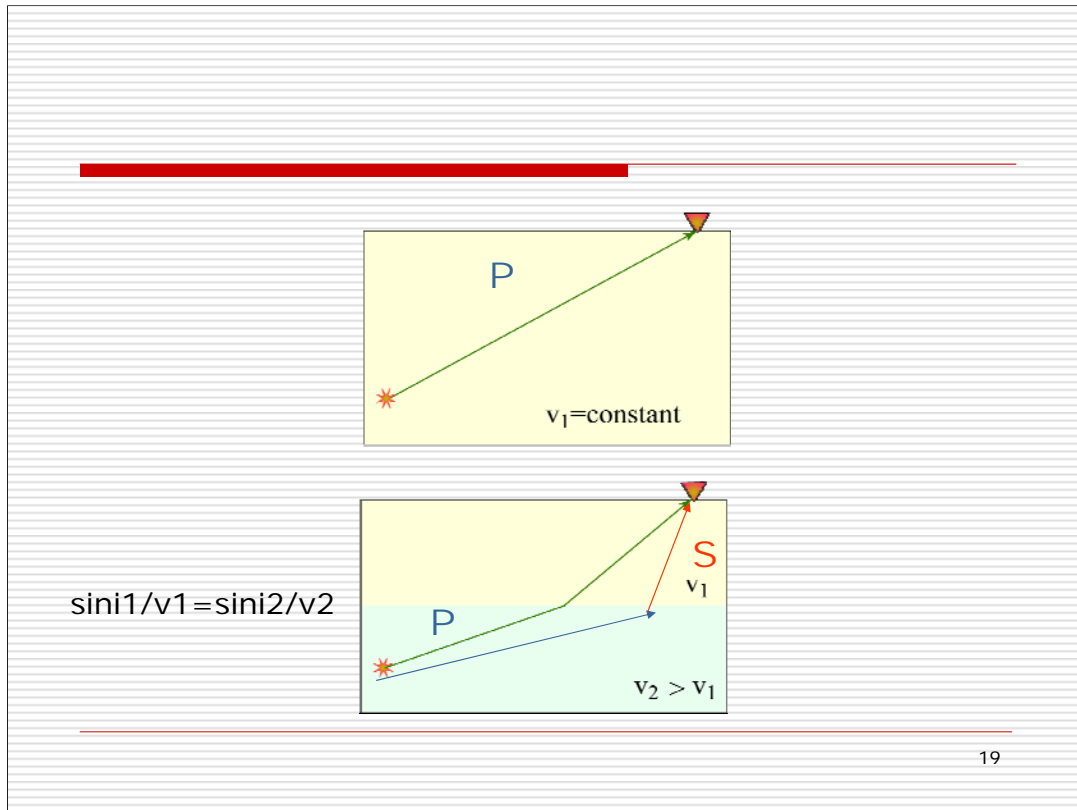
Refraction and conversion of waves at interfaces is a fundamental element in imaging. Here you see how a P wave incident from below at an interface is converted to an S wave. The rules for the incidence of refracted rays is called Snell's law and is given here.

Richard Oldham

Paths of seismic waves through the Earth assuming a core of radius 0.4R, in which the speed is 3 km/sec, while the speed outside it is 6 km/sec. [From Oldham, 1906.]

Time curves of first and second phases of preliminary tremors. The marks surrounded by circles are averages. [From Oldham, 1906.]

Bolt [1982]

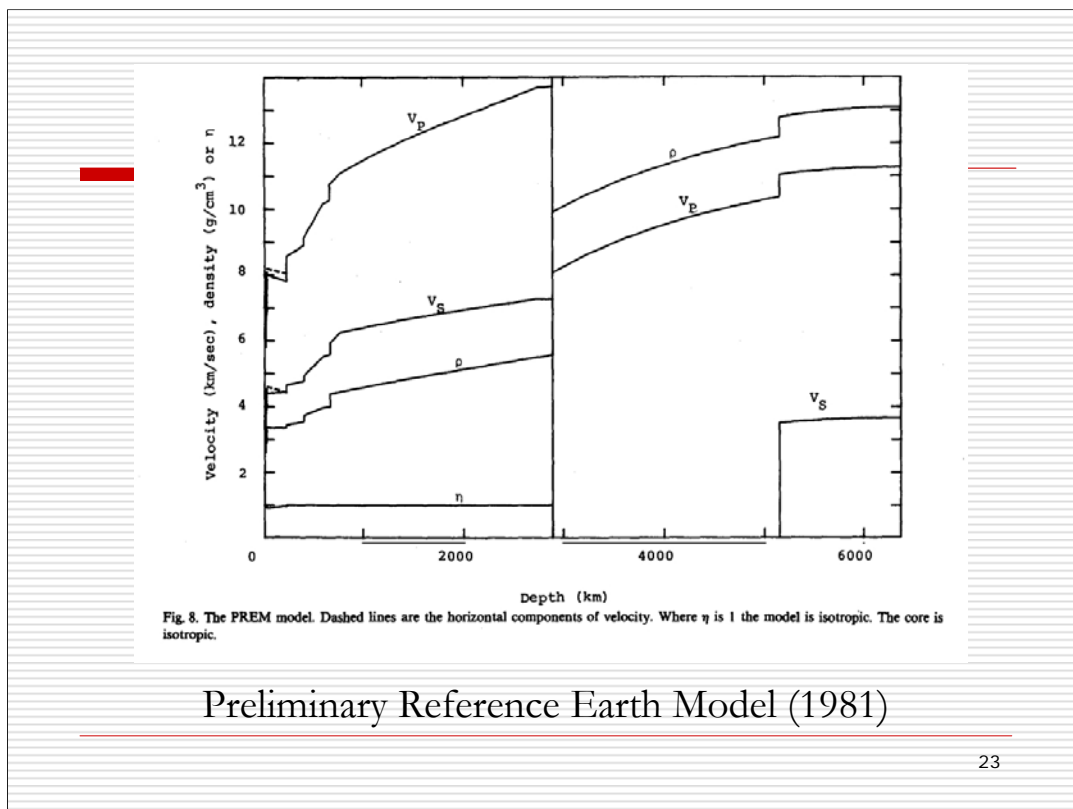Discovery of the Earth's (outer) core (1906)   20

Much of the early studies, and many of recent studies also, are based on ray theory, which is just the geometrical optics applied to propagation in solids. The major element inside the Earth is the presence of the core. This was recognized very early, and found by observing that the arrival time of P waves with distance was not continuous, but had a jump at about 120 degrees distance. Actually the jump is rather at 100 degrees and we know today that the core is larger than what Oldham had calculated, but the principle back his discovery is still valid.
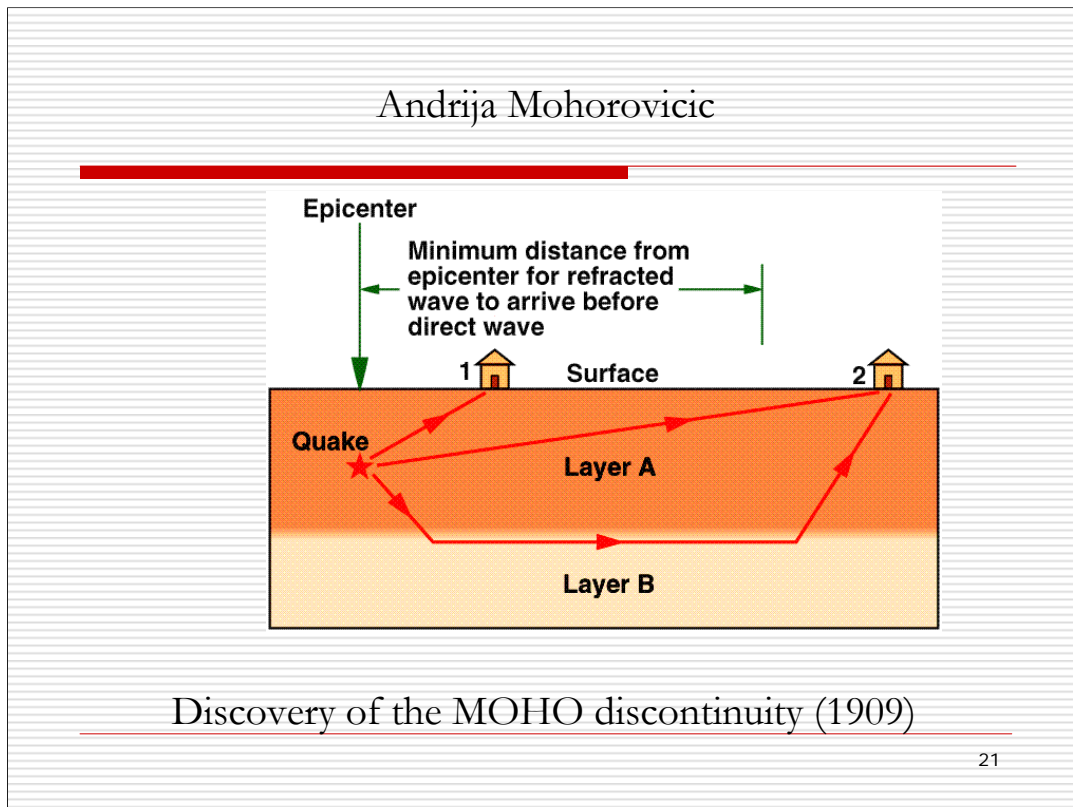
The other discontinuity to be discovered very early was the crust-mantle
boundary. This was discovered also by analyzing the arrival times as a function
of distance, but much closer to the earthquake. The first wave arrival time first
increases with distance with a given velocity, corresponding to the velocity in the
crust (layer A here), and then increases more sharply, when the wave refracted
in the mantle (layer B) starts to arrive first. This is also a simple example of how
the arrival time as a function of distance can be used to image the subsurface.
This technique, called refraction, is also used at smaller scale in seismics to find
the depth of sedimentary layers.

Traveltimes of
P waves at 60
to 100deg
distance:

600 to
1000sec
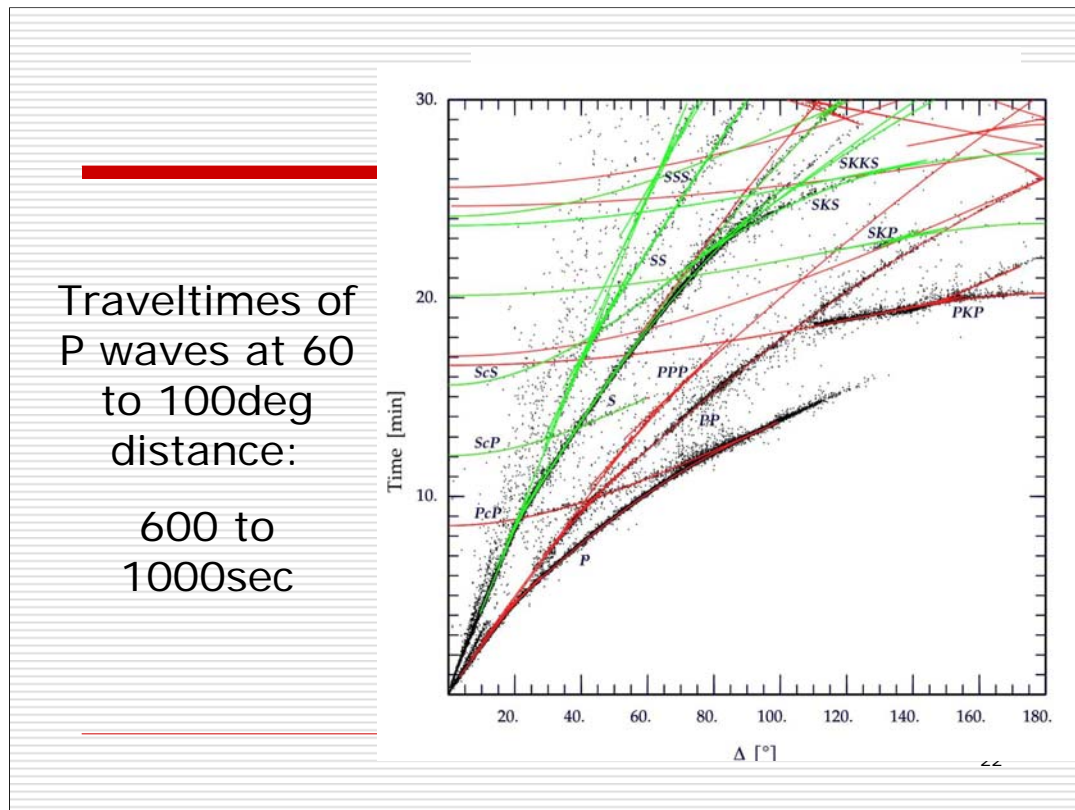
This figure shows the observed arrival times as a function of epicentral distance
for many data put together. You see that there is not only one P and one S wave,
as it would have been in a simple Earth structure, but many waves, and that they
are several branches in these waves. Each branch corresponds to a different
type of path within the Earth. You see also that the branches are curved. The
curvature tells us something about the velocity increase inside the Earth. By
combining all this information, we get some information about the distribution of
the waves velocity with depth with the Earth.

Fig. 8. The PREM model. Dashed lines are the horizontal components of velocity. Where η is 1 the model is isotropic. The core is isotropic.

Preliminary Reference Earth Model (1981)

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This is the result of such an analysis, showing the velocity of the P waves, the velocity of the S waves and the density with depth. You see that we have first an increase of velocity with depth from 0 to almost 3000 km depth. This first layer is the mantle, made of silicates. Below that you have the core, where velocity is lower, still increasing with depth.
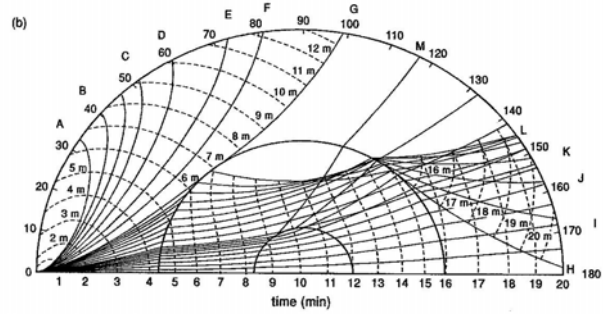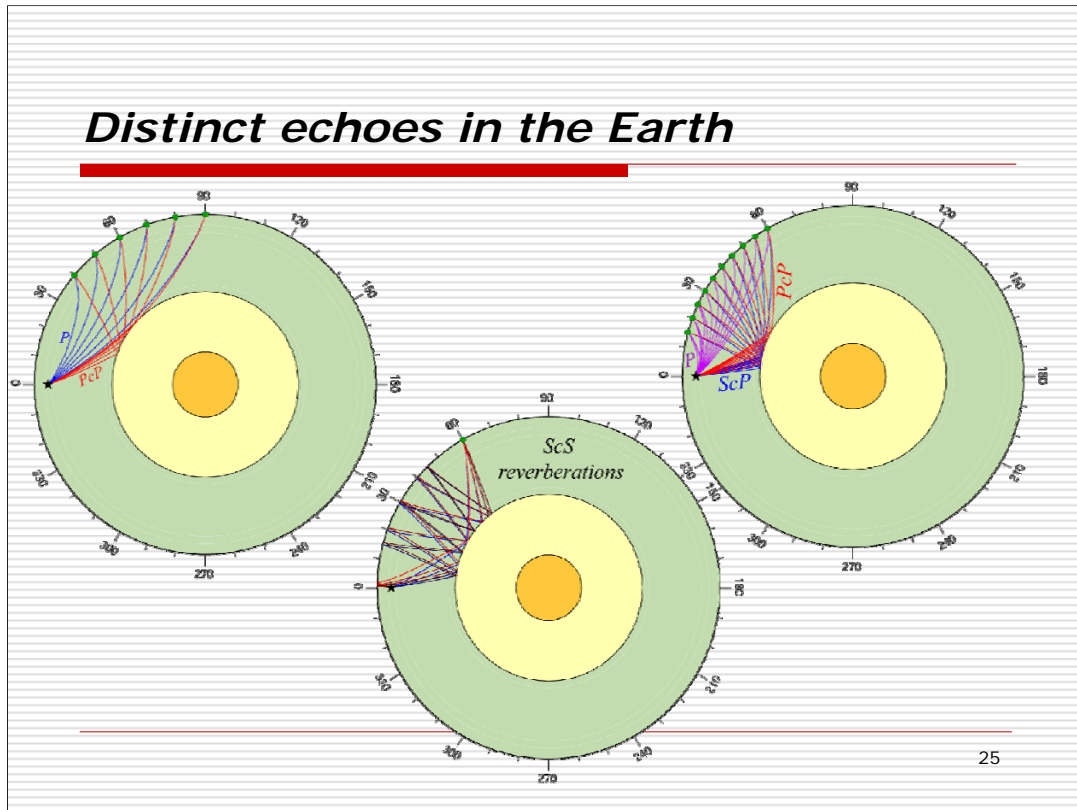
Wave paths for P waves


Figure 4.20 Ray paths in the Earth.

Due to the increase of velocity with depth, the rays are curved into the Earth. This is a major effect which cannot be neglected when we analyze the arrival times. All the imaging studies based on geometrical optics must take into account the curvatures of the rays inside the Earth.

Distinct echoes in the Earth

These are some examples of wave paths on Earth, for waves which propagate
deep into the mantle and are reflected by the core-mantle boundary.
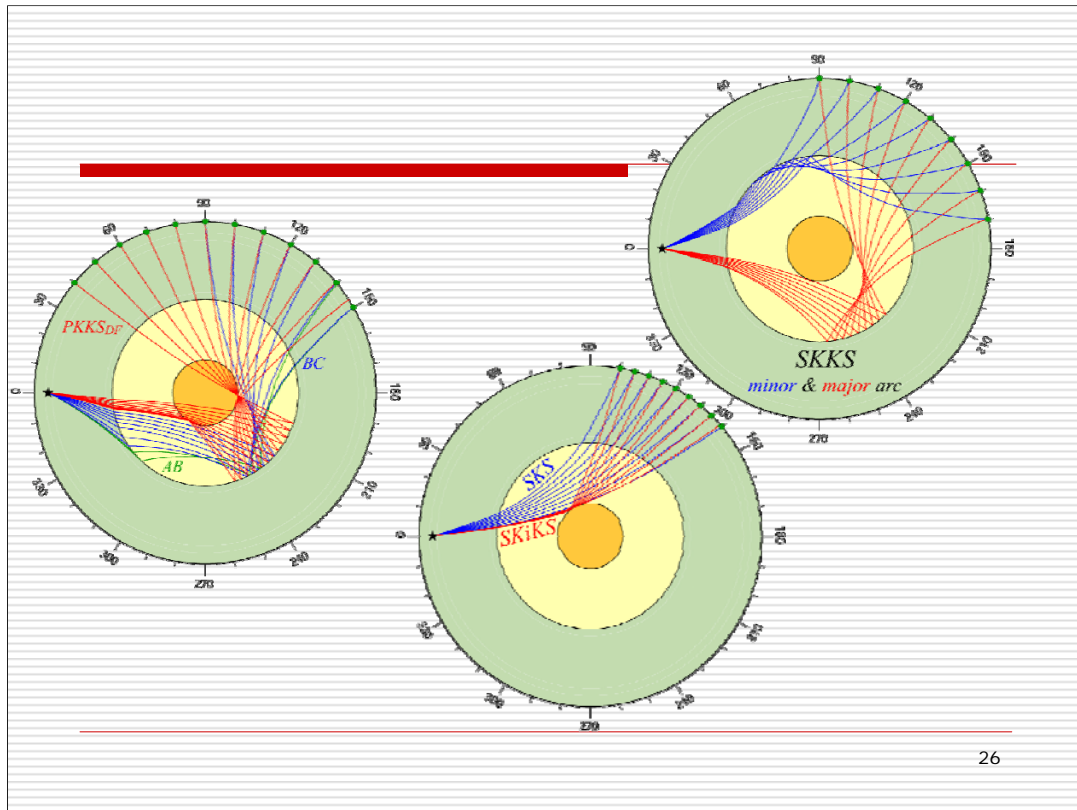
Examples of waves which propagate inside the core. Some are reflected several times inside the core. All the waves which propagate deep arrive early in the seismograms, as single arrivals, because the deeper layers have higher velocities than the upper layers. Some of the waves are quite large and observed at almost all distances from the earthquake. Some other are quite small and observed only in some special cases.
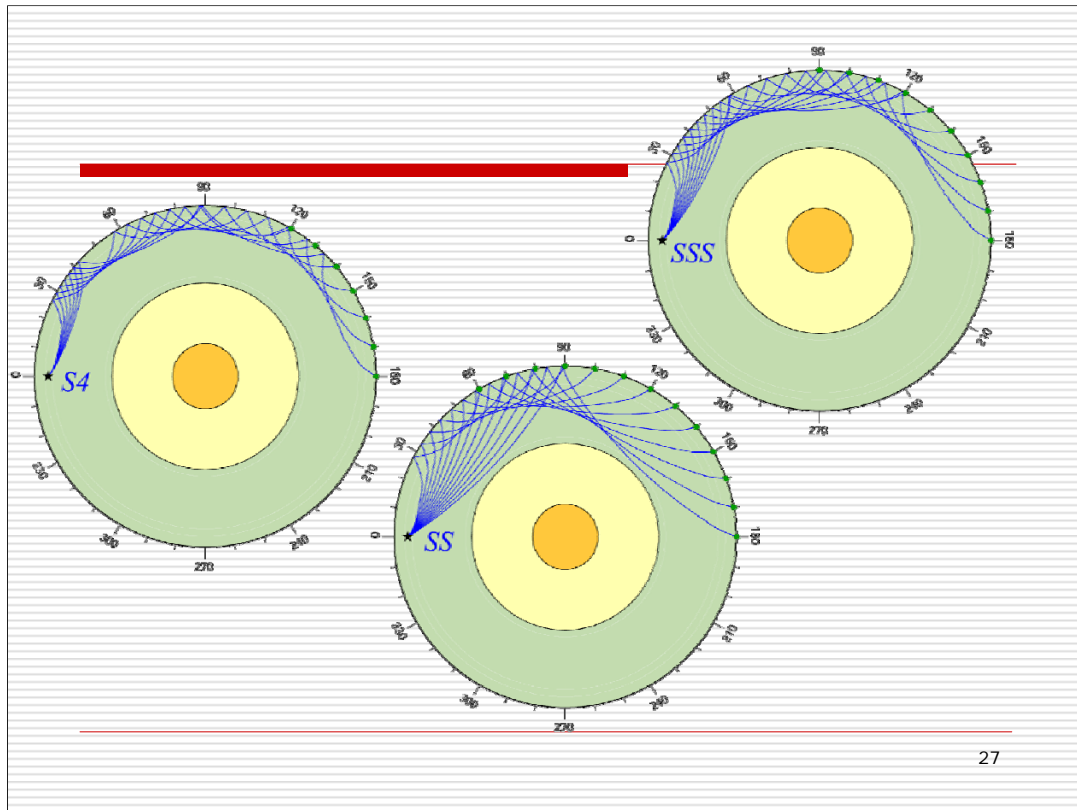
The waves which travel close to the surface and are reflected several times by the Earth's surface combine together to form what we call the surface waves. They are the largest waves on the seismograms and arrive last.
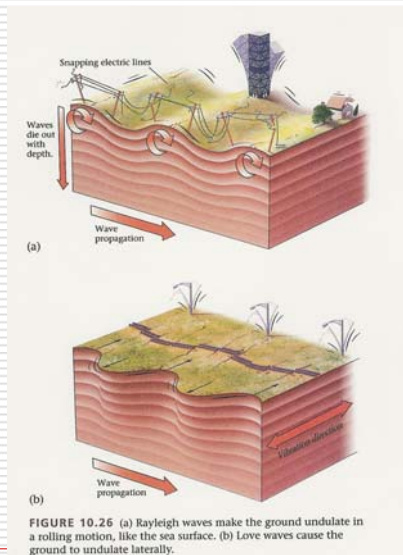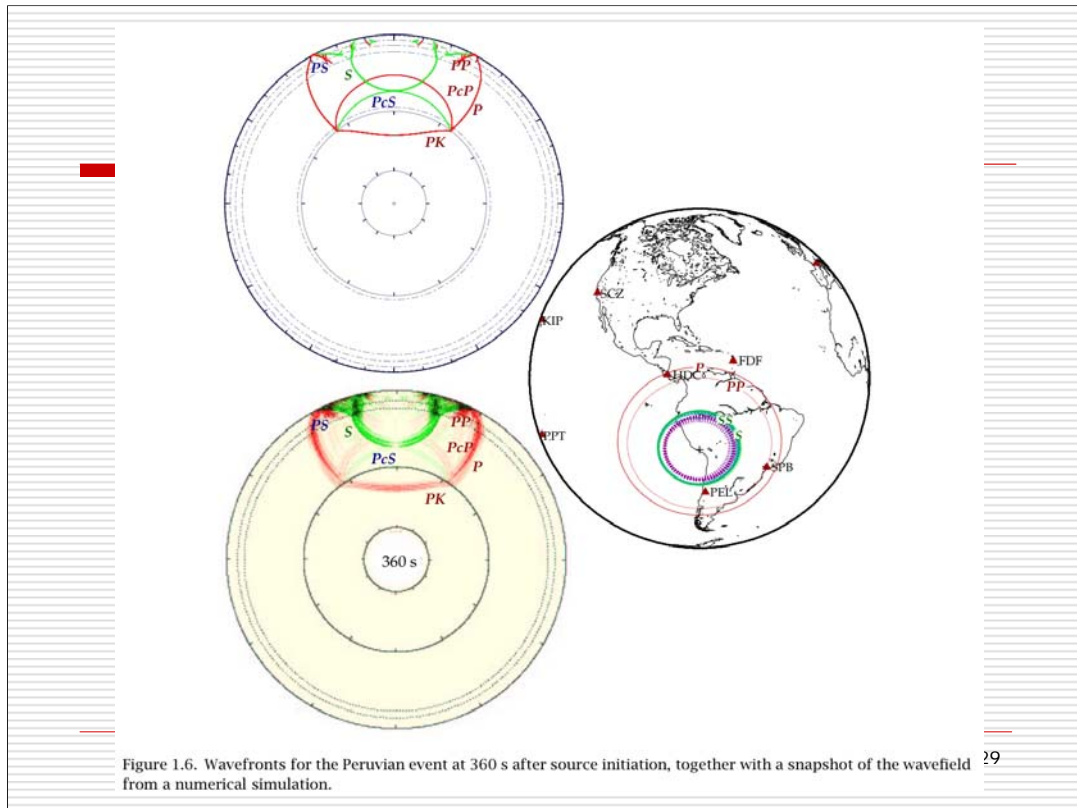
Surface waves: Rayleigh and Love waves

FIGURE 10.26 (a) Rayleigh waves make the ground undulate in a rolling motion, like the sea surface. (b) Love waves cause the ground to undulate laterally.

The surface waves can also been seen as waves which travel along the surface of the Earth. We have two kinds of surface waves: the Rayleigh waves which are very much like water waves or ground-roll, and Love waves which consist of a shear motion of the Earth's surface.

Figure 1.6. Wavefronts for the Peruvian event at 360 s after source initiation, together with a snapshot of the wavefield from a numerical simulation.

This shows all these waves propagate together both in depth and at the surface of the Earth at a given time after the earthquake. In red you have the P waves and in green the S waves.

Figure 1.10. Wavefronts for the Peruvian event at 1080 s after source initiation, together with a snapshot of the wavefield
from a numerical simulation.

And this is the same at a later time. The P waves have now reached the other side of the Earth. There are some S waves in the mantle and quite a lot of S waves propagating slowly close to the surface.

- What is an earthquake?
- Imaging: early studies and the mean structure of the Earth
- P-wave tomography
- Normal mode tomography
- Source location and time-reversal

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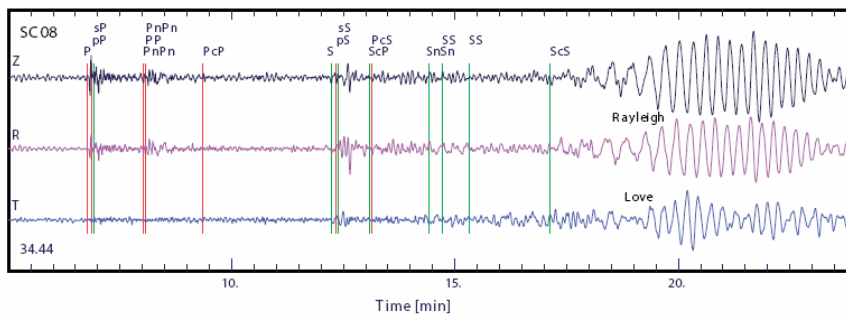
Figure 1: Example of a 3-component (Z, R, T), broadband seismic record, with particularly prominent surface wave signal; epicentral distance ∼ 34°. A few phases have been identified on the seismograms. I took this and some of the following images from http://wwwrses.anu.edu.au/seismology/SHon2002/

On this example seismogram that we have seen before, you can see that we have many different distinct arrivals at different times. The lines show the theoretical arrival times in a simple model of the Earth without lateral variations, only depth variations. You see that they correspond quite well to the real arrivals. This is because the dominant variation of the elastic parameters in the Earth is in the depth direction. The lateral variations are much smaller than the vertical ones.

Moonquake recording
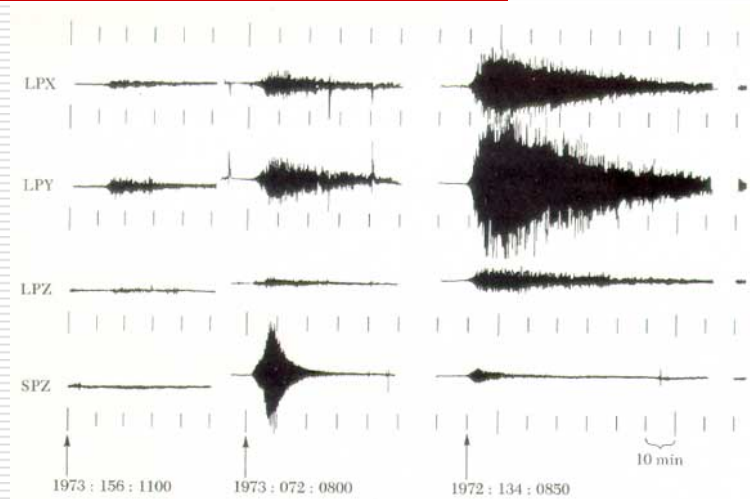
Deep and shallow moonquake and meteoroide impact
recording on the Moon (from NASA).

The moon is much more heterogeneous than the Earth. These plots show the
recordings of moonquakes and meteoroid impact at four different components of
a seismometer put on the Moon by NASA. As opposed to the Earth recordings,
you cannot identify different waves or echoes on these seismograms. You have
basically a very complicated record which does not allow for any sensible
imaging of precise structures.

Traveltimes of P waves at 60 to 100deg distance:

600 to 1000sec
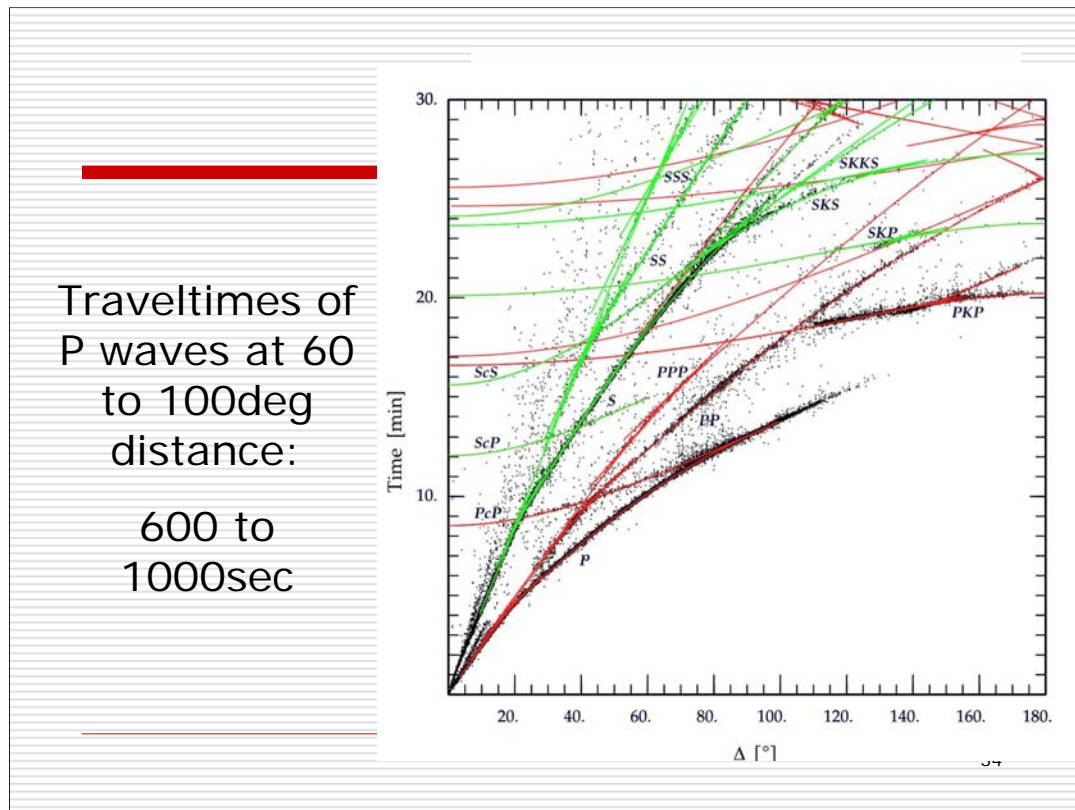
We have seen before that the traveltimes of the waves vary with distance and that this variation gives us information about the structure of the Earth. If the Earth was perfectly spherical with no lateral variations, all the arrivals would fit perfectly with the lines calculated for the Earth model here. You see that here are some points which fall outside. This is due to lateral variations of the velocities inside the Earth. The mean structure of the Earth is very well known now, and imaging studies concentrate on imaging the lateral variations.
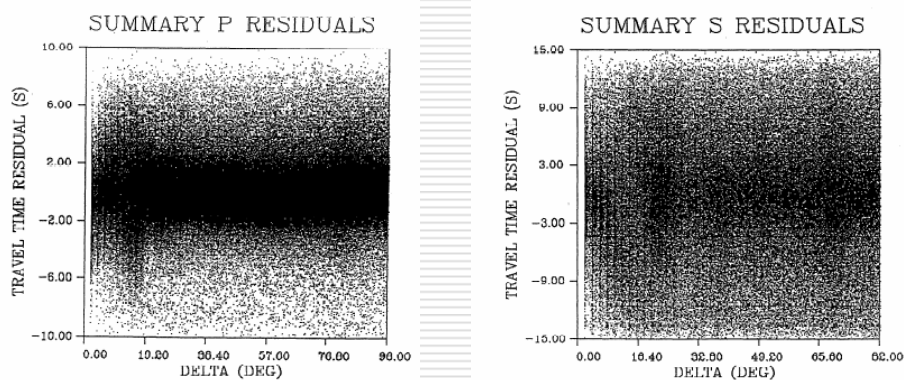
Amplitude of the time residuals

SUMMARY P RESIDUALS

SUMMARY S RESIDUALS

Figure 2. Distribution of P and S summary ray travel time residuals computed using a background velocity model of IASP91.

Vasco et al. 1994

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This shows the deviation of the arrival times of the P waves and of the S waves with respect to the theoretical ones calculated in the mean Earth model. You see that P waves arrival times are within 2 seconds of the theoretical one. This is not even 1% of the total propagation time, showing that the lateral variations are not large. For S waves, the variations are larger, but are still at most only of the order of a few percent of the total propagation time.

Traveltimes, origin times and locations

The arrival time at a station depends on:
- Velocity along the path
- Origin time of earthquake
- Epicenter of earthquake
- Depth of earthquake

In addition, errors due to
- Clock problems
- Reading errors
- Localized strong heterogeneities in the crust below the station

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ISC database

- About 2500 stations report to the ISC
  (International Seismological Centre):
  thousands of earthquakes/year,
  millions of traveltimes.
- Primary objective: earthquake
  location and catalogue
- Spin-off: global arrival time database

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Principle of tomography
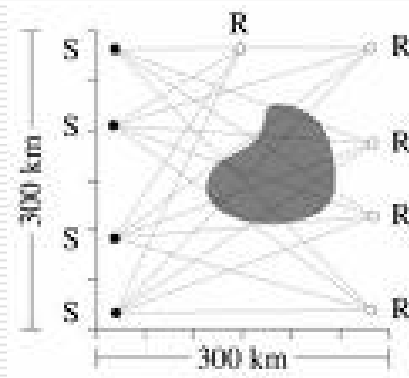
All the waves which travel through the slow region will arrive **late**.

The amount of delay on each ray depends on the strength of the heterogeneity and its size.

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Using the arrival times at the rays, we can reconstruct the distribution of the velocity inside the body.

Setting up the linearized inverse problem

from ray theory:

$$T = \int_{\text{ray path}} \frac{1}{v(\mathbf{r}(s))} ds$$

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The arrival time depends on distance divided by velocity along the whole
path

Setting up the linearized inverse problem

from ray theory:

$$T = \int_{\text{ray path}} \frac{1}{v(\mathbf{r}(s))} \mathrm{d}s$$

first-order Taylor expansion:

$$\delta T = - \int_{\text{ray path}} \frac{\delta v(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

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If we know the mean velocity V0, that the true velocity is V= V0 + deltaV and that deltaV is small compared to V0, then we can express the perturbation in traveltime deltaT by the equation above:

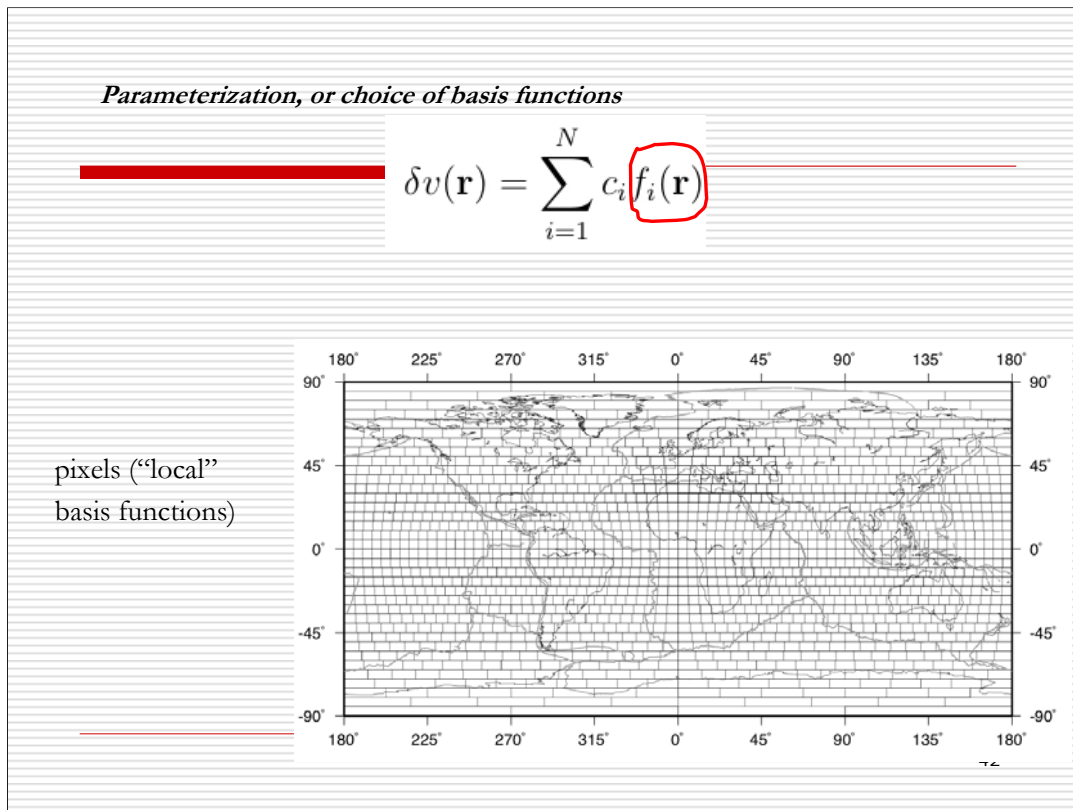*Setting up the linearized inverse problem*

from ray theory:

$$T = \int_{\text{ray path}} \frac{1}{v(\mathbf{r}(s))} ds$$

first-order Taylor expansion:

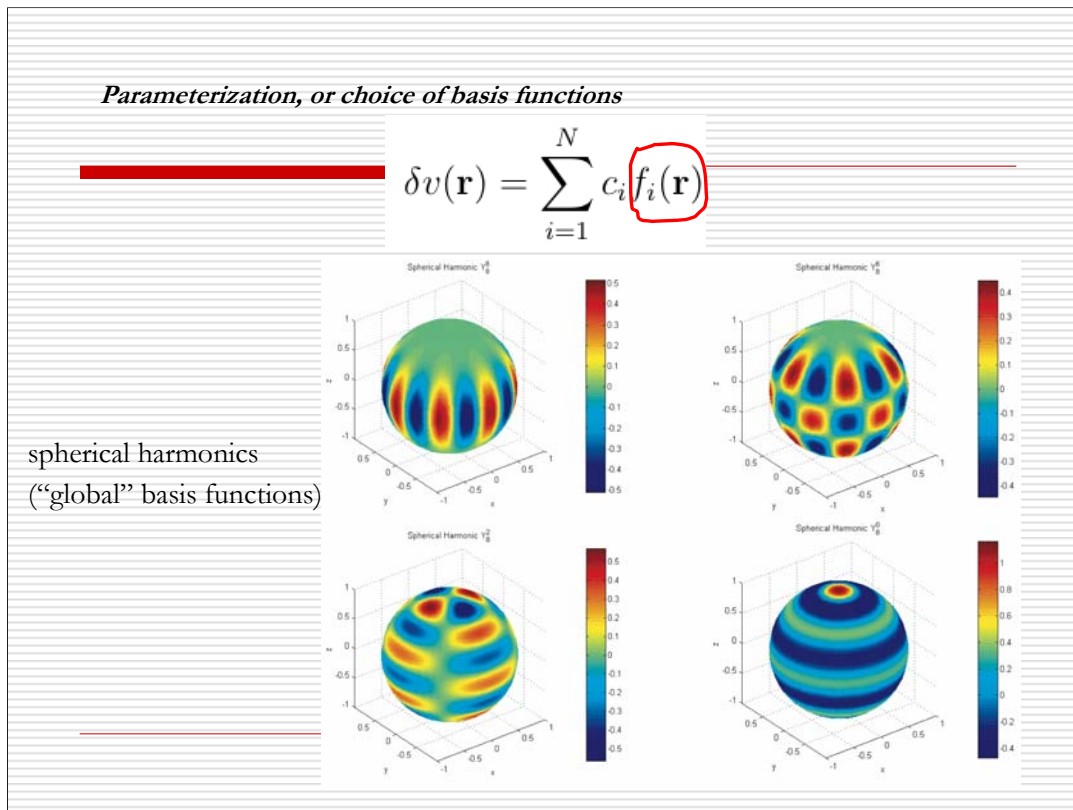$$\delta T = -\int_{\text{ray path}} \frac{\delta v(\mathbf{r})}{v_0^2(\mathbf{r}(s))} ds$$

"parameterization":

$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

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The velocity variations inside the structure is what we are trying to find. We have somehow to transform this three-dimensional quantity into a few parameters which can be handled by a computer. This can be done by expressing the velocities on a basis of functions.

Parameterization, or choice of basis functions

$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

pixels ("local"
basis functions)

One common basis of functions is to use small cells. This means that we assume that each cell has a constant velocity. The size of the cells has to be chosen such that it fits with the number of rays we have and the wavelength of the waves.

Parameterization, or choice of basis functions

$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

spherical harmonics
("global" basis functions)

Another possible parameterization is on functions like the spherical harmonics, which is equivalent to doing a 2D Fourier decomposition of the velocity. Although it is less intuitive than the cell decomposition, it is very well adapted in global studies.

Setting up the linearized inverse problem

from ray theory:

$$T = \int_{\text{ray path}} \frac{1}{v(\mathbf{r}(s))} \mathrm{d}s$$

first-order Taylor expansion:

$$\delta T = -\int_{\text{ray path}} \frac{\delta v(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

"parameterization":

$$\delta v(\mathbf{r}) = \sum_{i=1}^{N} c_i f_i(\mathbf{r})$$

Unknown coefficients are constant and don't need to be integrated:

$$\delta T = -\sum_{i=1}^{N} c_i \int_{\text{ray path}} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

known   44

Since the functions are known, the reference velocity and the ray paths are known, everything in the red ring is known and can be calculated. It is commonly assumed that the perturbations to the velocity are small and do not perturb significantly the ray path. This is of course true only at first order and some newer tomographic studies try to remove this assumption.

Setting up the linearized inverse problem

the same eq. Applies for many data and corresp. ray paths:

$$\delta T_j = - \sum_{i=1}^{N} c_i \int_{\text{ray path}_j} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} ds \qquad (j=1,\dots,M)$$

introduce matrix **A**:

$$\delta T_j = \sum_{i=1}^{N} c_i A_{ji}$$

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This integral can be calculated for each traveltime j and each cell i (or function i)
to build a matrix which then relates the traveltime anomalies to the velocity
perturbation in the different cells.

Setting up the linearized inverse problem

$$\delta T = -\sum_{i=1}^{N} c_i \int_{\text{ray path}} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s$$

the same eq. Applies for many data and corresp. ray paths:

$$\delta T_j = -\sum_{i=1}^{N} c_i \int_{\text{ray path}_j} \frac{f_i(\mathbf{r})}{v_0^2(\mathbf{r}(s))} \mathrm{d}s \qquad (j=1,...,M)$$

introduce matrix $\mathbf{A}$:
$$\delta T_j = \sum_{i=1}^{N} c_i A_{ji}$$

$$\delta \mathbf{T} = \mathbf{A} \cdot \mathbf{c}$$

46

And this can be written very simply in vector and matrix form.

Least-squares solution

given a linear inverse problem: $\delta \mathbf{T} = \mathbf{A} \cdot \mathbf{c}$

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We have A and the observed deltaT. What we want is to calculate the inverse of A in order to obtain the velocity perturbations c as a function of the traveltime anomalies. But A is almost never an invertible matrix! It is not even square. It does not have an exact inverse.

Least-squares solution

given a linear inverse problem:

$$\delta \mathbf{T} = \mathbf{A} \cdot \mathbf{c}$$

its least-squares solution:

$$\mathbf{c}_{LS} = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \delta \mathbf{T}$$

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We calculate therefore an approximate inverse in the least-squares sense. That will give us the velocity which fits the data best. Imaging is often called inversion in seismology, because the whole thing gets down to inverting a matrix.

Least-squares solution

:

As when you try to fit a line to points
which are not totally aligned with each other:
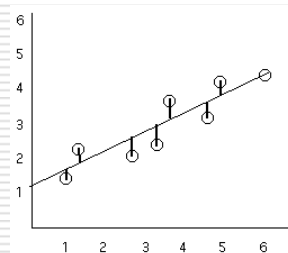
49

We calculate therefore an approximate inverse in the least-squares sense. That
will give us the velocity which fits the data best. Exactly as when you try to find to
find the best line which goes through points that are not perfectly aligned.

Least-squares solution

given a linear inverse problem: $\qquad \delta\mathbf{T} = \mathbf{A} \cdot \mathbf{c}$

its least-squares solution:

$$\mathbf{c}_{LS} = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \delta\mathbf{T}$$

is typically solved by either:

direct algorithms (SVD, Cholesky…)   iterative algorithms (CG, LSQR…)

50

In most cases, the A matrix is very big and you need special algorithms to perform the inversion.

The effect of errors in the data
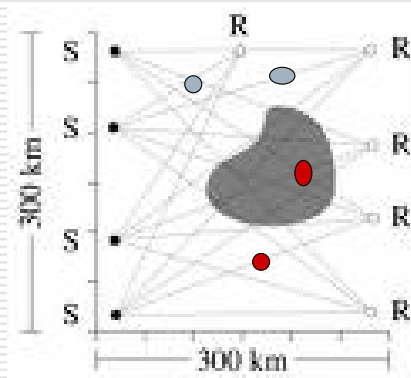
Small errors in the
data can be mapped
as small-scale
heterogeneities

What to keep?

Not a simple question.

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One very important question is the effect of the errors on the data on the model.
If you have errors on the data, they can get mapped into the structure.

In the example above, we have two rays with negative errors. If we don't know
that there are errors on these data and try to explain the data by heterogeneities
in the model, we have to put two small spots of low velocities (red) which are not
real features of the structure. Here we have also three rays with positive errors
which we have managed to explain with two spots (blue) of high velocity. In order
to avoid mapping errors in the model, we need to define the amplitude of the
errors we can expect on the data. This is a crucial step in the imaging process
which determines indirectly the size of the features that we can get in the model.

Tomography results are non-unique
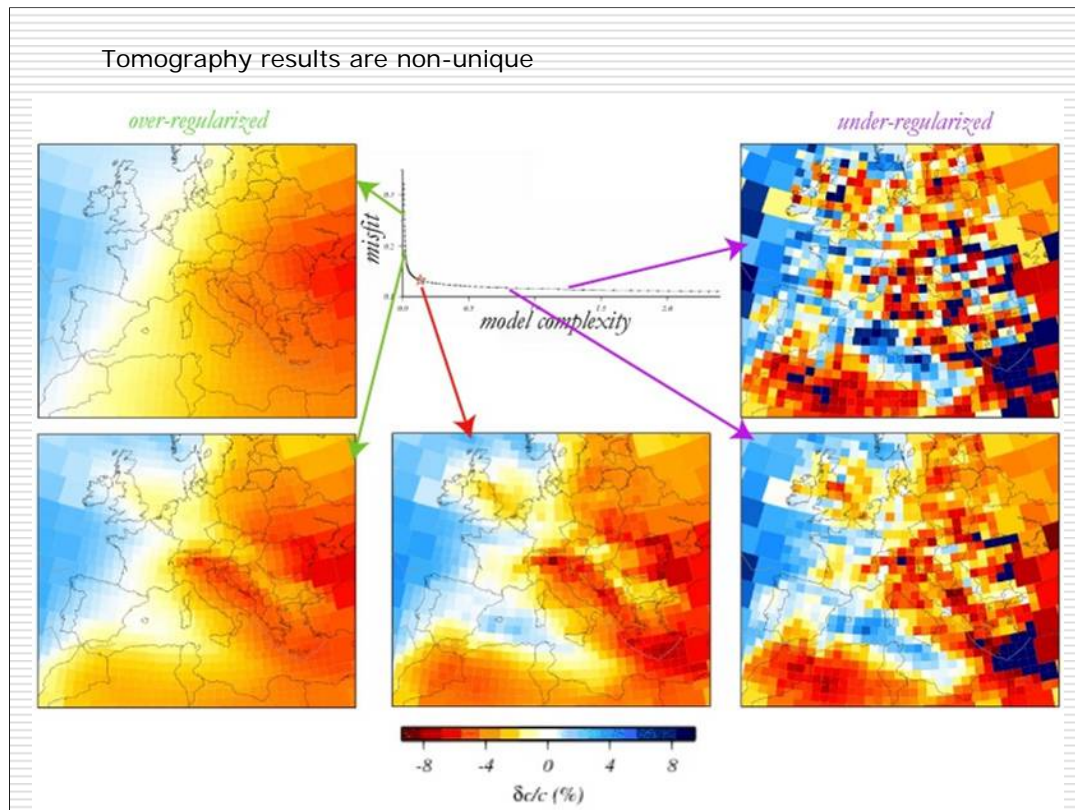
This is an example of different models derived from the same data set but using different values for the errors in the data. The curve in the middle show the misfit (that is how much of the data are not explained by the model) as a function of the model complexity. On the left, we have a very smooth model with high velocities to the left (in blue) and low velocities to the right (in red). As shown by the misfit, which is quite large, this model does not explain the data very well. Here we assumed that the errors on the data are large. If we get more confident in our data and try to explain them better, we get a smaller misfit and a model with higher complexity. In the end we get a model with very high complexity, where the velocities vary very much from one cell to the next. But the misfit doesn't get much smaller. Choosing between these models, also called choosing the damping, is the most difficult part in a tomographic inversion.

Effect of damping

- Small-scale heterogeneities
- Amplitude of the large-scale heterogeneities
- Basically the resolution is changed

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Damping reduces the amplitude of the smal heterogeneities in the final model. It also affects, although less severely; the amplitude of the large scale anomalies. It is basically changing what we call the resolution. In conclusion, we see very directly and mathematically here how the resolution of the imaging is related to the size of the errors on the measurements.

Resolution test

Marone et al, 2007
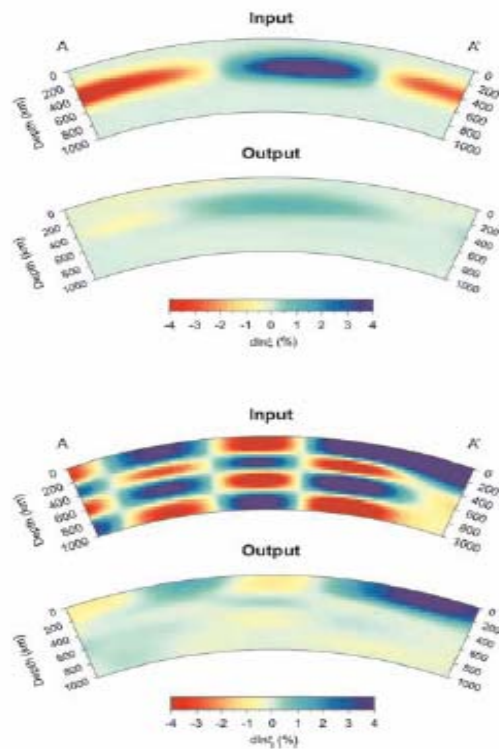
In order to test the resolution of the imaging, we often perform resolution tests. We build a model, calculate the data that we would have obtained if the Earth was like the model we have made, invert these data, and then compare the real initial model with the one obtained by the inversion. Here you see two examples of resolution tests, made with two different sizes of low and high velocity blobs. The input is the true model and the output is the result of the inversion. In the upper case, we see that the fast anomaly in the center of the model is resolved, but not the two slow anomalies on the borders. Probably there are few data propagating through the borders of the model. In the lower case, the resolution is also very poor, except for the fast anomaly close to the surface on the right hand side of the model. I have chosen here examples with low resolution, so that you see clearly the difference between the input and the output models. It is usually not as bad!

P-wave tomography (Spakman, vd Hilst)
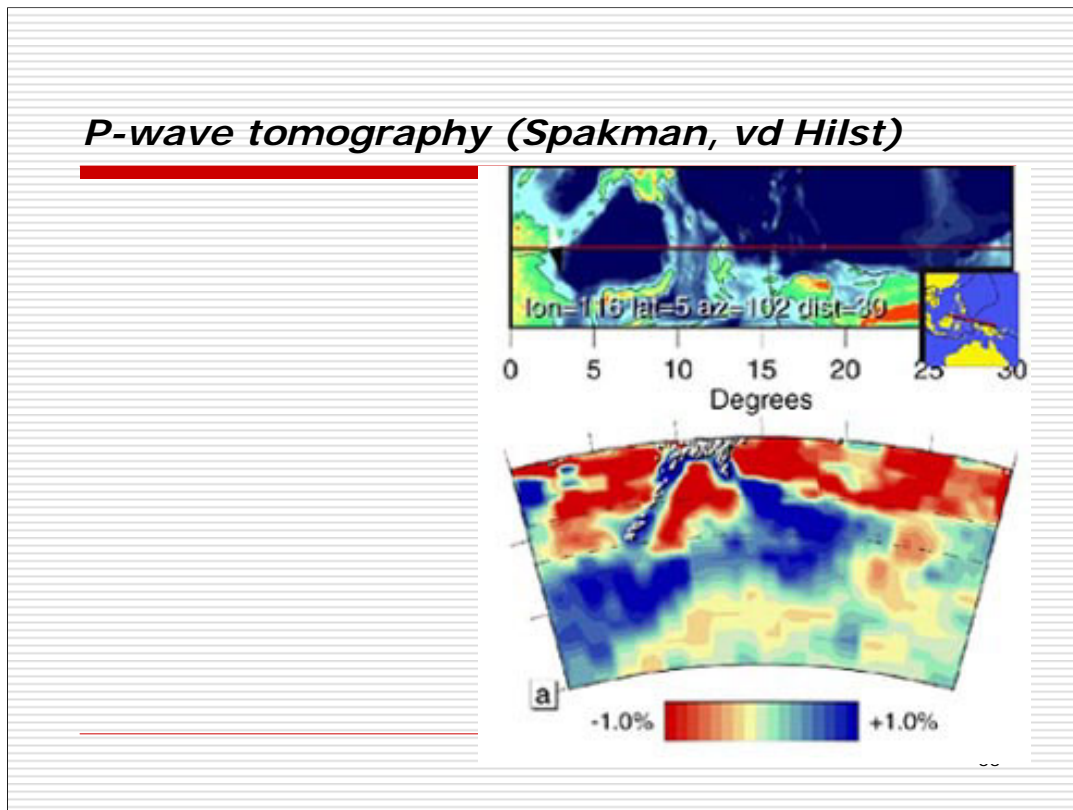
Example of result from a P-wave tomography. This shows a section with depth of a model obtained through Indonesia, down to 1500km depth. In the upper 500km, the model is on the average slow (red color), except in two regions, which also correspond to down going slabs (the earthquakes are shown by white dots, and you can see that they follow the fast anomalies). The slabs are faster than the surrounding because they are colder. Notice the color scale: Variations are only of -1 to 1%. It is really small variations that we are measuring. This requires very precise measurements of the waves arrival times and very precise earthquake locations.

P-wave tomography
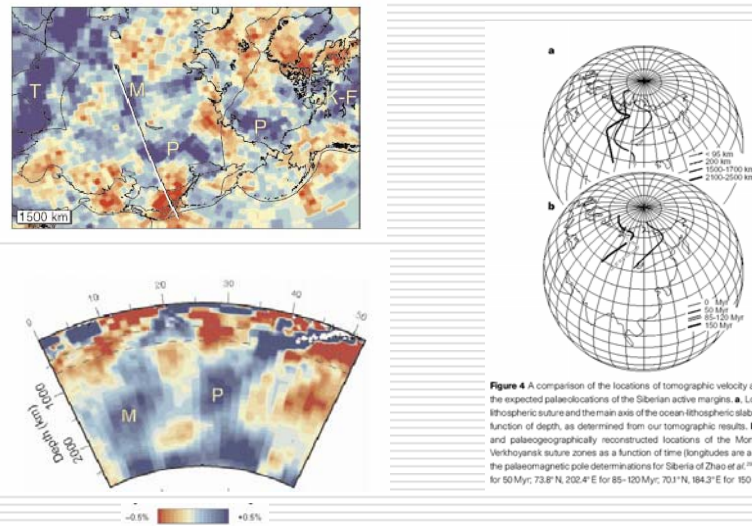
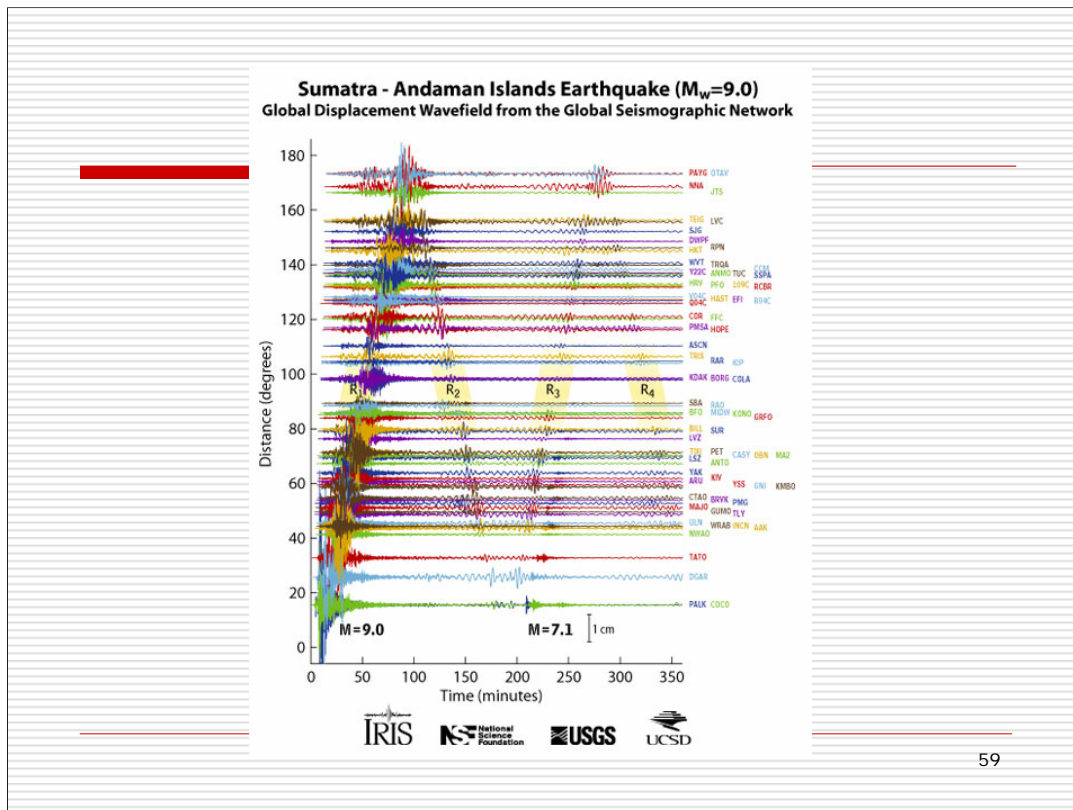Van der Voo et al., 1999 [56]

This is another example, through northern Asia and to a larger depth. To the right, you see the Japanese slab, with its seismicity. You see also that the fast material goes further down into the mantle than 700km (which is the maximum depth of the earthquakes). The fast material noted P is believed to come from the Japanese subduction. The fast material noted M is believed to come from an ancient subduction which occurred 200 to 150 Myears ago, at a time where East and West Asia were separated by an ocean.

- These models give us the P-wave velocity distribution in the Earth
- They are based on short-period P-waves (1Hz).

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- What is an earthquake?
- Imaging: early studies and the mean structure of the Earth
- P-wave tomography
- Normal mode tomography
- Source location and time-reversal

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Now we are going to see a rather different approach to imaging, not based on ray theory.

Sumatra - Andaman Islands Earthquake (Mw=9.0)
Global Displacement Wavefield from the Global Seismographic Network

We have seen before that the arrival times of waves as a function of distance between the earthquake and the station have a central position in the methods that we use to image the interior of the Earth. Now we are not going to look at the arrival time of individual waves, but look at the whole recording as a function of time and distance. The advantage of looking at the whole recording is that we keep much more information from the seismograms than when we pick just a few arrival times. In addition we use the information contained in the amplitudes of the waves, their frequency content, etc and we keep all the waves, even those with complicated wavepaths.

On the figure above, you see the recording of the Sumatra earthquake of 2004 at many stations, organized as a function of increasing distance from the earthquake. At the beginning of the traces, you see (but they are very small) the P waves that we have used in the P-wave tomography just before, then S waves, then the surface waves with the largest amplitudes. All these waves have traveltimes increasing with distance. Then you have a wave noted R2 which has an arrival time decreasing with distance, a wave called R3 which arrives with a constant delay with respect to the surface waves.

**Surface waves: late, long-period and large amplitude waves**
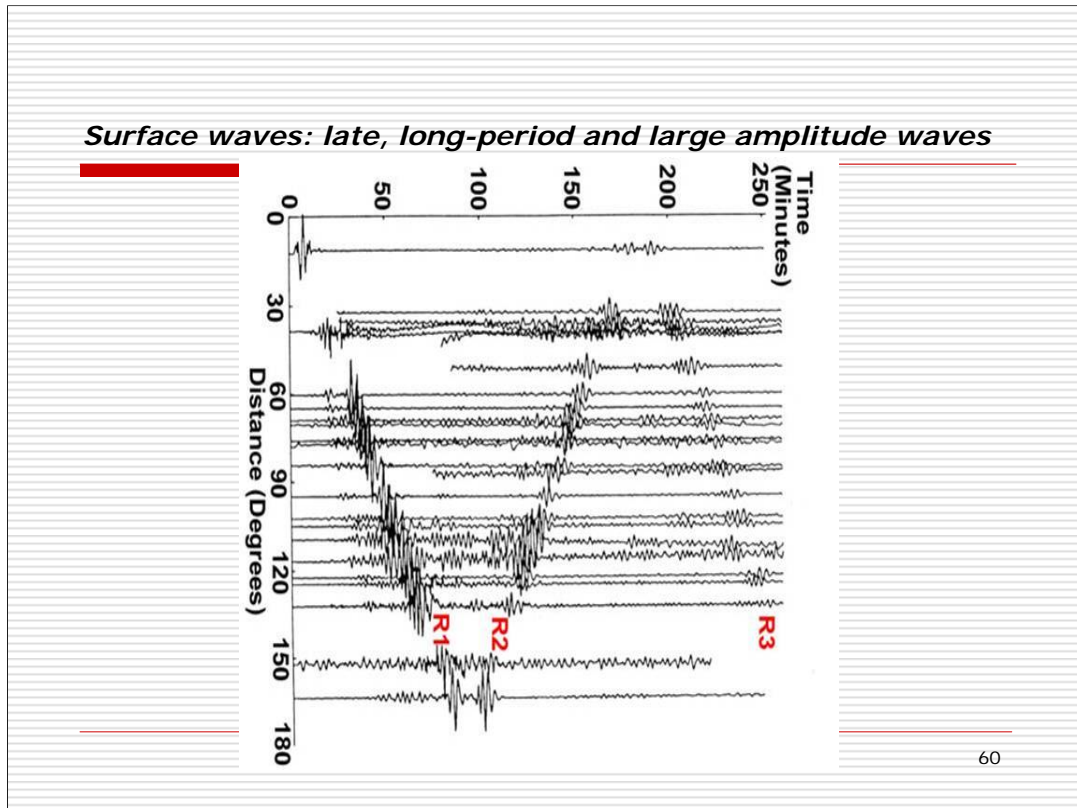
You see something similar here for another earthquake. Here the first surface wavetrain is called R1.
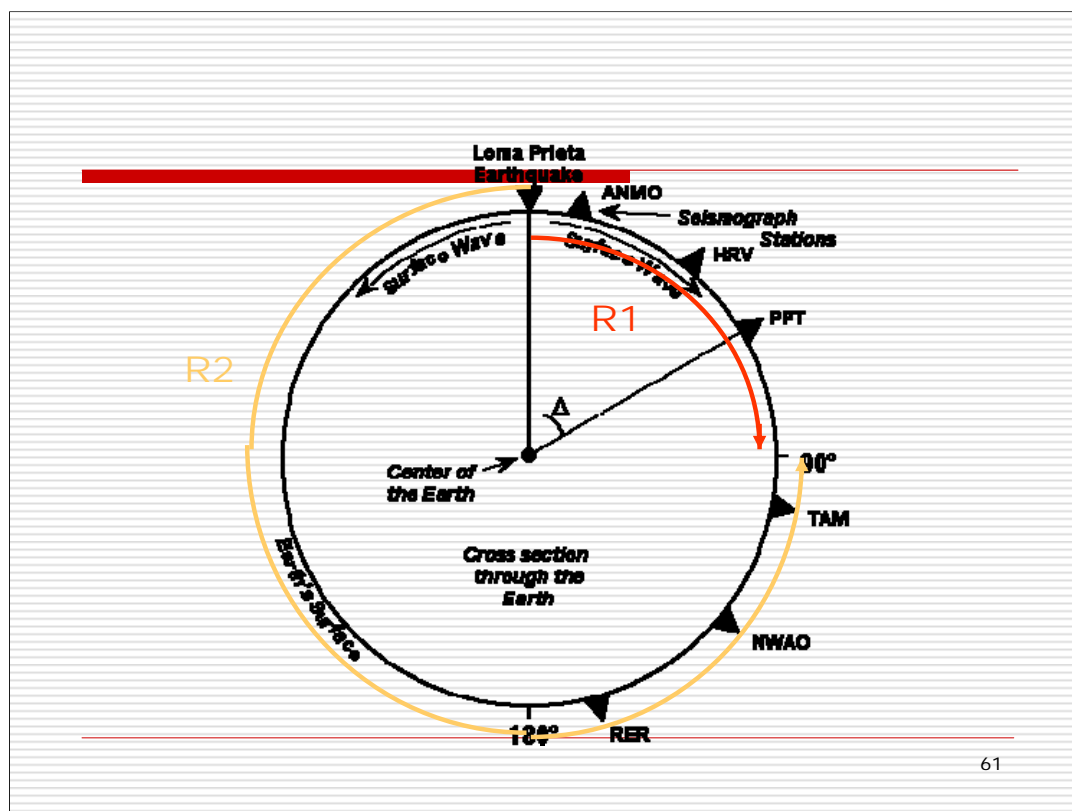
These different waves correspond to waves which propagate to the station from different sides of the Earth. R1 travels directly from the earthquake to the station. R2 travels around and arrives at the station from the other side. R3 is R1 after it has gone one time around the Earth and back to the station. One full round trip lasts about 3 hours. For very large earthquakes, the waves continue to travel around for many days.

If you see that in 3D, you see that the waves are propagating away from the earthquake, converge at the antipode, propagate back to the earthquake and then diverge again etc. You see that you have a sort of pulsation of movement going back and forth on the planet. This can be seen as the eigenvibrations of the Earth.

Fundamental
vibration

Overtones
or
harmonics

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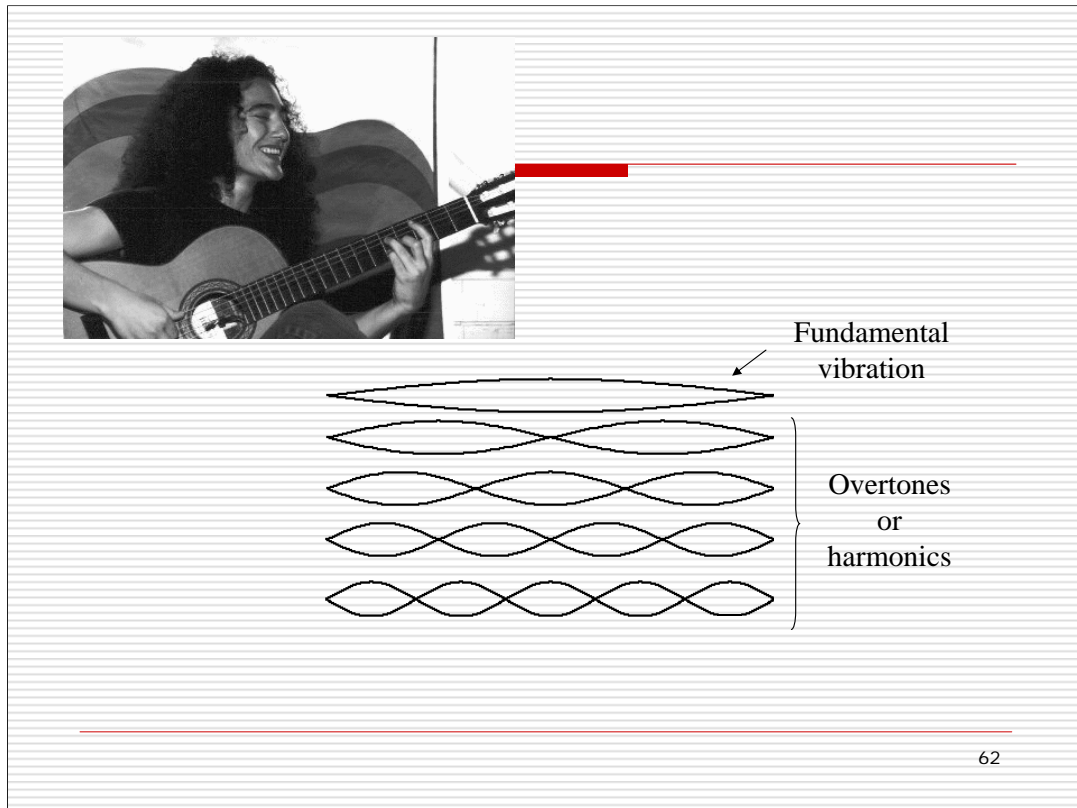
Exactly as when you get the string of a guitar to vibrate, the earthquake makes the Earth vibrating. The vibrations decompose on the different vibrations of the string. For a simple string, the vibrations are very simple, with the fundamental mode and the different overtones.
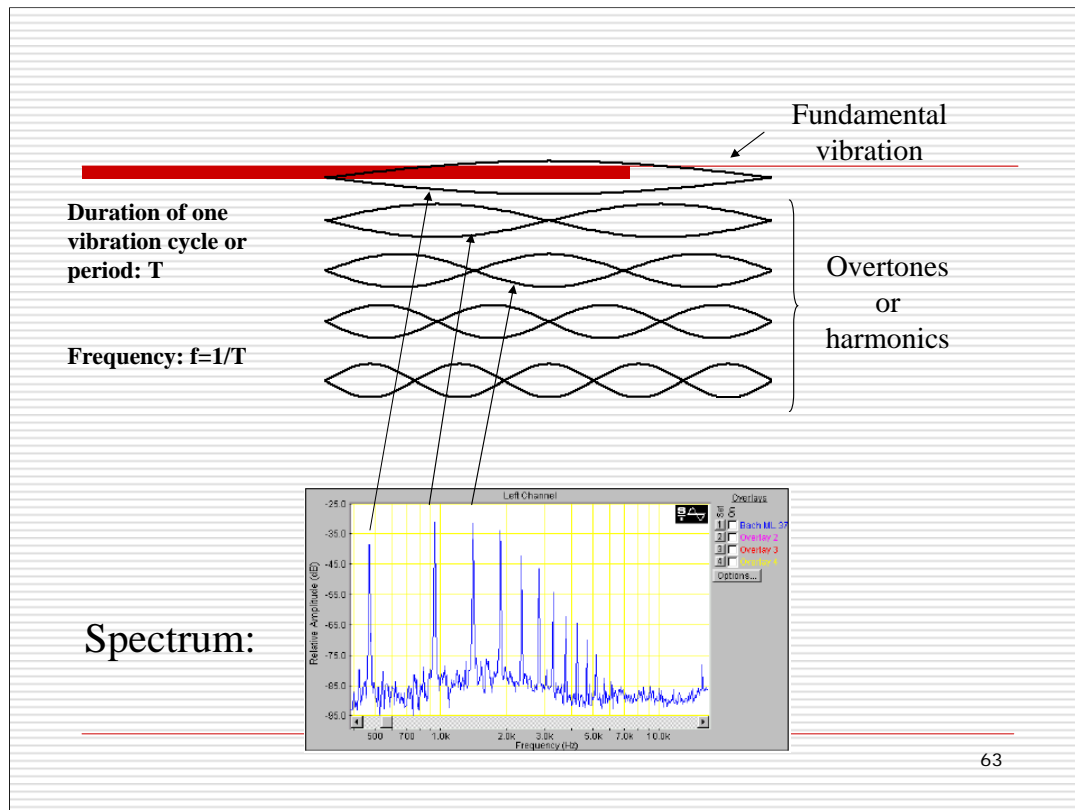
If you take the spectrum of the sound made by the guitar, you will see that the spectrum has some peaks, corresponding to the different modes of the string.

Spectrum of the Earth
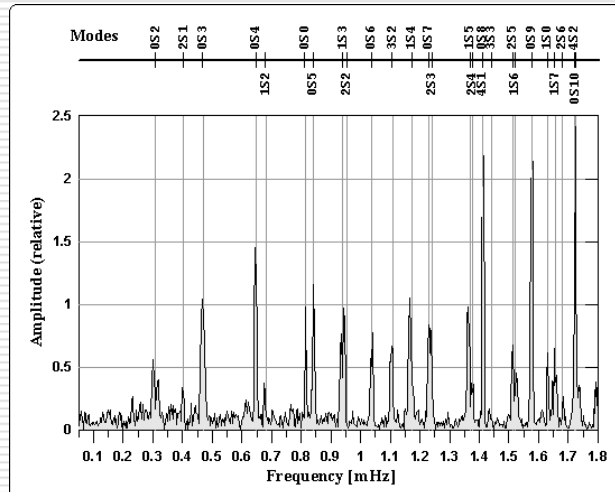
It is the same with the Earth. If you take the spectra of the seismograms, you find that they have a number of peaks at particular frequencies. These frequencies correspond to the frequencies of different modes of the Earth. The lowest frequency is for mode 0S2 and corresponds to a period of 54mn.

Normal modes

Toroidal modes $_0T_2$ (44.2 min), $_1T_2$ (12.6 min) and $_0T_3$ (28.4 min)

Spheroidal modes $_0S_0$ (20.5 min), $_0S_2$ (53.9 min) and $_0S_3$ (25.7 min)

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Each mode has a number and a particular way of deforming the Earth. The toroidal modes do not have any vertical component. They only produce a shear at the surface of the Earth. Three examples of toroidal modes are shown here in blue, together with their periods. Spheroidal modes have a vertical component. 0S0 corresponds to a uniform dilatation and contraction of the Earth. 0S2 to a rugby-ball type deformation.
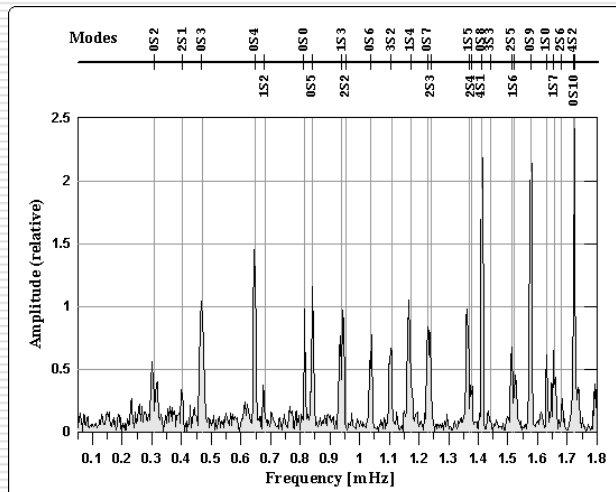
Sensitivity kernels of normal modes
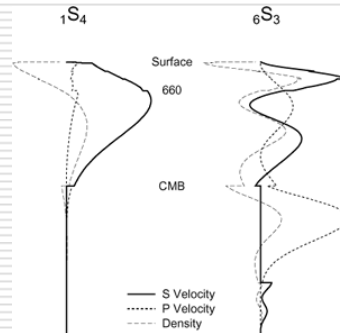
The deformation is not only at the surface of course. Each mode has its own amplitude of deformation with depth. Mode 1S4 for example deforms the whole mantle, with a maximum at about 600km depth. Mode 6S3 has a more complex deformation involving the mantle and the core.

Spectrum of the Earth

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If we now go back to the spectrum, we see that each mode has its own frequency. By analyzing all the frequencies of the modes, we can infer the elastic properties of the Earth, very much as a musician can tell you where you have pinched the string of your guitar by listening to the sound you made with it, or as you can tell if a C-note has been played on a flute or a violin by analyzing the overtone content of the sound.

PREM model

Dziewonski and Anderson, 1981)

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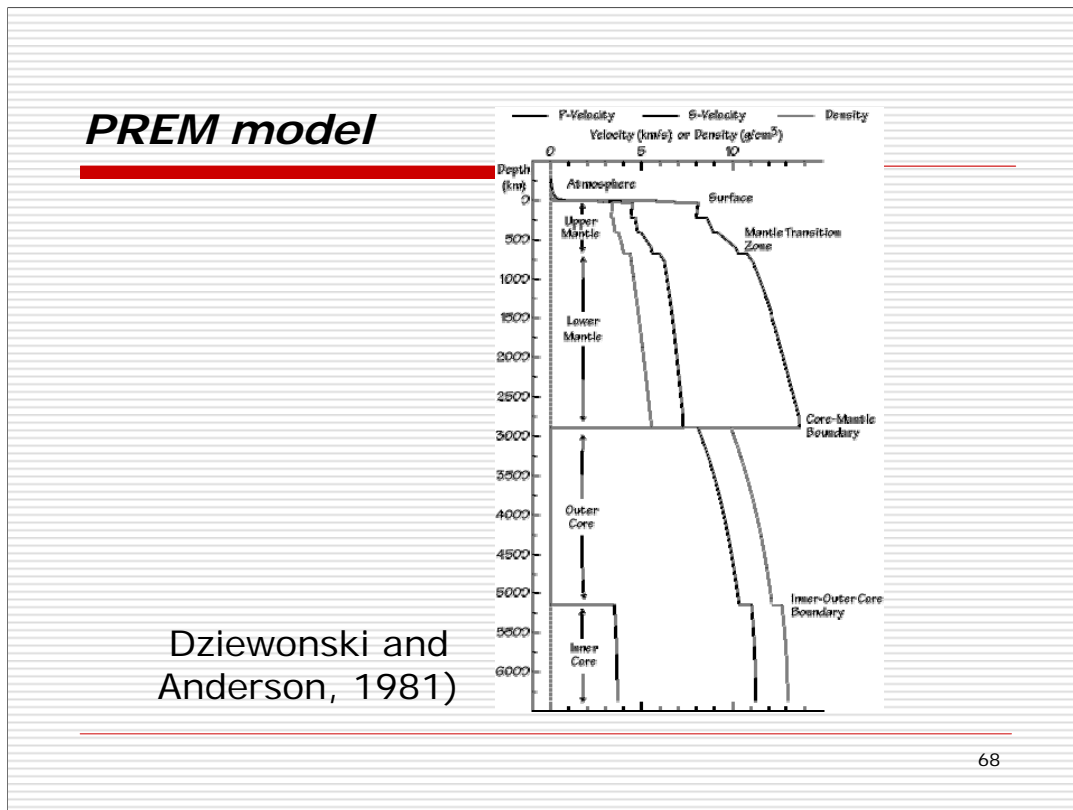
This is how our most used reference model PREM has been made. This is a better way of doing a reference model for the Earth than by analyzing many many rays because here we let the waves travel around many times, sample every single corner of the planet and average all the properties. If we take only direct P waves from earthquakes to stations, we sample some regions (with many earthquakes for example) more than others and the risk of getting a bias is larger. But we can also use the normal modes to analyze the heterogeneities of the Earth as we will see now.
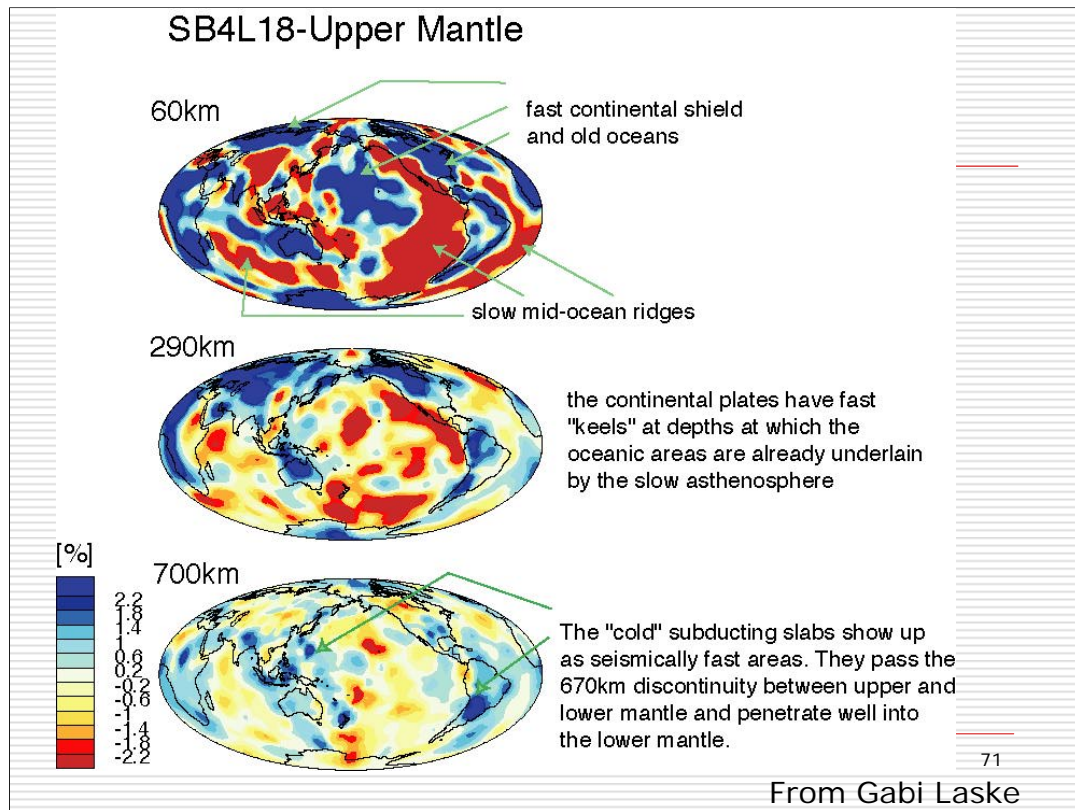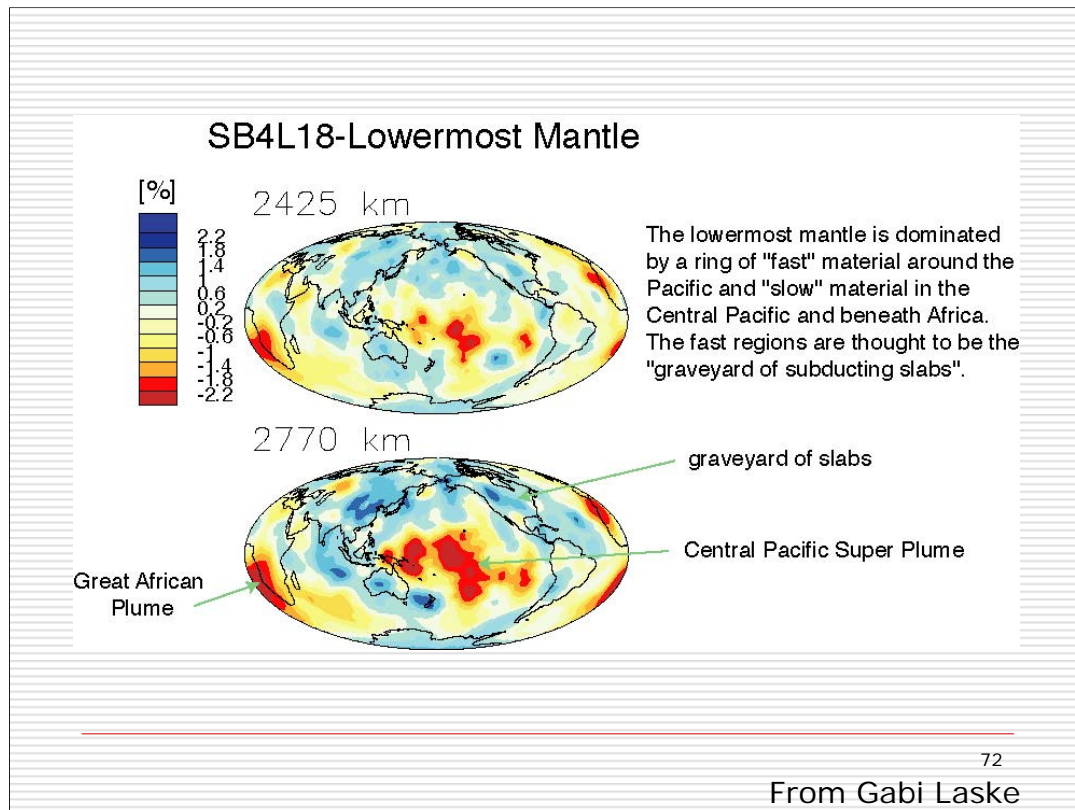
Splitting of the frequencies
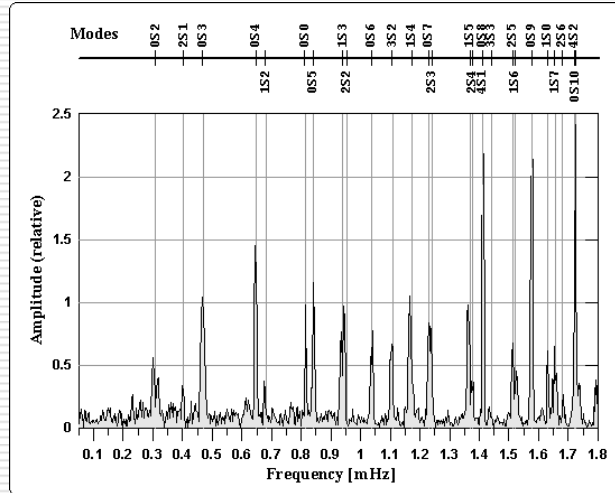
If the Earth was laterally homogenous, each mode would have a single frequency. You may see here that for some modes you have two peaks or more, for example for 0S2. This is due to the lateral heterogeneity inside the Earth, which results basically in the fact that the modes do not oscillate at exactly the same frequencies at all locations.

From Resovsky 1997
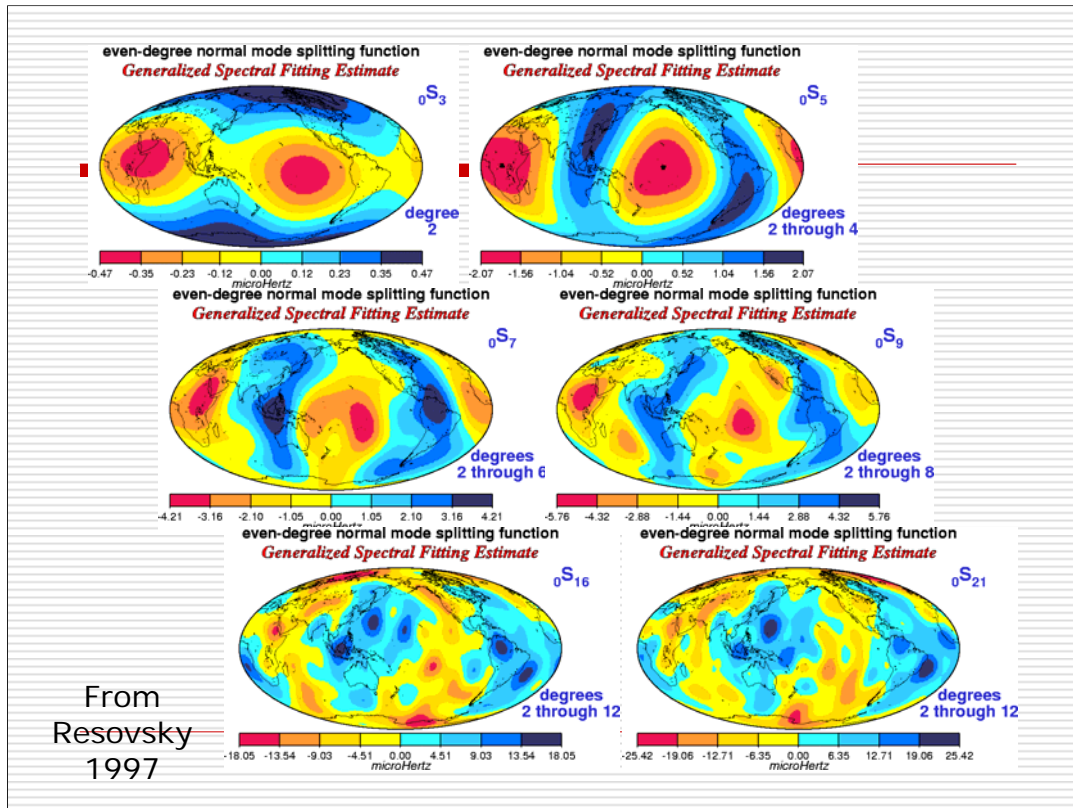
This is a map of the deviation from the central frequency for different modes. For example mode 0S3 oscillates at a slightly smaller frequency below the Pacific Ocean and Africa. This is because the wave velocity is slightly slower at these places, and it takes slightly more time for the vibration to travel at that place. The higher the order of the mode, the more detailed the maps get, because the waves get higher frequencies and have therefore a better resolution. But you see basically that they all have lower and higher frequencies in the same regions. These correspond to regions of the Earth with faster or slower material, corresponding to regions which are warmer or colder.

SB4L18-Upper Mantle

60km — fast continental shield
and old oceans

slow mid-ocean ridges

290km — the continental plates have fast
"keels" at depths at which the
oceanic areas are already underlain
by the slow asthenosphere

[%]

700km — The "cold" subducting slabs show up
as seismically fast areas. They pass the
670km discontinuity between upper and
lower mantle and penetrate well into
the lower mantle.

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From Gabi Laske

I don't have time here to show you more in details how the imaging is done, from the spectrum of individual seismograms to the map of the frequency shifts, and then to the imaging of the structure of the Earth with depth. But I can say that this is done iteratively by taking an approximate known structure for the Earth and calculating, by least-squares, which perturbations are the best ones to explain the observed data. Many different people do this kind of imaging and we now have a good idea of the structure of the Earth, even if the different studies do no agree yet on the details. What is clear is that we have most heterogeneities in the upper 700km and in the 300kn above the core-mantle boundary. We see that the continents are usually faster than oceanic regions, that the oceanic regions get faster when they get older (because they cool). We see fast anomalies around the Pacific Ocean, related to the cold subduction slabs.

SB4L18-Lowermost Mantle

[%]

2425 km

The lowermost mantle is dominated
by a ring of "fast" material around the
Pacific and "slow" material in the
Central Pacific and beneath Africa.
The fast regions are thought to be the
"graveyard of subducting slabs".

2770 km

graveyard of slabs

Central Pacific Super Plume

Great African
Plume

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From Gabi Laske

At the core-mantle boundary, we see the remnants of ancient subductions and
we see two regions of slow material below Africa and below Central Pacific.
There is no completely accepted explanation for these slow anomalies.

Plate tectonics ⟺ Mantle Convection
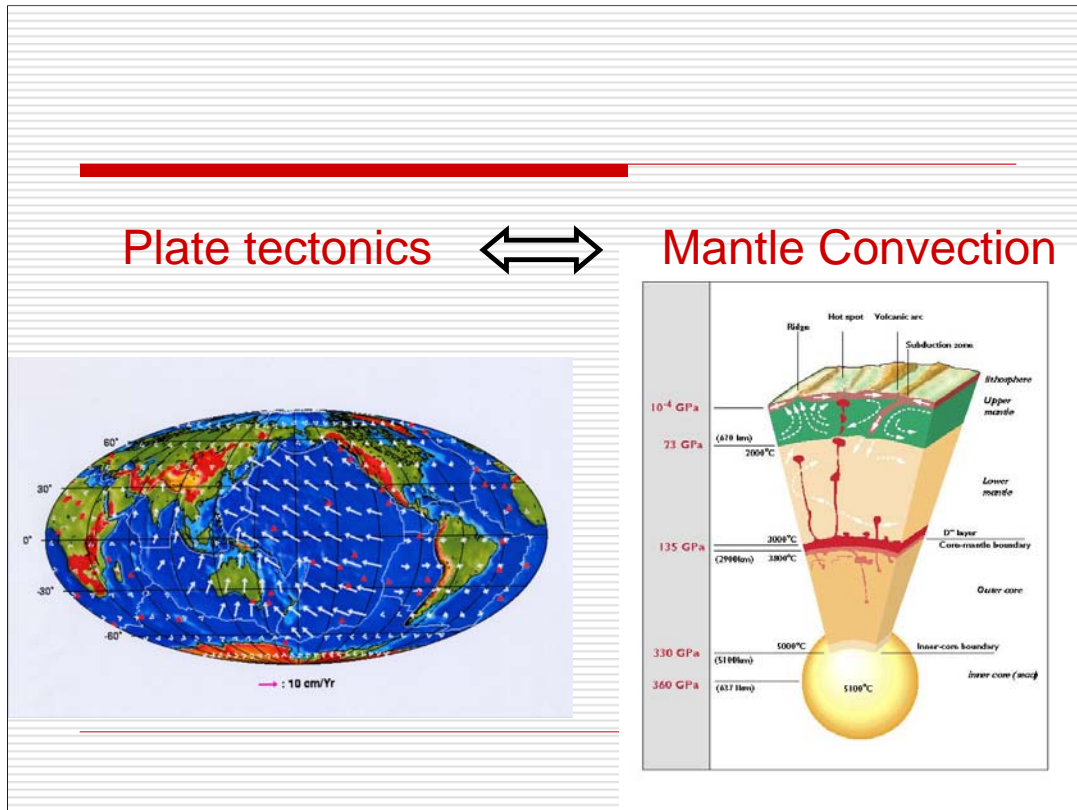
All this information is used to make models of how the Earth is working, what is
the driving force of plate tectonics, volcanism, hotspots etc..

- What is an earthquake?
- Imaging: early studies and the mean structure of the Earth
- P-wave tomography
- Normal mode tomography
- Source location and time-reversal

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In this last part, we are going to look at another aspect of imaging in seismology, which is the location and the study of earthquakes.

FIGURE 10.18
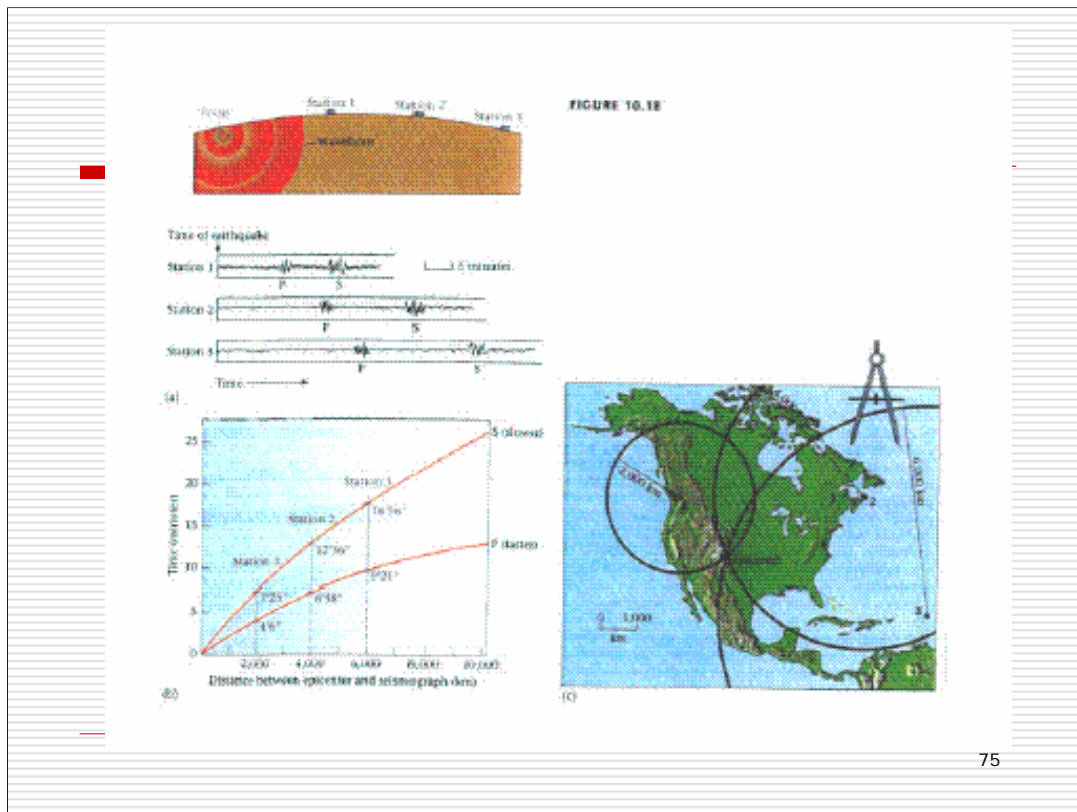
The basic principle behind the location of earthquakes is that the difference in arrival time between the first P wave and the first S-wave is dependent on the distance to the earthquake, but not on the origin time of the earthquake. By measuring this time difference, we can therefore determine for each station a distance to the earthquake, that is a circle at the surface of the Earth where the earthquake must be located. If we have at least three stations, we have at least three such circles. The point where they cross is the location of the earthquake.

station B

5600 km

1500 km
station A

Epicenter

8700 km
station C

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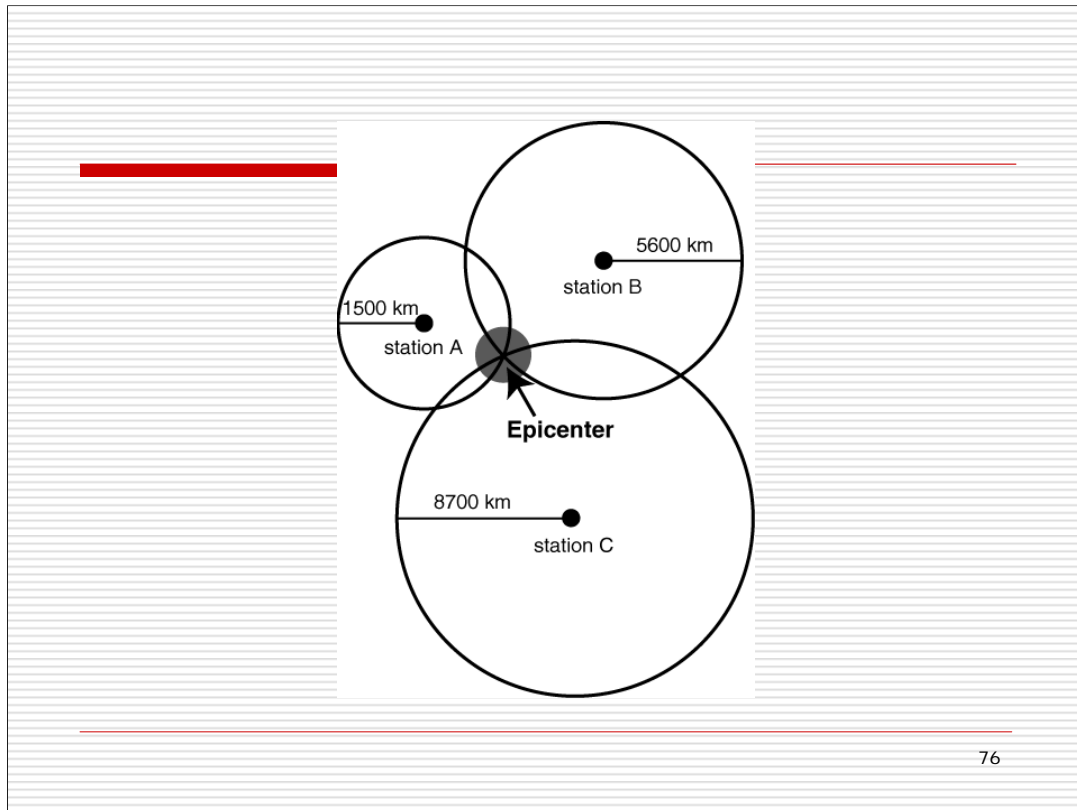
When you have the distance from each station, you can easily estimate the location of the earthquake

We can read the arrival time of the P wave t_p.

If we knew the origin time of the earthquake t_0, we could write:

$$t_p = t_0 + d / V_p$$

which implies for the distance:

$$d = V_p * (t_p - t_0)$$

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Mathematically, and assuming that we are not far from the earthquake and can assume a flat Earth model and a constant velocity, this can be expressed as the following:

The arrival times of the P and S waves are:

$$t_p = t_0 + d / V_p$$

$$t_s = t_0 + d / V_s$$

which implies: $t_s - t_p = d / V_s - d / V_p$

$$= d (1/V_s - 1/V_p)$$

$$= d (V_p - V_s)/(V_s V_p)$$

This gives: $d = (t_s - t_p) V_s V_p / (V_p - V_s)$

or about $d = 8 (t_s - t_p)$ for d in km and t in s

and local earthquakes

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- This is the simple way you can use for local events. In practice we use more than three stations, make a first guess of the location and then revise the location and the origin time using global traveltime tables until we have a location which explains the data at the best from a least-squares sense.

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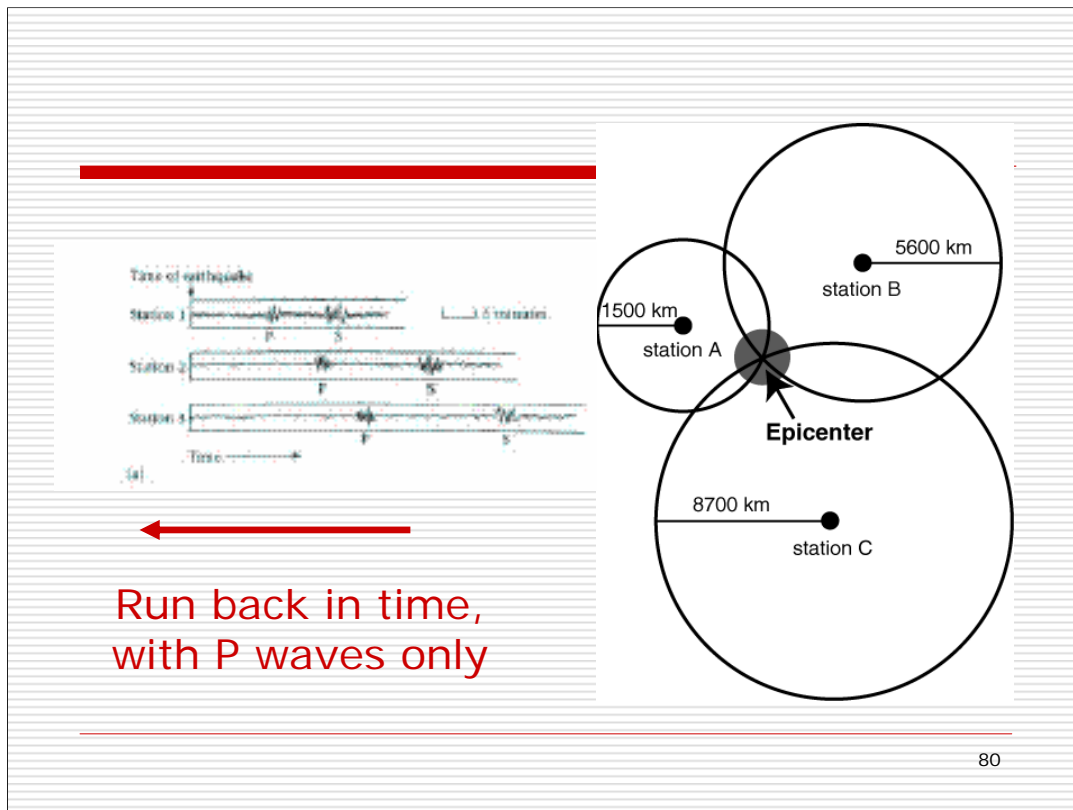
Another way to look at the location of the earthquake is by back-propagation. Assume that you have simple P wave arrivals at the three stations shown here. You run back the time and you back-propagate away from the station the energy you have registered at any station. At a given time, the three circles cross each other at the epicenter location, at origin time t0 of the earthquake. This is a procedure very similar to the migration procedure that we use in seismics to find interfaces where reflections come from: by back-propagating the wavefield until it makes a sharp image.
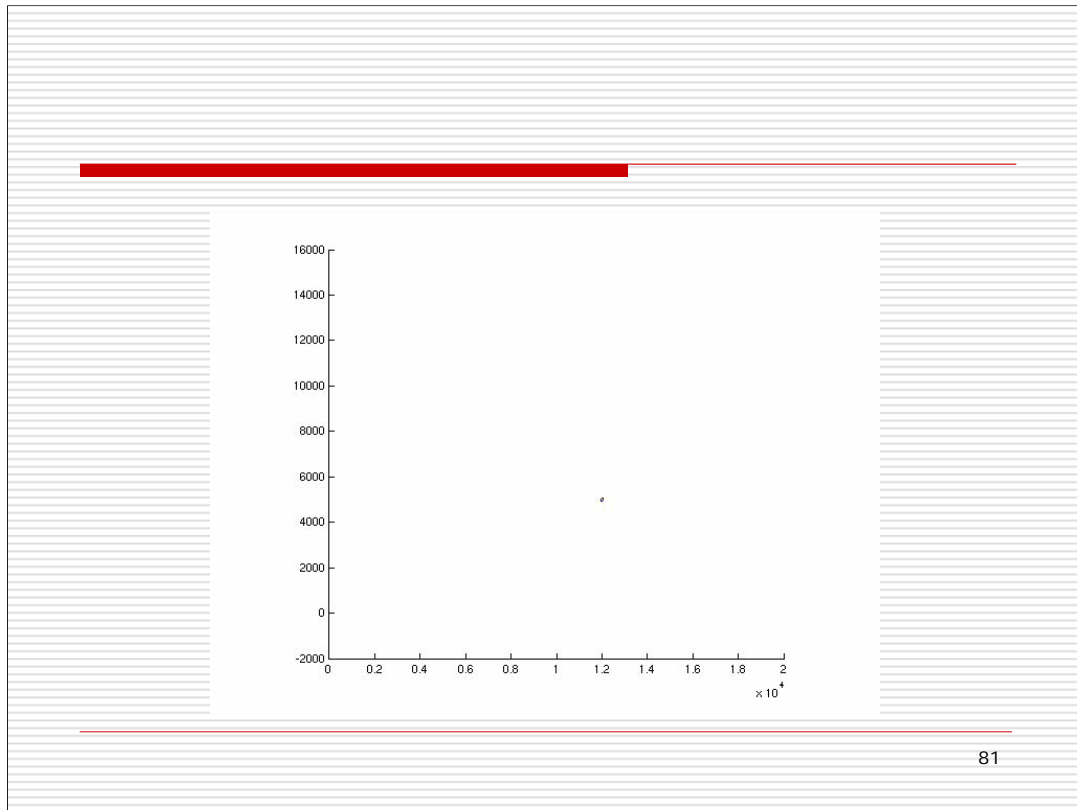
This is an animation showing the back-propagation of P waves from the three stations. You see that the three backpropagated waves cross each other at the location of the earthquake.
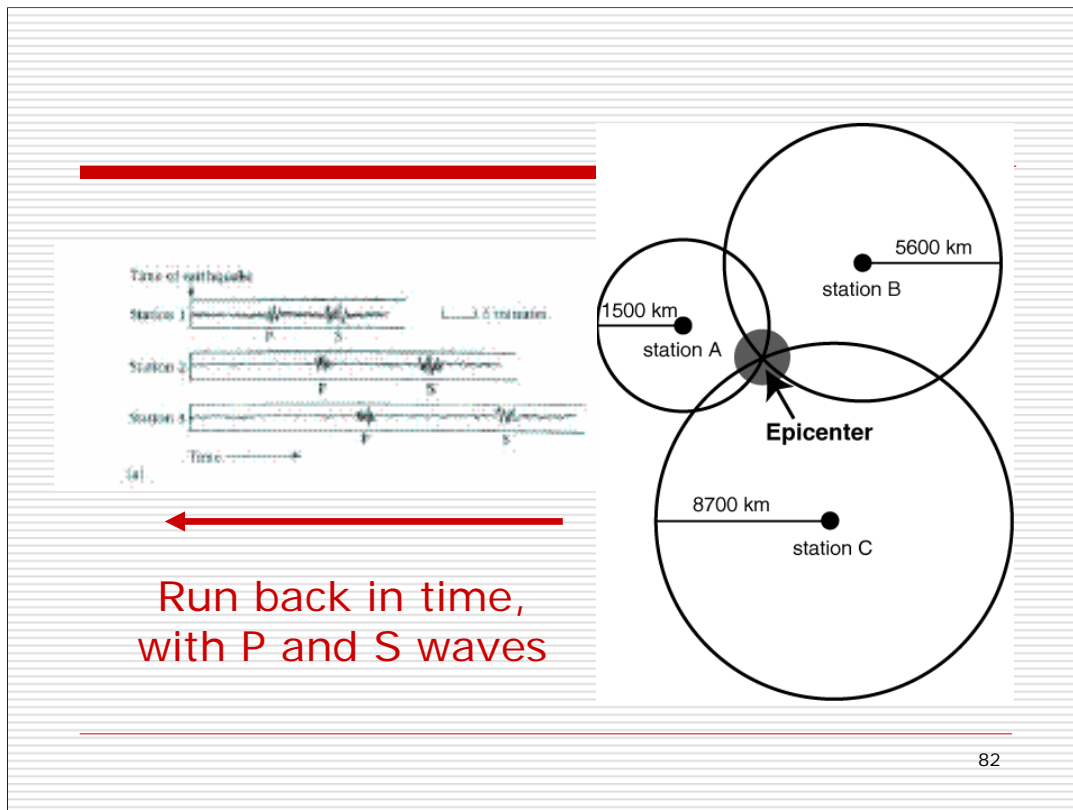
If you can identify the P and the S waves on the seismogram, you can also backpropagate the P waves with the P wave velocity and the S waves with the S wave velocity.
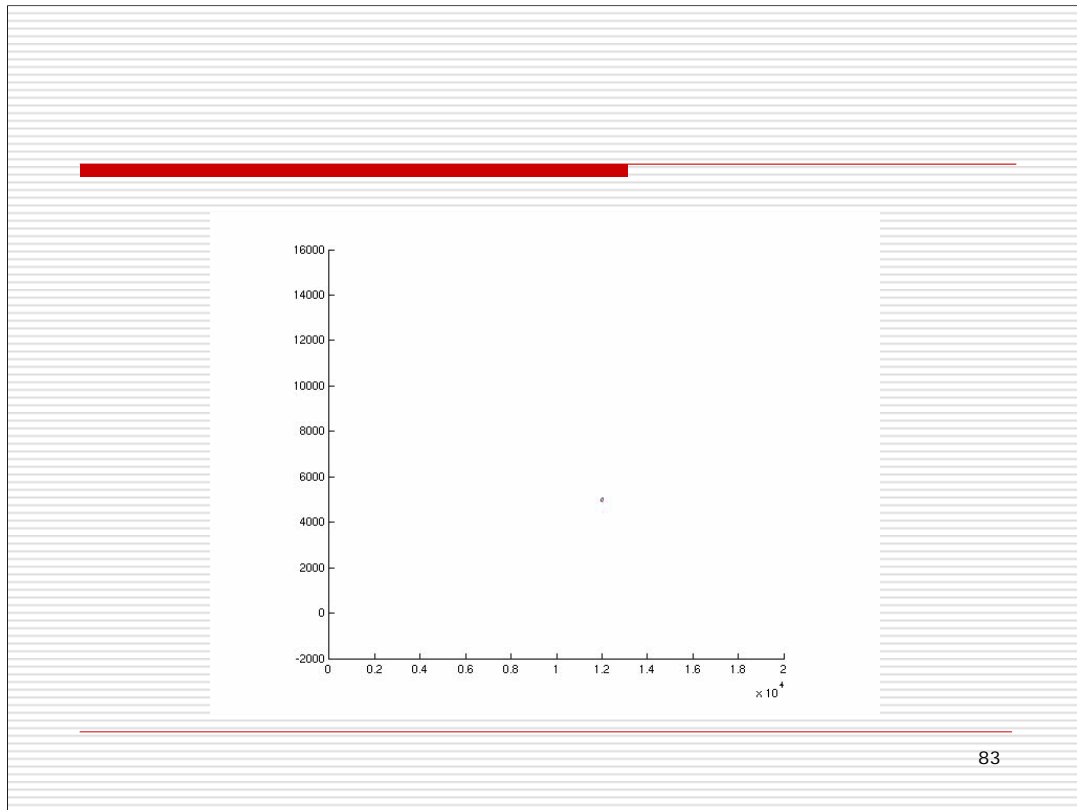
This is the same as before but with backpropagation of the P and S waves. The
S waves are in red and the P waves in blue. When you run back in time, first you
get the S waves which start to backpropagate. Then the P waves. You see that
all the wavefronts cross at the location of the earthquake. Errors in the velocity
model induce errors or uncertainties in the location of the earthquakes.
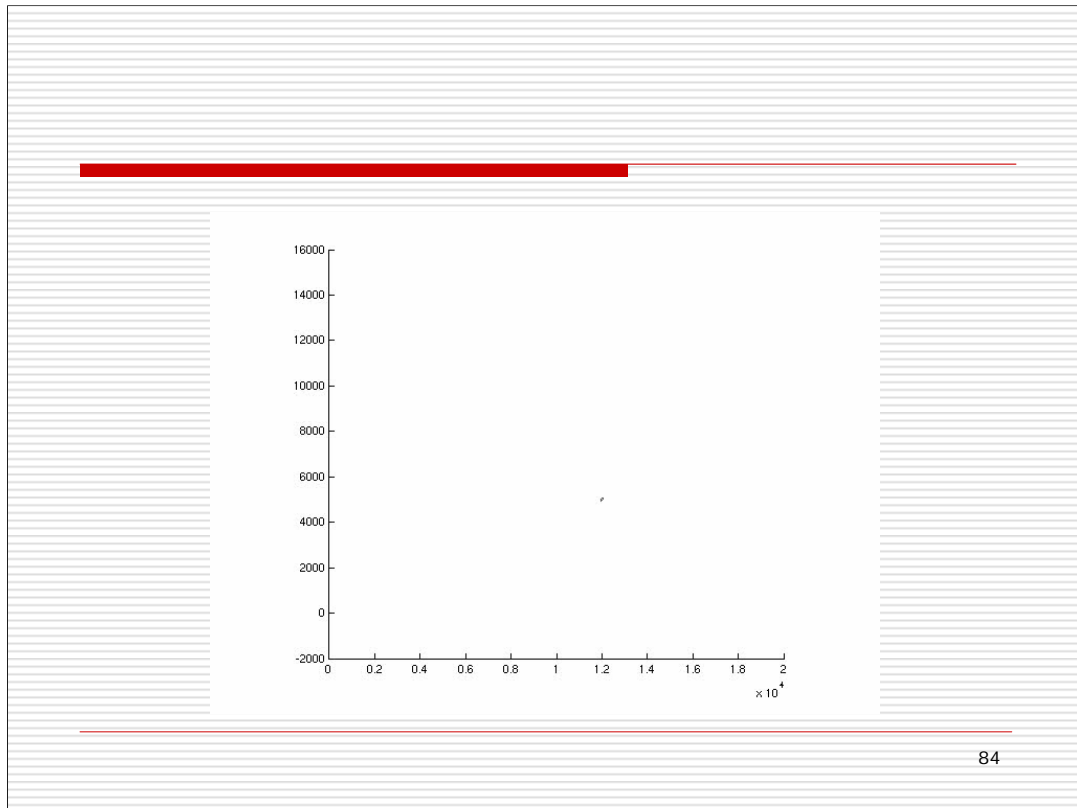
This is the same as before but I have used wrong velocities to backpropagate: I have used 5.5km/s instead of 5km/s for the P waves, and 2.5km/s instead of 3km/s for the S waves. You see that in this case the backpropagated wavefronts do not cross at a single point. In seismology, since both structure and source time and location are unknown, we should ideally image both at the same time. In practice, it is difficult to do it. In the best case, one can iterate: guess a location, find the structure, then refine the location, then refine the structure etc. Or one can try to use data like traveltimes for pairs of waves which are not very sensitive to the source location when imaging the structure.
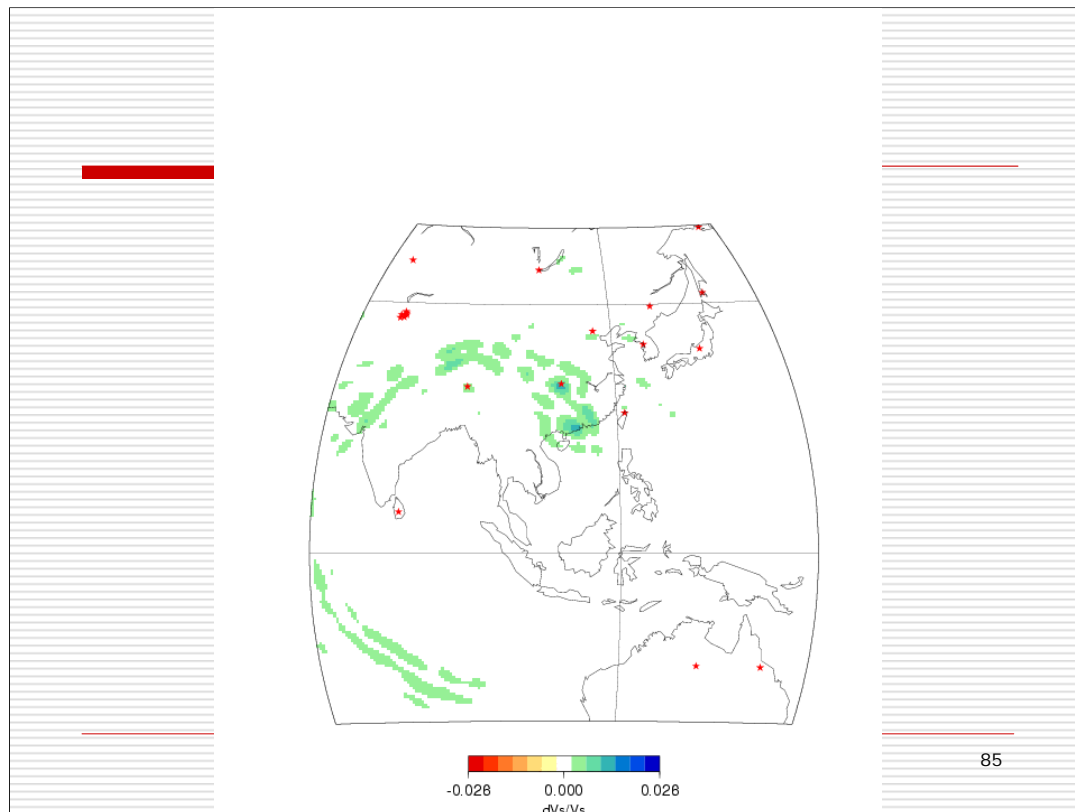
In the examples before, we have identified what are the P waves and what are
the S waves to backpropagate them with their respective velocities. In real
seismograms, some waves are a mixing of P and S waves and you cannot
separate that easily what is P and what is S waves. Still you can backpropagate
the full seismograms using the complete equation of elastic motion. The property
of reciprocity in the equation of motion ensures that the backpropagation
converges towards the source. This is an example of the backpropagation of the
recordings of the large Sumatra event of 2004, using seismograms filtered to
keep only periods larger than 150s. The stations are shown by red dots and the
wavefield at each station is used back in time as source of motion. You see that
the motion is converging towards the tip of Sumatra. This gives a correct location
for the Earthquake, but not a very sharp image because of the long periods
involved.
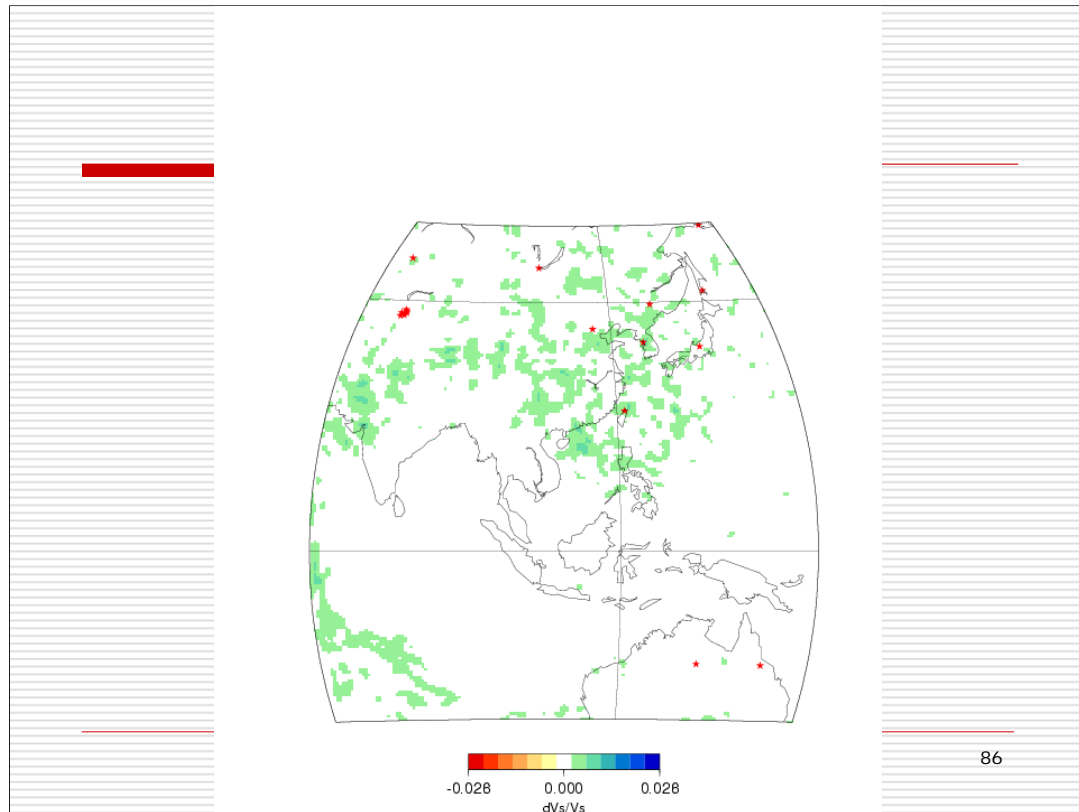
This is the same but including shorter periods, down to 60s. The focusing doesn't
work that well anymore because these waves are more sensitive to the
heterogeneities within the Earth, and that these heterogeneities are not known
precisely enough for the back-propagation to focus onto the earthquake.

Conclusion

- To give you an overview of imaging in seismology.
- Imaging of the structure and of the earthquakes
- Ray theory is a good approximation in many cases
- Waveform inversion gives more information
- Compared to other branches of imaging:
- → much is based on transmission
- → the sources are natural and we have no influence on where they are and when they occur
- → the number of stations is also limited
- → we have developed methods to use all the information contained in the seismograms.

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