INF4171 / INF3170
Exercises, week 36

Exercise 1

Prove that all the rules of LK preserve falsifiability.

Exercise 2

An α-rule preserves falsifiability if the assumption sequent is falsifiable whenever the conclusion sequent is. An α-rule preserves validity if the conclusion sequent is valid whenever the assumption sequent is.

Use the results of exercise 1 to prove that all the α-rules of LK preserves validity.

Exercise 3

Generalize the notion of preservation of validity to β-rules (using the general statement of preservation of falsifiability) and prove that all LK-rules preserve validity.

Exercise 4

Define the semantics of addition and multiplication modulo 3. For example, $(2 \cdot_{\mod 3} 2) = 1$ and $(2 +_{\mod 3} 1) = 0$. Use the notation of sequent calculus to create a formal system for evaluating expressions this basic modulo 3 arithmetic.

Try to add rules that allow the removal of several others.

Note: Although this system share some notation with sequent calculus, it is quite different and the comparisons should not be taken too far.

Exercise 4: Hints and pointers

For an expression containing $0 +_{\text{mod } 3} 1$ as a subexpression, write $\text{Exp}_i(0 +_{\text{mod } 3} 1)$. One reasonable rule would then be

$$\frac{\text{Exp}_1(0)}{\text{Exp}_1(1 +_{\text{mod } 3} 2)}$$

In an application of the rule, Exp_1 would be written out as a normal arithmetic expression.

Norwegian translations

Completeness Kompletthet[1]

Contradiction Selvmotsigelse

Counter model Motmodell

Falsifiable Falsifiserbar

Propositional formula Utsagnslogisk formel

Propositional logic Utsagnslogikk

Satisfiable Oppfyllbar

Sequent Sekvent

Sequent calculus Sekventkalkyle

Soundness Sunnhet

Tautology Tautologi

Valid Gyldig

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[1]Here, completeness is translated to "kompletthet". In this setting, completeness refers to a calculus' ability to proove every valid formula. Where completeness refers to every sentence being either a tautology or a contradiction, completeness is translated "fullstendighet".