INF-MAT 5360 Mathematical optimization

Compulsory Project 1
Completed by October 16, 2011

The compulsory project shall be made and handed in individually (paper version or as a single pdf file). You may discuss the problems with fellow students, but copying other students answers is not permitted; see the general rules for compulsory projects at Ifi. To pass you should solve the following exercises.

1. Let $D = (V, E)$ be a directed graph, with integer lower capacities $l \in Z_+^E$ and integer upper capacities $c \in Z_+^E$. Let $b \in Z^V$ be an integer divergence vector satisfying $\sum_{v \in V} b_v = 0$ and let $w \in \mathbb{R}^E$. Consider the min-cost flow problem

$$\begin{array}{lll} \min & \sum_{uv \in E} w_{uv} x_{uv} & \\ s.t. & & \\ (i) & \sum_{uv \in \in \delta^+(v)} x_{uv} - \sum_{vu \in \in \delta^-(v)} x_{vu} = b_v & (v \in V) \\ (ii) & l_{uv} \leq x_{uv} \leq c_{uv} & (uv \in E) \end{array} \tag{1}$$

   Let P denote the polyhedron of the feasible solutions to the above linear program. Show that every vertex of P is integer. (*Hint: use the fact that $x \in P$ is a vertex of P if and only if there do not exist two distinct points $x^1, x^2 \in P$, such that x can be obtained as a convex combination of x^1 and x^2*).

2. An IP network can be represented by a directed graph $G = (V, A)$: routers correspond to vertices and a link from router u to router v is the directed edge $(u, v) \in A$. In the OSPF protocol packets are sent from a router s to a router t by following the shortest path from s to t in the IP network. In addition, each link add a delay to the transmission, and the sum of the delays introduced by the links of a path cannot exceed a given threshold. I will provide an instance of this problem (which is called *constrained shortest path problem.* The student has to solve the constrained shortest path problem for such instance, possibly by using OPL studio (but any other means is allowed).

   At the course web-page you find an instance $G = (V, A)$ of the problem with 10 vertices (numbered from 0 to 9) and 30 arcs, where s and t are vertex 0 and vertex 9, respectively, and the **maximum time delay is 10**. You may choose your favorite format between two. The first (called *Graph Compact*) is a compact representation of the graph. The first row contains the the number of vertices and the number of arcs of the graph. Next rows are in one-to-one

correspondence with the arcs. Each row contains, for each arc, and in this order, the tail vertex, the head vertex, the arc delay and the arc length.

The other format is an extended one and contains three matrices. The first is the *tail-head* incidence matrix of the graph, a 0,1 square matrix M of order $|V|$, where $M_{ij} = 1$ if and only if $(i, j) \in A$. Associated with M you have the delay matrix D (where D_{ij} is the delay introduce by arc (i, j)) and the length matrix L (where L_{ij} is the length of the arc (i, j)).

Good Luck.