

SOLUTION SKETCH 2014

QUERY PROCESSING

(a) The order is not guaranteed to be optimal. Consider three terms with postings list sizes $s_1 = 100$, $s_2 = 105$ and $s_3 = 110$. Suppose the intersection of s_1 and s_2 has length 100 and the intersection of s_1 and s_3 length 0. The ordering s_1, s_2, s_3 requires $100+105+100+110=315$ steps through the postings lists. The ordering s_1, s_3, s_2 requires $100+110+0+0=210$ steps through the postings lists.

(b) (i) Applying the intersection algorithm on the standard postings list, comparisons will be made unless either of the postings list end, i.e., till we reach 47 in the upper postings list, after which the lower list ends and no more processing needs to be done. Number of comparisons = 11.

(b) (ii) Using skip pointers of length 4 for the longer list and of length 1 for the shorter list, the following comparisons will be made:

1. 4 & 47
2. 14 & 47
3. 22 & 47
4. 120 & 47
5. 81 & 47
6. 47 & 47

Number of comparisons = 6

HEAPS' LAW

(a) Heaps' law is a purely empirical law where:

M = the size of the vocabulary

k, b = empirically determined constants

T = the number of tokens in the collection

(b) In log-log space we have:

$$\log(M) = \log(k) + b \cdot \log(T)$$

We have two data points in log-log space we can use to estimate b :

$$\begin{aligned} b &= (\log(3 \cdot 10^4) - \log(3 \cdot 10^3)) / (\log(10^6) - \log(10^4)) \\ &= (\log 3 + 4 - \log 3 - 3) / (6 - 4) \\ &= 1 / 2 \\ &= 0.5 \end{aligned}$$

Since $T^{0.5} = \sqrt{T}$ we have:

$$\begin{aligned}
k &= M/\sqrt{T} \\
&= 3000/\sqrt{10000} \\
&= 30000/\sqrt{1000000} \\
&= 30
\end{aligned}$$

So, $M = 30 \cdot \sqrt{T}$.

In our example, we have $T = 2 \cdot 10^{10} \cdot 200$ tokens, so:

$$\begin{aligned}
M &= 30 \cdot \sqrt{2 \cdot 10^{10} \cdot 2 \cdot 10^2} \\
&= 30 \cdot \sqrt{4 \cdot 10^{12}} \\
&= 30 \cdot 2 \cdot 10^6 \\
&= 60 \cdot 10^6 \\
&= 60000000 \\
&= 60 \text{ million}
\end{aligned}$$

LOOKUP FUN

(a) sting -> sting\$, ting\$s, ing\$st, ng\$sti, g\$stin, \$sting. Lookup on key ng\$s\$, i.e., a prefix lookup.

(b) Suffix array (using a 0-based indexing scheme):

10: i
 7: ippi
 4: issippi
 1: ississppi
 0: mississippi
 9: pi
 8: ppi
 6: sippi
 3: sissippi
 5: ssippi
 2: ssissippi

Binary search on the prefix is*, then a sequential scan (if match). Can exploit precomputed LCP values.

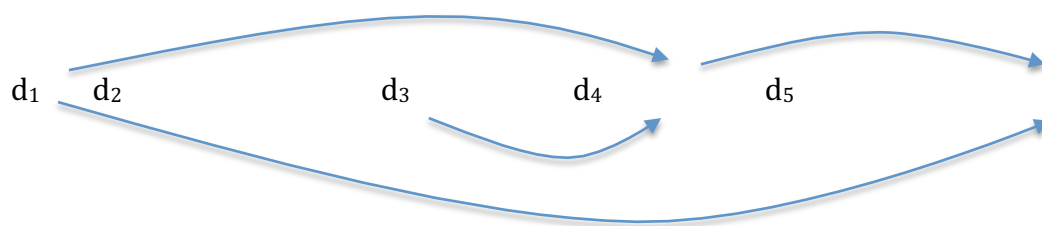
COSINE SCORES, PAGERANK AND CLASSIFICATION

(a) Vectors and inner product score:

	$w_{t,q}$	$w_{t,d1}$	$w_{t,d2}$	$w_{t,d3}$	$w_{t,d4}$	$w_{t,d5}$
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speech	$\frac{1}{4}$	2	0	1	1	2
dialogue	$\frac{1}{4}$	2	1	1	1	0
system	$\frac{1}{4}$	2	0	2	1	1
Score:		1.5	0.25	1	0.75	0.75

(b) Links between documents:



Transition matrix:

P =

0.1	0.1	0.35	0.1	0.35
0.1	0.1	0.6	0.1	0.1
0.1	0.1	0.1	0.1	0.6
0.2	0.2	0.2	0.2	0.2
0.2	0.2	0.2	0.2	0.2

Assuming we start at document 1, the first PageRank estimate is:

$$[1 \ 0 \ 0 \ 0 \ 0] P = [0.1 \ 0.1 \ 0.35 \ 0.1 \ 0.35]$$

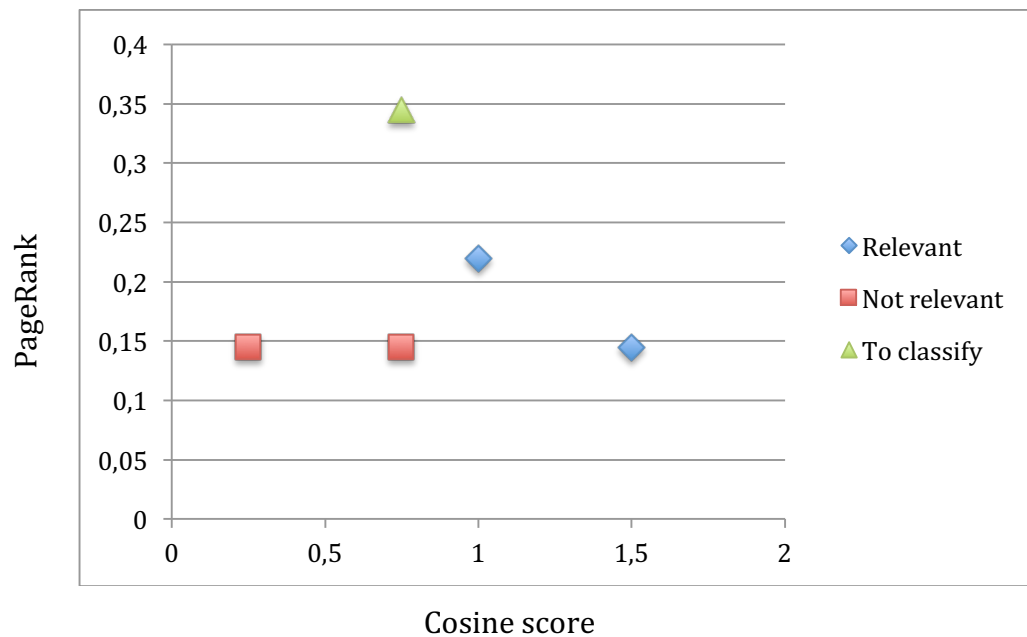
And a second iteration yields:

$$[0.1 \ 0.1 \ 0.35 \ 0.1 \ 0.35] P = [0.145 \ 0.145 \ 0.22 \ 0.145 \ 0.345]$$

(c) The five vectors are:

d1	1.5	0.145
d3	1	0.22
d2	0.25	0.145

d4	0.75	0.145
d5	0.75	0.345



(d) (i) Rocchio classifier:

class	centroid	distance to d5
relevant	[1.25, 0.1825]	0.6625
not-relevant	[0.5, 0.145]	0.45

→ document d₅ classified as not relevant

(d) (ii) 3-NN classifier:

The three closest documents are d₂, d₃ and d₄, which means that d₅ is classified as non-relevant.

(d) (iii) Linear SVM classifier:

classification function: $y = \text{sign}(w \cdot x + b)$

where $w = \sum \alpha_i y_i x_i \rightarrow$ in this case $[20 \ 4.4]^T - [15 \ 2.9]^T = [5 \ 1.5]^T$

for d₅, we have $y = \text{sign}(5 \cdot 0.75 + 1.5 \cdot 0.145 - 4.65) = -1 \rightarrow$ not relevant

(e) Examples of features that can be useful in relevance judgments are, e.g., query-term proximity, document age, document length, etc.