

INF3480 - spring 2015

Compulsory assignment 2

Deadline: Friday, April 17th 2015 (before midnight)

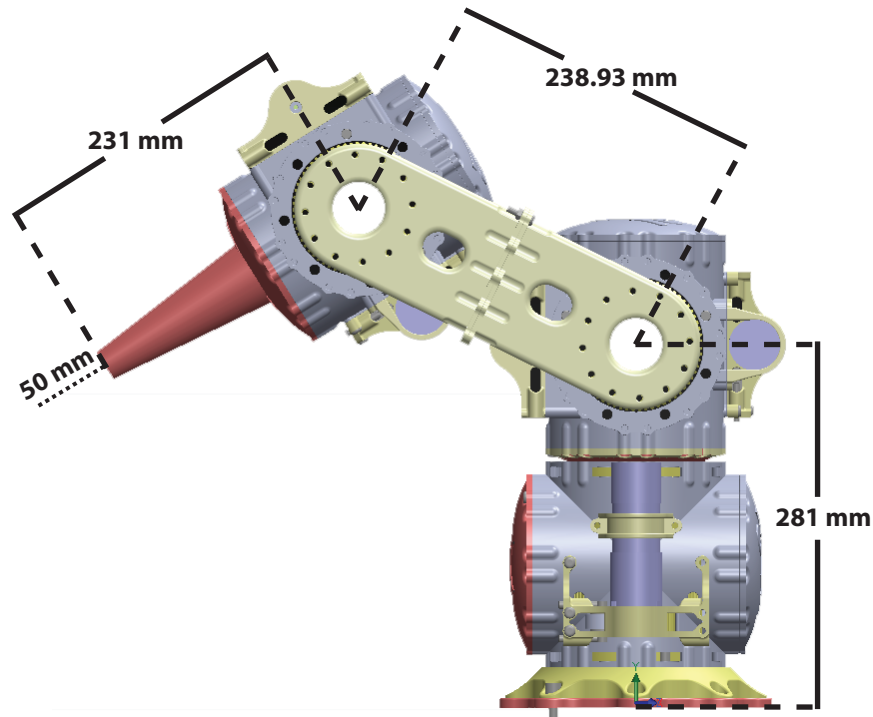


Figure 1: The X2 robot with measurements

Introduction

In this exercise we will keep working on the X2 robot, displayed in Figure ???. As was explained in the previous exercise: “The robot consists of three identical parts, the first one is attached to the surface beneath the robot, and does not rotate. The next part is attached on top of the first part, and rotates along the vertical axis. The second and third parts are connected with a link. In each end of this link, there is a revolute joint. A pen will be attached to the tip of the outermost part.”

Figure ??? shows the necessary measurements to work with the robot. The dotted line at the end of the robot denotes the pen that will be attached there.

1 Forward and inverse kinematics

Implement the forward kinematics and the inverse kinematics as Matlab functions.

(You derived the equations in compulsory exercise 1. If for some reason you do not have the correct equations, you may ask Lars at the group seminar.)

- a) The forward kinematics function takes 3 joint angles as input, and gives the corresponding cartesian coordinates for the tip of the pen as output. The matlab function shall look like this:

```
function cart_cord = forward(joint_angles)
```

where both `cart_cord` and `joint_angles` are column vectors of size 3.

- b) The inverse kinematics function takes the position of the tip of the pen as input, and gives the corresponding joint configuration(s) as output. The matlab function shall look like this:

```
function joint_angles = inverse(cart_cord)
```

where both `cart_cord` and `joint_angles` are column vectors of size 3.

- c) Use the matlab functions to show how you can verify that the inverse and forward kinematics are correctly derived.

2 Jacobian

- a) Derive the Jacobian for the X2 robot.
- b) Use the Jacobian matrix to find the singular configurations of the robot.
- c) Give an evaluation of the results from exercise 2b, based on the practical understanding of the anatomy of the robot (in other words: Did your results in 2b match what you expected to be singular configurations for the robot? Why / Why not?).

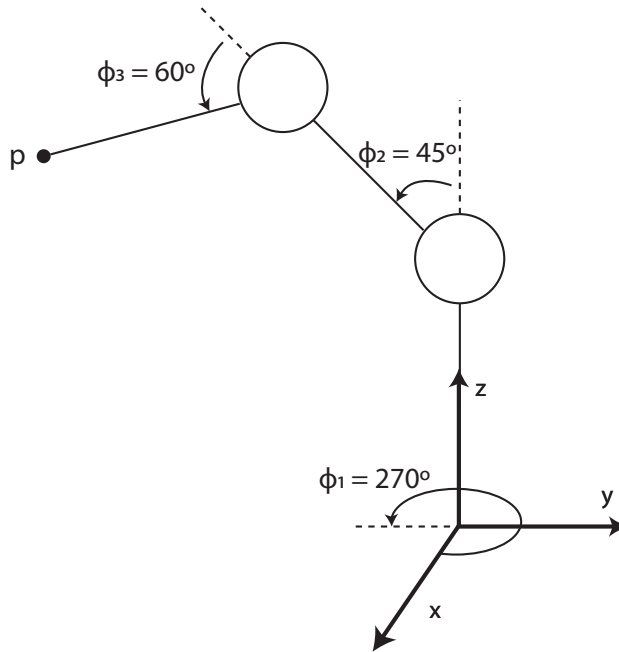


Figure 2: Robot configuration for 3b)

3 Jacobian

- a) Implement the Jacobian matrix as a matlab function. This function takes the instant joint angles and joint velocities as input, and gives a 3-dimensional vector of cartesian velocities of the tip of the pen as output. The matlab function shall look like this
- ```
function cart_velocities = jacobian(joint_angles, joint_velocities)
```
- where both `joint_angles`, `joint_velocities` and `cart_velocities` are column vectors of size 3.

- b) Point  $p$  is located at the end-effector of the robot (the tip of the pen). We adjust the robot as displayed in figure ??, where  $\phi_1 = 270^\circ$ ,  $\phi_2 = 45^\circ$ ,  $\phi_3 = 60^\circ$ . (These angles are not to be used directly, you have to figure out the correct  $\theta$ -angles corresponding to your placement of the joint coordinate frames yourself.)

Given the configuration in figure ??, and the joint speed vector  $\dot{q} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]$ , where  $\dot{\theta}_1 = 0.1 \text{ rad/s}$  and  $\dot{\theta}_2 = \dot{\theta}_3 = 0.05 \text{ rad/s}$ , use the matlab function for the Jacobian matrix to find the speed of the end-effector at point  $p$  (tip of the pen).

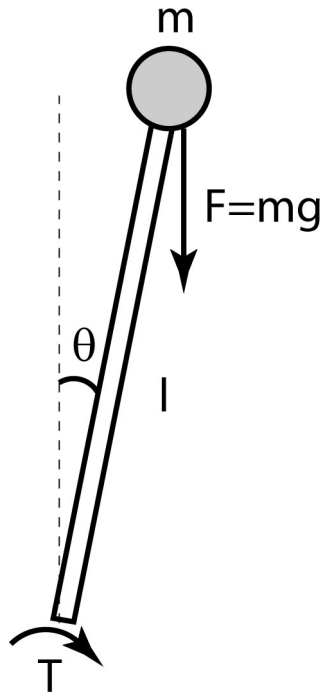


Figure 3: The inverted pendulum

## 4 Dynamics

In this exercise we will model joint 2 of the X2 robot as an inverted pendulum. The pendulum has a mass  $m$  which is located on top of the pendulum (see Figure ??). The distance from the rotation point to the mass is  $l$ . The moment of inertia  $I$  of the pendulum is  $ml^2$ . The motor in the joint provides a torque  $\tau_m$ .

- a) (optional) Derive the dynamics of the inverted pendulum using *Newton-Euler formulation*. It will be useful to use the fact that torque is defined as the cross product between the force and length of the arm, i.e.  $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ . In our case this simplifies to  $\tau = rF \sin \theta$ . The rotation counterpart to Newton's second law is  $\sum \tau_i = I\ddot{\theta}$ .
- b) Derive the dynamics of the inverted pendulum using *Euler-Lagrange Equations*. Generally the kinetic energy is  $\mathcal{K} = \frac{1}{2}mv^2$  and the potential energy is  $\mathcal{P} = mgh$ . Set zero potential energy to the rotation point.
- c) (optional) Are the two models equal? Why or why not?

**Requirements:**

Each student must hand in their own assignment, and you are required to have read the following requirements to student submissions at the department of informatics: <http://www.mn.uio.no/ifi/english/studies/admin/mandatory-assignments/assignments-guidelines.html>

Submit your assignment on <https://devilry.ifi.uio.no/>  
Your submission must include:

- A pdf-document with answers to the questions.
- The matlab source code

Remember to put your name in all your files.

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