Bandgap References and Discrete Time Signals (chapter 8 + 9)

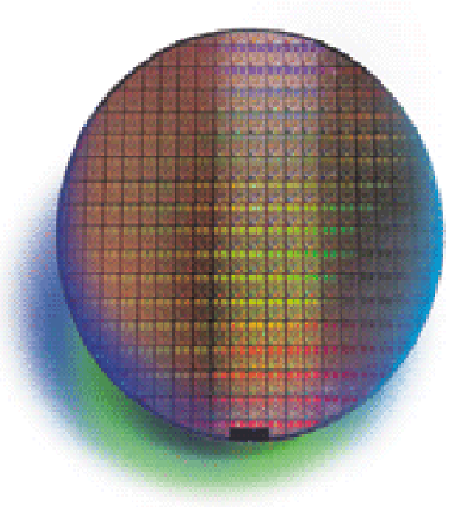
Tuesday 9th of February, 2010

Snorre Aunet, sa@ifi.uio.no
Nanoelectronics Group, Dept. of Informatics
Office 3432

UNIVERSITETET I OSLO

Last time – Tuesday 2nd of February, and today, February the 9th:

- 8.1 performance of Sample-and-Hold Circuits
- 8.2 MOS Sample-and-Hold circuits
- 8.3 Examples of CMOS S/H circuits
- 8.5 Bandgap Voltage Reference Basics
- 8.6 Circuits for Bandgap References
- Chapter 9 Discrete-Time Signals
- 9.1 Overview of some signal spectra
- 9.2 Laplace Transforms of Discrete-Time Signals
- 9.3 Z-transform



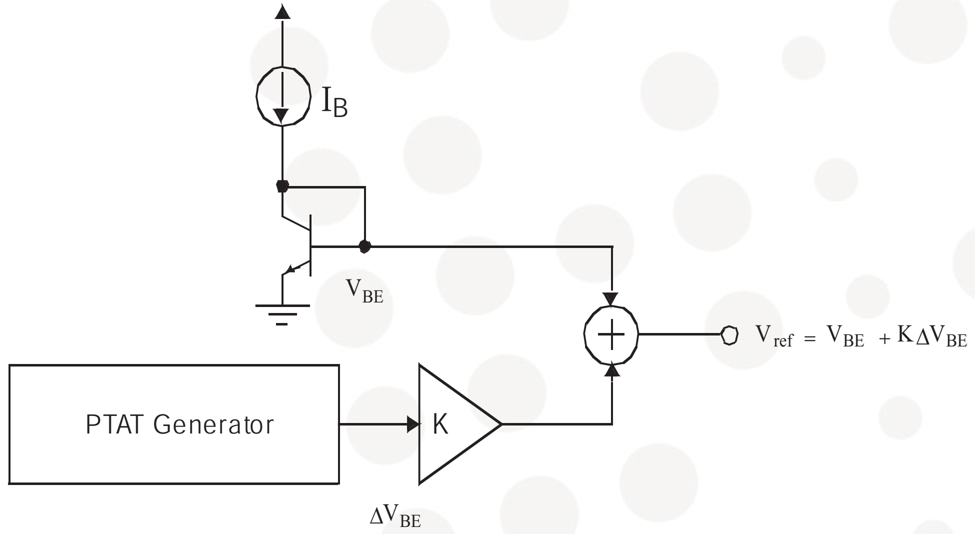
Voltage references (chapter 8.5)

- Purpose:
  - Generate a constant on-chip voltage which is independent of temperature, supply voltage, aging etc.
- Different approaches:
  - 1) Breakdown voltage of a reverse-biased zener diode
    - Too high voltage for CMOS
  - 2) Threshold voltage difference between CMOS enhancement and depletion transistors
    - Depletion-mode transistors unavailable in most CMOS processes
  - 3) Bandgap references: Canceling the negative temperature dependence of a forward-biased pn-junction (CTAT) with a positive temperature dependence (PTAT) (proportional-to-absolute-temperature) circuit
    - Most commonly used
    - CTAT: Conversely proportional to temperature
    - PTAT: Proportional to temperature

More about todays Bandgap Reference agenda (including, but not limited to):

- About the fundamental equations giving the relationship between the output voltage of a bandgap reference and temperature.
- How to design a bandgap reference for a ”most stable” reference voltage at a particular temperature.
- How to estimate temperature dependence at another temperature that the BG reference was designed for.
- Practical implementations

Basic principle



- The voltage V_{BE} is CTAT
- The voltage ΔV_{BE} is PTAT
- ΔV_{BE} is scaled by K to get the same slope as V_{BE}
- By adding V_{BE} and K ΔV_{BE}, the output V_{ref} becomes independent of temperature

Bandgap reference example

A High Precision Curvature Compensated Bandgap Reference without Resistors

Jianghua Chen*, Xuewen Ni, Bangxian Mo and Zhanfei Wang
Institute of Microelectronics, Peking University, Beijing 100871, P. R. China
* Email: chenjianghua@ime.pku.edu.cn

A High Precision Curvature Compensated Bandgap Reference with
Jianghua Chen; Xuewen Ni; Bangxian Mo; Zhanfei Wang;
Solid-State and Integrated Circuit Technology, 2006. ICSICT '06. 8th Inte
23-26 Oct. 2006 Page(s):1748 - 1750
Digital Object Identifier 10.1109/ICSICT.2006.306414
AbstractPlus | Full Text: PDF(124 KB) IEEE CNF
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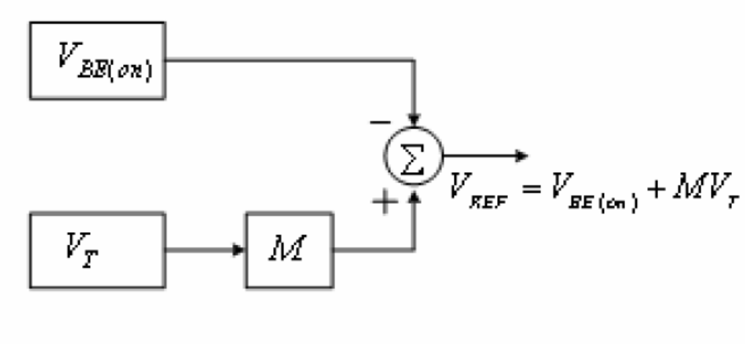


Figure 1 General bandgap reference architecture.

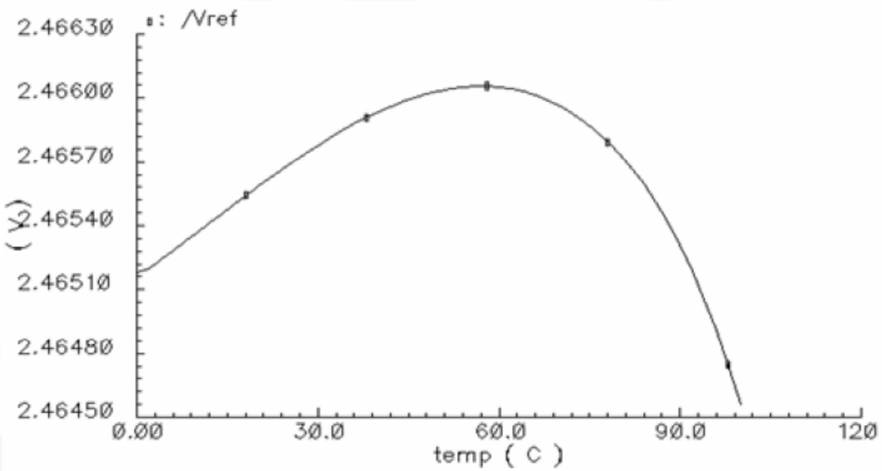


Figure 4 Output reference Verf vs. temperature.

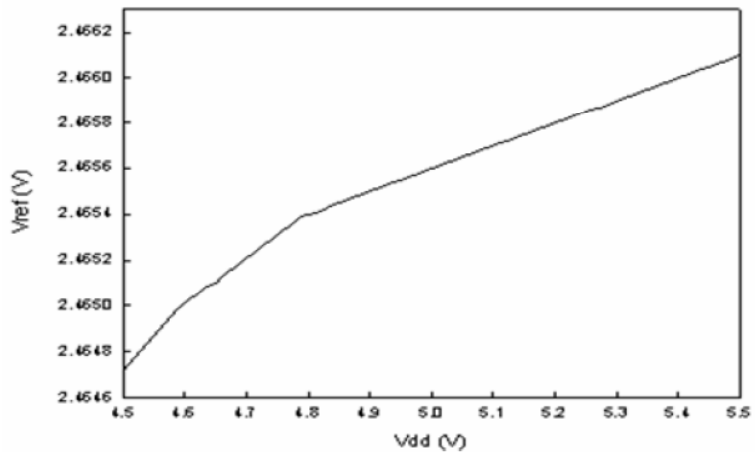


Figure 5 Output reference Vref vs. power supply Vdd.

Theory

- Collector current

$$I_C = I_s e^{V_{BE}/((kT)/q)}$$

- Solved with respect to V_{BE}:

$$V_{BE} = V_{G0}\left(1 - \frac{T}{T_0}\right) + V_{BE0}\frac{T}{T_0} + \frac{mkT}{q}\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\ln\left(\frac{J_C}{J_{C0}}\right)$$

- The junction current equals the effective area of the base-emitter junction times the junction current density, Jc:

-

$$I_C = A_E J_C$$

The difference between two base-emitter junctions biased at different densities (proportional to temperature):

$$\Delta V_{BE} = V_2 - V_1 = \frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)$$

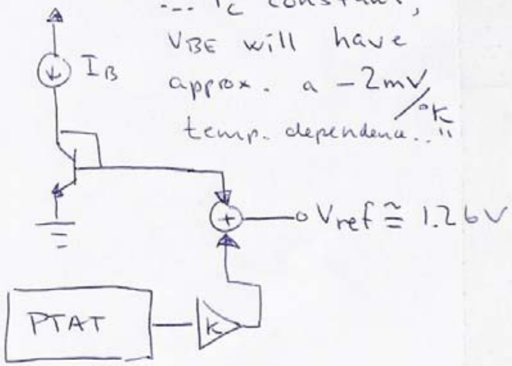
Example 8.3

Ex. 8.3 Assume two trans. biased at current-density ratio of 10:1 at 300 °K. What is the difference in their base-emitter voltages and what is its temperature dependence?

likn. 8.12 : $\underline{\Delta V_{BE}} = \frac{kT}{q} \ln\left(\frac{J_2}{J_1}\right)$ ← eq. 8.12 p. 354

$$= \frac{1.38 \cdot 10^{-23}\,(300)}{1.602 \cdot 10^{-19}} \ln(10) = \underline{\underline{59.5\ mV}}$$

page 354: "... I_C constant, V_{BE} will have approx. a $-2mV/^\circ K$ temp. dependence.."



$V_{ref} = V_{BE2} + K\,\Delta V_{BE}$

Since this voltage is proportional to absolute temperature, after a 1 °K temp. increase, the voltage difference will be $\Delta V_{BE} = 59.5\ mV \cdot \frac{301}{300} = 59.7mV$

Thus, the voltage dependence is $59.5\ mV/300\,^\circ K$ or $0.198\ mV/^\circ K$.

Since the temperature dependence of a single V_{BE} is $-2mV/^\circ K$, if it is desired to cancel the temp. dependence of a single V_{BE} then ΔV_{BE} should be amplified by about a factor of 10.

Theory

- Assuming that:

$$\frac{J_i}{J_{i0}} = \frac{T}{T_0}$$

- V_{ref} can then be written as:

$$V_{ref} = V_{BE2} + K\,\Delta V_{BE}$$
$$= V_{G0} + \frac{T}{T_0}(V_{BE0\text{-}2} - V_{G0}) + (m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right) + K\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)$$

- For a given temperature V_{ref} may be independent of changes in the temperature if a proper vaule of K is assigned
- This (equation 8.16) is the fundamental equation giving the relationship between the output voltage of a bandgap voltage reference and temperature.

From V_{BE} as a function of collector current and temperature to Vout for BG ref. (part 1 of 2)

Pp 354–355

$$V_{BE} = V_{G0}\left(1 - \frac{T}{T_0}\right) + V_{BE0}\frac{T}{T_0} + \frac{mkT}{q}\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\ln\left(\frac{J_C}{J_{C0}}\right) \quad (8.10)$$

$\Updownarrow$ (using $\frac{J_C}{J_{C0}} = \frac{T}{T_0}$)

$$V_{BE} = V_{G0} - V_{G0}\frac{T}{T_0} + V_{BE0}\frac{T}{T_0} + \frac{mkT}{q}\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\ln\left(\frac{T}{T_0}\right)$$

$\Updownarrow$

$$V_{BE} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + \frac{mkT}{q}\left[\ln T_0 - \ln T\right] + \frac{kT}{q}\left[\ln T - \ln T_0\right]$$

$\Updownarrow$

$$V_{BE} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + \frac{mkT}{q}\ln T_0 - \frac{mkT}{q}\ln T + \frac{kT}{q}\ln T - \frac{kT}{q}\ln T_0$$

$\Updownarrow$

$$V_{BE} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + \frac{mkT}{q}\ln T_0 - \frac{mkT}{q}\ln T - \left(\frac{kT}{q}\ln T_0 - \frac{kT}{q}\ln T\right)$$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b = -\ln b + \ln a = -(\ln b - \ln a)$

$\Updownarrow$

$$V_{BE} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + \frac{mkT}{q}\ln T_0 - \frac{mkT}{q}\ln T - \frac{kT}{q}\ln\frac{T_0}{T}$$

OBS! "Unneccesary rewriting from 2nd to 3rd eq.

$\Updownarrow$

$$V_{BE} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + \frac{mkT}{q}\ln\frac{T_0}{T} - \frac{kT}{q}\ln\frac{T_0}{T}$$

$\Updownarrow$

$$V_{BE} = V_{G0} + \frac{T}{T_0}\left(V_{BE0} - V_{G0}\right) + (m-1)\frac{kT}{q}\ln\frac{T_0}{T}$$

$\Longrightarrow$ SIMILAR TO ALL BUT LAST PART OF EQ 8.16.

From V_{BE} as a function of collector current and temperature to Vout for BG ref. (part 2 (of 2))

8.16 $\quad V_{ref} = V_{BE2} + K\,\Delta V_{BE}$

$$= V_{G0} + \frac{T}{T_0}\left(V_{BE0\text{-}2} - V_{G0}\right) + (m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right) \quad (8.16)$$

$$+ K\left[\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)\right] \quad (\text{using } 8.12)$$

This equation (8.16) is the fundamental equation giving the relationship between the output voltage of a bandgap reference and temperature.

Differentiating eq. 8.16 with respect to temperature, getting eq. 8.17

$$\left[V_{G0} + \frac{T}{T_0}\left(V_{BE0-2} - V_{G0}\right) + (m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right) + K\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)\right]'$$

$y = u + v$, $y' = u' + v'$
$y = uv$, $y' = u'v + uv'$
$y = \ln x$, $y' = \frac{1}{x}$

$$= \left(V_{G0}\right)' + \left(\frac{T}{T_0}\right)'\left(V_{BE0-2} - V_{G0}\right) + \frac{T}{T_0}\left(V_{BE0-2} - V_{G0}\right)' + (m-1)'\left[\frac{kT}{q}\ln\left(\frac{T_0}{T}\right)\right] + (m-1)\left[\frac{kT}{q}\ln\left(\frac{T_0}{T}\right)\right]'$$

$$+\left(K\frac{kT}{q}\right)'\ln\left(\frac{J_2}{J_1}\right) + K\frac{kT}{q}\left(\ln\left(\frac{J_2}{J_1}\right)\right)'$$

$$\frac{\partial V_{ref}}{\partial T} = \left(\frac{T}{T_0}\right)'\left(V_{BE0-2} - V_{G0}\right) + \underbrace{(m-1)\left[\frac{kT}{q}\ln\left(\frac{T_0}{T}\right)\right]'}_{A} + \underbrace{\left(K\frac{kT}{q}\right)'\cdot\ln\left(\frac{J_2}{J_1}\right)}_{B}$$

Noen mellomregninger:

$$\left(\frac{T}{T_0}\right)' = \left(T\cdot\frac{1}{T_0}\right)' = 1\cdot\frac{1}{T_0} + T\cdot 0 = \frac{1}{T_0}$$

$$y = \ln\frac{T_0}{T} = \ln T_0 - \ln T$$
$$y' = (\ln T_0)' - (\ln T)' = -\frac{1}{T}$$

$$\underline{A} = (m-1)\cdot\left(\frac{kT}{q}\right)'\cdot\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\left(\ln\frac{T_0}{T}\right)' = (m-1)\frac{k}{q}\cdot\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\left(-\frac{1}{T}\right) = (m-1)\frac{k}{q}\cdot\ln\left(\frac{T_0}{T}\right) - \frac{k}{q}$$

$$= \underline{(m-1)\frac{k}{q}\left(\ln\left(\frac{T_0}{T}\right) - 1\right)}$$

$$B = \left[\left(K\frac{k}{q}\right)'\cdot T + K\frac{k}{q}\cdot T'\right]\cdot\ln\left(\frac{J_2}{J_1}\right) = K\frac{k}{q}\cdot\ln\left(\frac{J_2}{J_1}\right)$$

$$\Rightarrow\quad \frac{\partial V_{ref}}{\partial T} = \frac{1}{T_0}\left(V_{BE0-2} - V_{G0}\right) + K\frac{k}{q}\cdot\ln\left(\frac{J_2}{J_1}\right) + (m-1)\frac{k}{q}\left[\ln\left(\frac{T_0}{T}\right) - 1\right] \quad \text{eq. 8.18}$$

Setting equation 8.17 = 0, and $T = T_0$ getting eq. 8.18, giving the needs for zero temperature dependence at the reference temp.

$$\text{Setter} \quad \frac{\partial V_{ref}}{\partial T} = 0 \quad \wedge \quad T = T_0 \qquad \boxed{\ln 1 = 0}$$

$$0 = \frac{1}{T_0}\left(V_{BE0-2} - V_{G0}\right) + K\frac{k}{q}\ln\left(\frac{J_2}{J_1}\right) + (m-1)\frac{k}{q}\left[\ln\left(\frac{T_0}{T_0}\right) - 1\right]$$

$$\Updownarrow$$

$$0 = \frac{1}{T_0}\left(V_{BE0-2} - V_{G0}\right) + K\frac{k}{q}\ln\left(\frac{J_2}{J_1}\right) + (m-1)\frac{k}{q}(-1)$$

$$\Updownarrow$$

$$0 = \left(V_{BE0-2} - V_{G0}\right) + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right) + (m-1)\frac{kT_0}{q}(-1)$$

$$\Updownarrow$$

$$V_{BE0-2} + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right) = V_{G0} - (m-1)\frac{kT_0}{q}(-1)$$

$$\Updownarrow$$

$$V_{BE0-2} + K\frac{k}{q}T_0\ln\left(\frac{J_2}{J_1}\right) = V_{G0} + (m-1)\frac{kT_0}{q} \qquad (8.18)$$

Setting $T=T_0$ in eq. 8.16 gives the left side of eq. 8.18

8.16 : $V_{ref} = V_{BE2} + K\Delta V_{BE}$

$$= V_{G0} + \frac{T}{T_0}\left(V_{BE0-2} - V_{G0}\right) + (m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right) + K\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)$$

Letting $T = T_0$:

$$V_{ref} = V_{G0} + \frac{T_0}{T_0}\left(V_{BE0-2} - V_{G0}\right) + (m-1)\frac{kT_0}{q}\ln\left(\frac{T_0}{T_0}\right) + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right)$$

$$= V_{G0} + \left(V_{BE0-2} - V_{G0}\right) + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right)$$

$$= V_{BE0-2} + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right)$$

For $T = T_0$, the left side of eq 8.18 equals the result above.

8.18 : $$V_{BE0-2} + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right) = V_{G0} + (m-1)\frac{kT_0}{q}$$

For zero temperature dependence at T=T0. At 300 K (8.18, 8.19, 8.20):

$$V_{BE0-2} + K\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right) = V_{G0} + (m-1)\frac{kT_0}{q} \quad (8.18)$$

$[^\circ C] = [K] - 273.15$

$300[K] - 273.15[K] \approx 27\,^\circ C$

The left side of eq 8.18 is the output voltage V_{ref} at $T = T_0$ (as we have shown).

For zero temperature dependence at $T = T_0$, we need

$$V_{ref-0} = V_{G0} + (m-1)\frac{kT_0}{q} \quad (8.19)$$

For the special case of $T_0 = 300\,K$ and $m = 2.3$ (8.19) implies that, for zero temperature dependance

$$V_{ref-0} = V_{G0} = 1{,}206\,V + (2.3-1)\cdot\frac{1.38\times10^{-23}(300)}{1.602\times10^{-19}}$$

$$= 1.206\,V + 1.3\times 25.8\,mV$$

$$\underline{= 1.24\ V}$$

Note that this value is independent of the current densities chosen.

Required value for K at 300K (eq. 8.21):

From eq (8.20) we got $V_{ref-0} = 1.24\ V$ for zero temp. dependence at 300°K. This value is independent of the current densities chosen.

K from equation 8.18:

$$K = \frac{V_{G0} + (m-1)\frac{kT_0}{q} - V_{BE0-2}}{\frac{kT_0}{q}\ln\left(\frac{J_2}{J_1}\right)} = \frac{1.24\,V - V_{BE0-2}}{25.8\,mV \cdot \ln\left(\frac{J_2}{J_1}\right)}$$

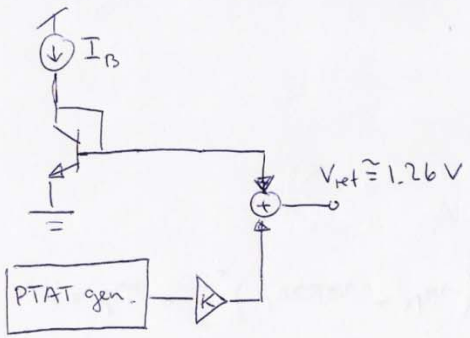
The output of a bandgap reference is given by the bandgap voltage, V_{G0}, plus a small correction to account for 2nd-order effects.

Output voltage for temperatures different from the reference; get (8.22) and then differentiate …

The fundamental equation giving the relationship between the output voltage of a bandgap reference ant temperture is equation 8.16, page 355:

$$V_{ref} = V_{BE2} + K\,\Delta V_{BE}$$

$$= V_{G0} + \frac{T}{T_0}\left(V_{BE0-2} - V_{G0}\right) + (m-1)\frac{kT}{q}\ln\left(\frac{T_0}{T}\right) + K\frac{kT}{q}\ln\left(\frac{J_2}{J_1}\right)$$



The output voltage of the reference for temperatures different from the reference is found after backsubstituting (8.18) and (8.19) into (8.16) and some manipulation:

$$V_{ref} = V_{G0} + (m-1)\frac{kT}{q}\left[1 + \ln\left(\frac{T_0}{T}\right)\right] \quad (8.22)$$

(8.22) differentiated with respect to T, getting (8.23):

Differentiating 8.22 :

$\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$

$y = uv$
$y' = u'v + uv'$

$y = \ln x$
$y' = \frac{1}{x}$

$$\frac{\partial V_{ref}}{\partial T} = (V_{G0})' + \underbrace{\left[(m-1)\frac{kT}{q}\right]'}_{u'}\underbrace{\left[1+\ln\left(\frac{T_0}{T}\right)\right]}_{v} + \underbrace{(m-1)\frac{kT}{q}}_{u}\underbrace{\left[1+\ln\left(\frac{T_0}{T}\right)\right]'}_{v'}$$

$$= (m-1)\frac{k}{q}\left[1+\ln\left(\frac{T_0}{T}\right)\right] + (m-1)\frac{kT}{q}\cdot\left(\frac{-1}{T}\right)$$

$$= \text{———} \;\; " \;\; \text{———} + (-m+1)\frac{k}{q}$$

$$= \text{———} \;\; " \;\; \text{———} \div (m-1)\frac{k}{q}$$

$$= (m-1)\frac{k}{q}\left[1+\ln\left(\frac{T_0}{T}\right) - 1\right] = (m-1)\frac{k}{q}\ln\left(\frac{T_0}{T}\right) \quad (8.23)$$

Equations 8.22 and 8.23 may be used to estimate the temperature dependence at temperatures different from the reference temperature

Example 8.4

Ex. 8.4 p. 357 Estimate the temperature dependence at 0°C for a bandgap reference that was designed to have zero temperature dependence at 20°C. Present the result as ppm/°K.

Using eq. 8.23 :
$$\frac{\partial V_{ref}}{\partial T} = (m-1)\frac{k}{q}\ln\left(\frac{T_0}{T}\right)$$

0°K corresponds to −273°C. $T_0 = 293°K$, $T = 273°K$

$$\frac{\partial V_{ref}}{\partial t} = (2.3-1)\frac{1.38\cdot 10^{-23}}{1.6\cdot 10^{-19}}\ln\left(\frac{293}{273}\right)\left[\mu V/°K\right] = \underline{8\ \mu V/°K}$$

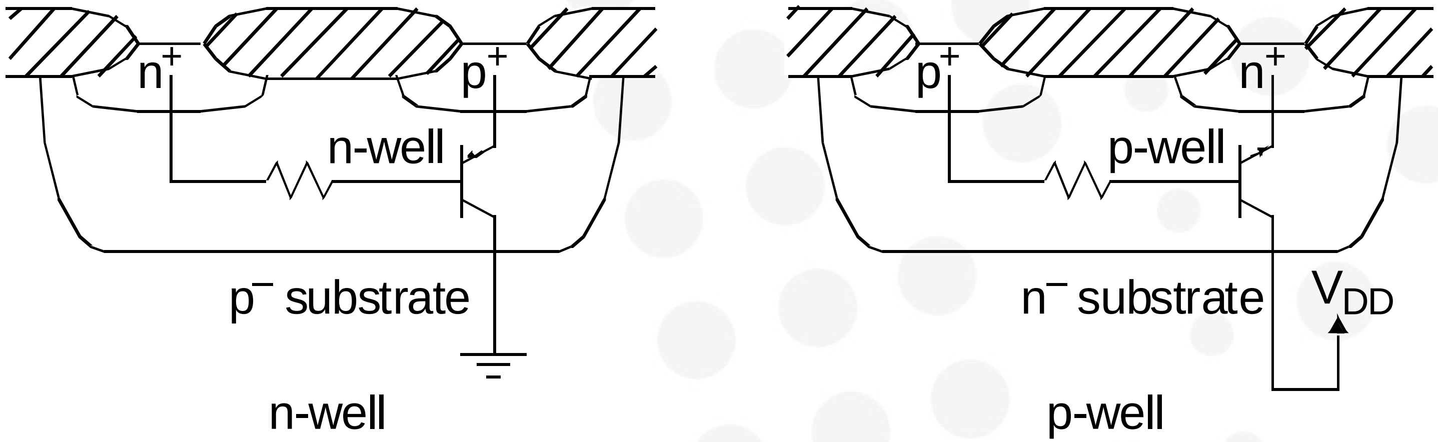
For a reference voltage of 1.24 V, a dependency of 8 μV/°K results in
$$\frac{8\mu V/°K}{1.24\,V} = 6.5\cdot 10^{-6}\ \text{parts}/°K$$

$$\underline{= 6.5\ \text{ppm}/°K}$$

"ppm": parts per million

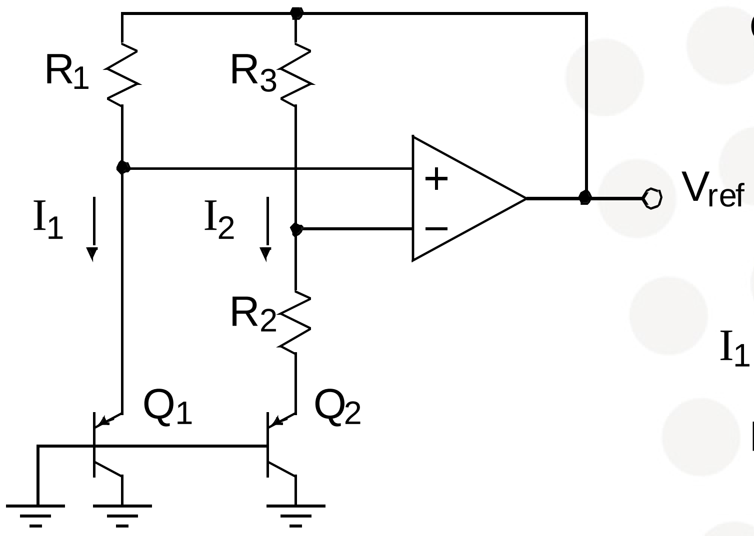
NB! Ideally: 0 ppm/°K, typically 4 – 10 × the value here.

CMOS Bandgap References

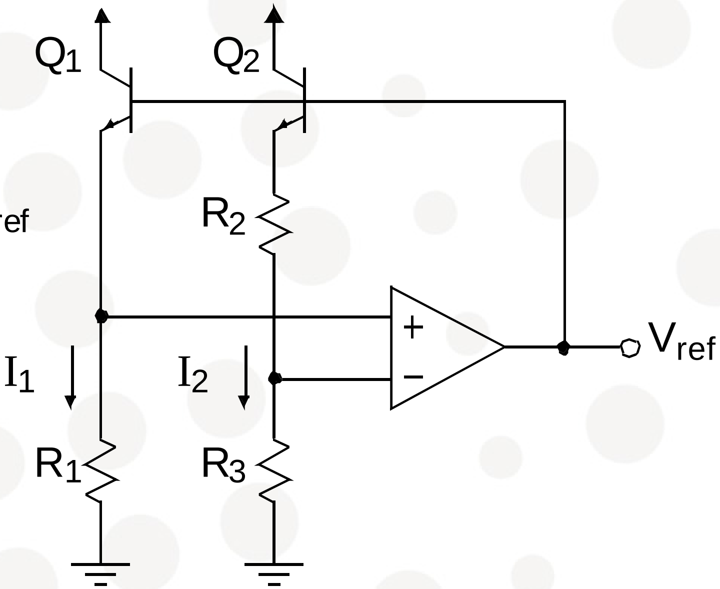


- Vertical CMOS well transistors in an n-well and p-well process (pnp in –well, npn in p-well)

CMOS BGR Circuits



n-well



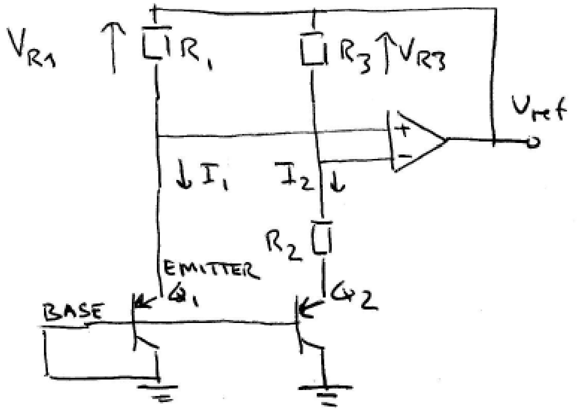
p-well

- CMOS bandgap references implemented with well transistors

Design equations, BG ref.

DESIGN EQUATIONS

Fig. 8.27



The voltage drop due to I_1:

$$V_{ref} = V_{EB1} + V_{R1} \qquad (8.35)$$

Since $V_{R1} = V_{R3}$, assuming an ideal OP-AMP:

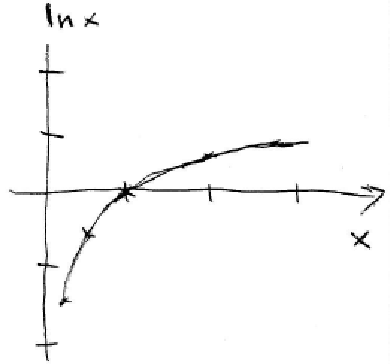
$$V_{R2} = V_{EB1} - V_{EB2} = \Delta V_{EB} \qquad (8.36)$$

I_2 runs through both R_3 and R_2, so that

$$(8.37) \quad V_{R3} = \frac{R_3}{R_2} V_{R2} = \frac{R_3}{R_2} \cdot \Delta V_{EB} \qquad (V_{R2} = \Delta V_{EB})$$

ln x

x	0,2	0,5	0	1	2	e
ln x	−1.61	−0.69	invalid	0	0,69	1



Substituting (8.37) into (8.35):

$$V_{ref} = V_{EB1} + V_{R1} = V_{EB1} + V_{R3} = V_{EB1} + \frac{R_3}{R_2} \Delta V_{EB} \qquad (8.38)$$

In integrated realizations the bipolar transistors are often taken the same size, and different current densities are realized by taking $R_3 > R_1$, causing $I_1 > I_2$. In this case

$$\cancel{\frac{I_1}{I_2}} \quad \frac{J_1}{J_2} = \frac{R_3}{R_1} \qquad (8.39)$$

Recalling from 8.12 that $\Delta V_{EB} = V_{EB1} - V_{EB2} = \frac{kT}{q} \ln \frac{J_2}{J_2}$ (8.40)

(8.39) into (8.38): $V_{ref} = V_{EB1} + \frac{R_3}{R_2} \frac{kT}{q} \ln\left(\frac{R_3}{R_1}\right)$. $K = \frac{R_3}{R_2}$ (8.42)

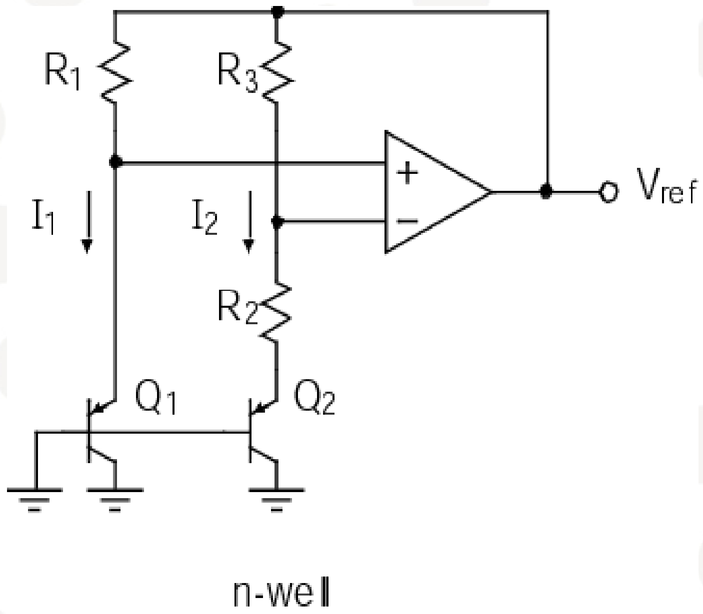
Design Equations

$$V_{ref} = V_{EB1} + V_{R1}$$

$$V_{R2} = V_{EB1} - V_{EB2} = \Delta V_{EB}$$

$$V_{R3} = \frac{R_3}{R_2} V_{R2} = \frac{R_3}{R_2} \Delta V_{EB}$$

$$V_{ref} = V_{EB1} + \frac{R_3}{R_2} \Delta V_{EB}$$



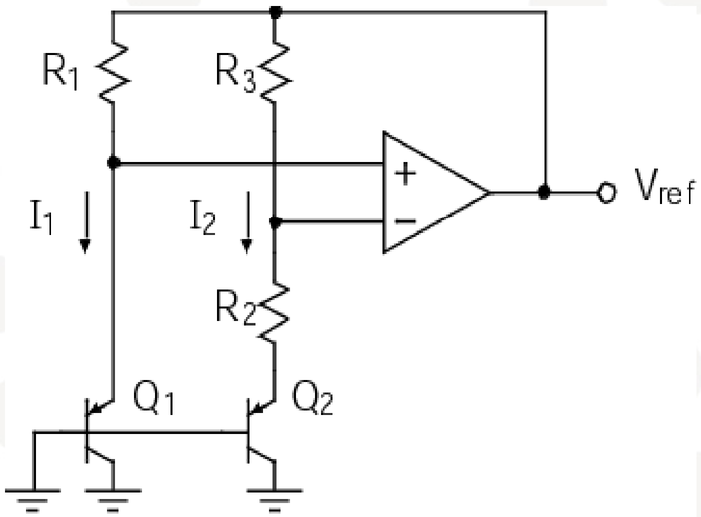
Design Equations

$$\frac{J_1}{J_2} = \frac{R_3}{R_1}$$

$$\Delta V_{EB} = V_{EB1} - V_{EB2} = \frac{kT}{q} \ln\left(\frac{J_1}{J_2}\right)$$

$$V_{ref} = V_{EB1} + \frac{R_3}{R_2}\frac{kT}{q} \ln\left(\frac{R_3}{R_1}\right)$$

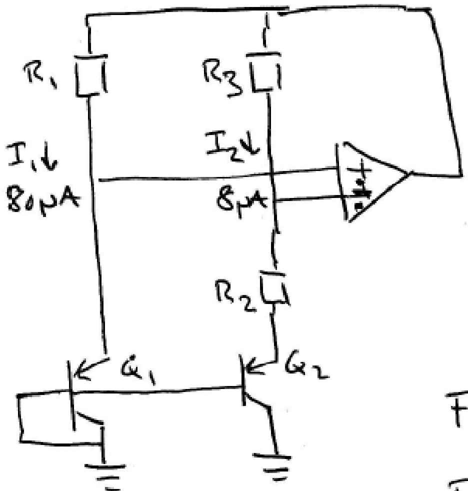
$$K = \frac{R_3}{R_2}$$



n-well

EXAMPLE 8.5

Find the resistances of a bandgap reference based on fig. 8.27 a), where $I_1 = 80\ \mu A$, $I_2 = 8\mu A$, and $V_{EB1-0} = 0.65V$ at $T = 300°K$



Assuming that the sizes of Q_1 and Q_2 are the same

$$\Delta V_{EB} = \frac{kT_0}{q} \ln \frac{I_1}{I_2} = \frac{kT_0}{q} \cdot 2.30259 = 59.4\ mV$$

$$R_2 = \frac{V_{R2}}{I_2} = \underline{\underline{7.44\ k\Omega}} = \frac{59.4\ mV}{8\mu A}$$

From (8.20) we know that $V_{ref-0} = 1.24\ V$ at $T_0 = 300°K$

From (8.35) we get $V_{R1} = V_{ref-0} - V_{EB1-0} = 1.24V - 0.65V$

$\Downarrow$

$V_{R1} = 0.59\ V$ $\quad \underline{\underline{R_1}} = \frac{0.59V}{80\mu A} = \underline{\underline{7.38k\Omega}}$, $\underline{\underline{R_3}} = \frac{0.59V}{8\mu A} = \underline{\underline{73.8k\Omega}}$

Example 8.5 (2)

$V_{R2} = \Delta V_{EB} = \frac{kT}{q} \ln(10)$, since the sizes of Q_1 and Q_2 are assumed to be the same. (equations 8.36 and 8.40)

$\Rightarrow V_{R2} = \frac{1.38 \cdot 10^{-23}}{1.62 \cdot 10^{-19}} (300) \cdot \ln(10) = 59.5 \text{ mV}$ $\quad (V_{R2} = \Delta V_{EB})$

$U = RI \Leftrightarrow R = \frac{59.5 \text{ mV}}{8 \mu A} = 7.44 \text{ k}\Omega$ $\quad \underline{R_2 = 7.44 \text{ k}\Omega}$

Remember from 8.20 that for 300°K and $m = 2.3$, (8.19) implies that $V_{ref-0} = V_{G0} + (m-1)\frac{kT_0}{q} = 1.24\text{V}$ $\quad (V_{G0} = 1.206\text{V})$

Then we can get K from (8.21):

$$K = \frac{V_{G0} + (m-1)\frac{kT_0}{q} - V_{EB1-0}}{\frac{kT_0}{q} \ln\left(\frac{J_1}{J_2}\right)} = \frac{1.24\text{ V} - 0.65\text{ V}}{0.0258 \ln(10)} = 9.93$$

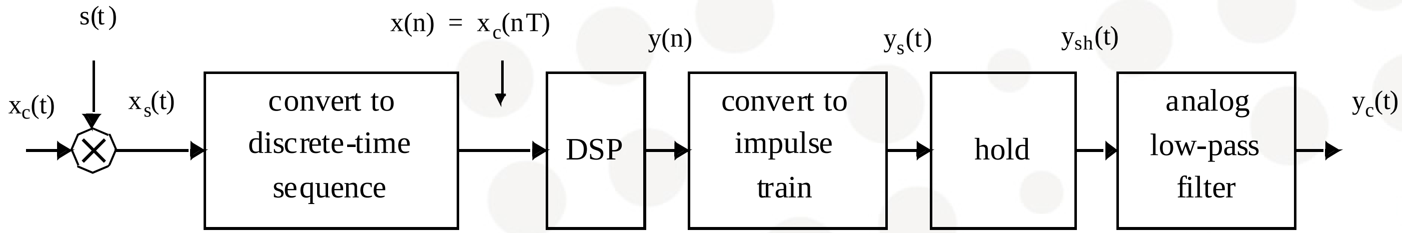
8.42 says that $K = \frac{R_3}{R_2} \Leftrightarrow \underline{R_3} = K \cdot R_2 = \underline{73.9 \text{ k}\Omega}$

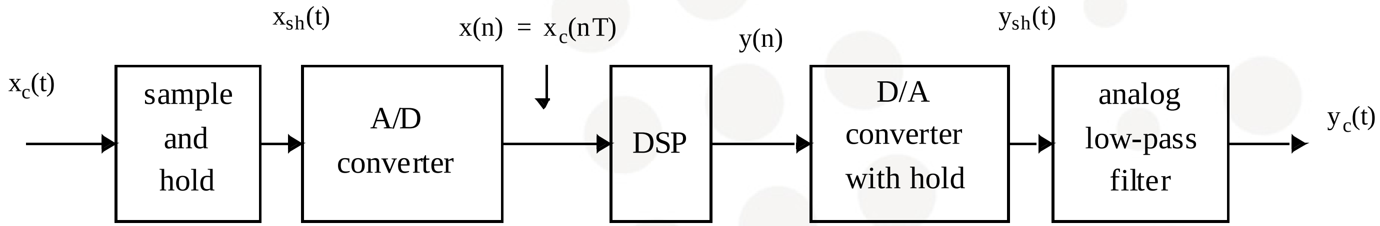
8.39: $\frac{J_1}{J_2} = \frac{R_3}{R_1} \left(= \frac{I_1}{I_2}\right) \Leftrightarrow \frac{R_3}{R_1} = \frac{80 \mu A}{8 \mu A} \Leftrightarrow \underline{R_1} = \frac{R_3}{10} = \underline{7.39 \text{ k}\Omega}$

Chapetr 9; Discrete-time signals

- Discrete-time signal processing is heavily used in the design and analysis of oversampling A/D and D/A converters as well as switched capacitor filtering ;"SC-circuits".
- Switched Capacitor filters are classified as analog, since they use continous time analog values.

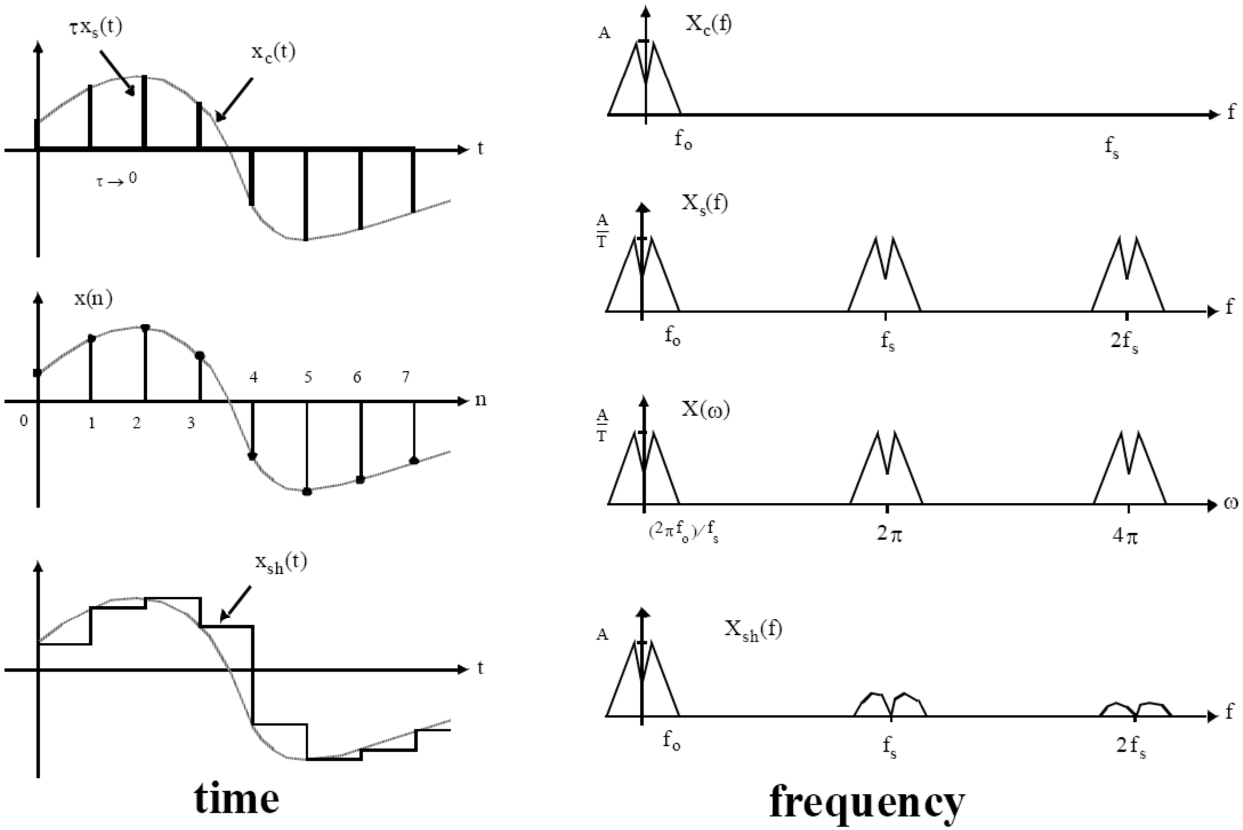
Overview of signal spectra – conceptual and physical realizations





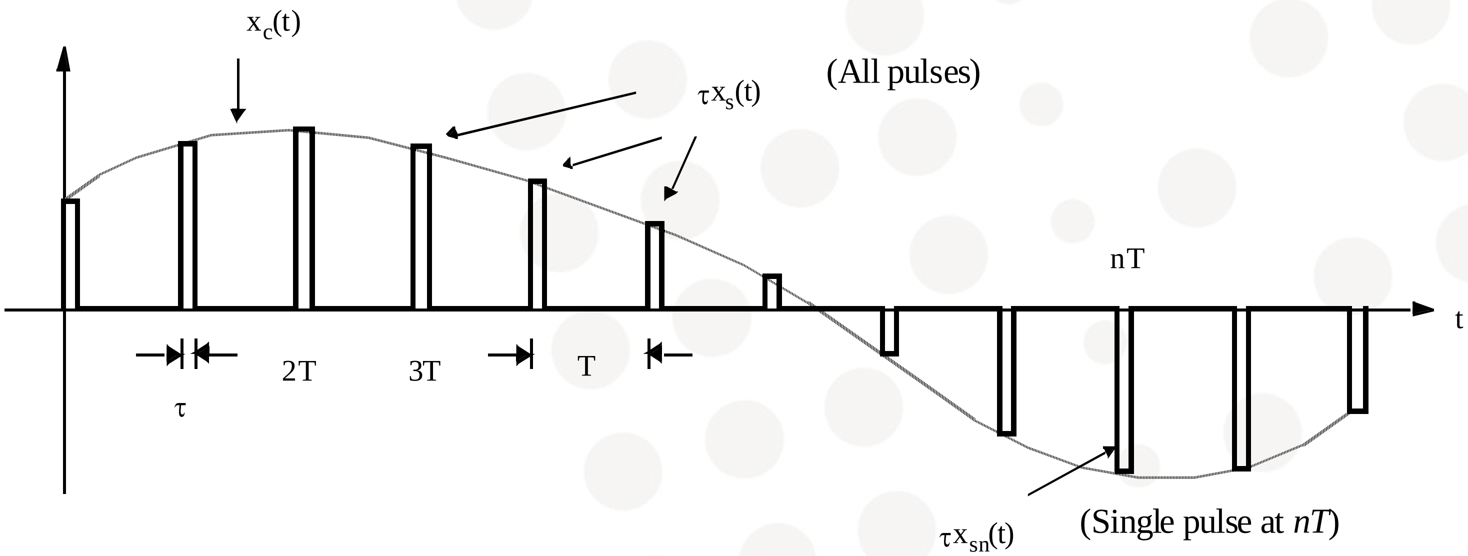
- An anti-aliasing filter (not shown) is assumed to band limit the continous time signal, $x_c(t)$.
- DSP (”discrete-time signal processing”) may be accomplished using fully digital processing or discrete-time analog circuits (ex.: SC-circ.).

Signals in time, and frequency spectra



- S(t): periodic impulse train with period T (T=1/f_s)
- $x_s(t)$ has the same frequency spectrum as $x_c(t)$, but the baseband spectrum repeats every f_s (assuming no aliasing)
- x(n) has the same frequency spectrum as $x_c(t)$, but the sampling frequency is normalized to 1
- The frequency spectrum of $x_{sh}(t)$ is equal to that of $x_s(t)$ multiplied by the sin(x)/x response of the S/H.

Laplace Transform of Discrete-Time Signals (1/3)



- The signal must be defined for all time
- For t=nT:

$$x_s(nT) = \frac{x_c(nT)}{\tau}$$

- τ is chosen such that the area under $x_s(nT)$ equals the value of $x_c(nT)$
- As τ approaches 0, the height of $x_s(nT)$ goes to ∞

Laplace Transform of Discrete-Time Signals (2/3)

- A single pulse at t=nT may be defined as:

- $\vartheta(t)$ is the step function:

$$\vartheta(t) \equiv \begin{cases} 1 & (t \geq 0) \\ 0 & (t < 0) \end{cases}$$

- $x_s(t)$ may then be rewritten as a linear combination of a series of pulses, xfs(t), where xsn(t) is zero everywhere except for a single pulse at nT:

$$x_{sn}(t) = \frac{x_c(nT)}{\tau}[\vartheta(t-nT) - \vartheta(t-nT-\tau)]$$

$x_s(t)$ is now defined for all time:

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$$

Laplace Transform of Discrete-Time Signals (3/3)

- The Laplace transform for $x_{sn}(t)$ is:

$$X_{sn}(s) = \frac{1}{\tau}\left(\frac{1-e^{-s\tau}}{s}\right)x_c(nT)e^{-snT}$$

- Since there is a linear relationship between $x_s(t)$ and $x_{sn}(t)$, the Laplace transform of $x_s(t)$ is:

$$X_s(s) = \frac{1}{\tau}\left(\frac{1-e^{-s\tau}}{s}\right)\sum_{n=-\infty}^{\infty} x_c(nT)e^{-snT}$$

- When τ approaches 0, the term before the sum equals 1 (eq. 9.7):

$$X_s(s) = \sum_{n=-\infty}^{\infty} x_c(nT)e^{-snT}$$

$$z \equiv e^{sT}$$

$$X(z) \equiv \sum_{n=-\infty}^{\infty} x_c(nT)z^{-n}$$

Spectra of Discrete-Time Signals (1/2)

- The frequency spectrum of $x_s(t)$ may be found by replacing s by $j\omega$ in the Laplace transform (eq. 9.7).
- Another more intuitive approach is to use the property that multiplication in the time domain equals convolution in the frequency domain. Using this and $\tau \rightarrow 0$, Xs(t) can be rewritten

$$x_s(t) = x_c(t)s(t)$$

- Define a pulse-train:

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

- The sampled signal is now:
- The Fourier-transform of s(t) is:

$$S(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$

Spectra of Discrete-Time Signals (2/2)

- Writing (9.8) in the frequency domain:

$$X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) \otimes S(j\omega)$$

- The frequency spectrum of $x_s(t)$ is then (eq. 9.12):

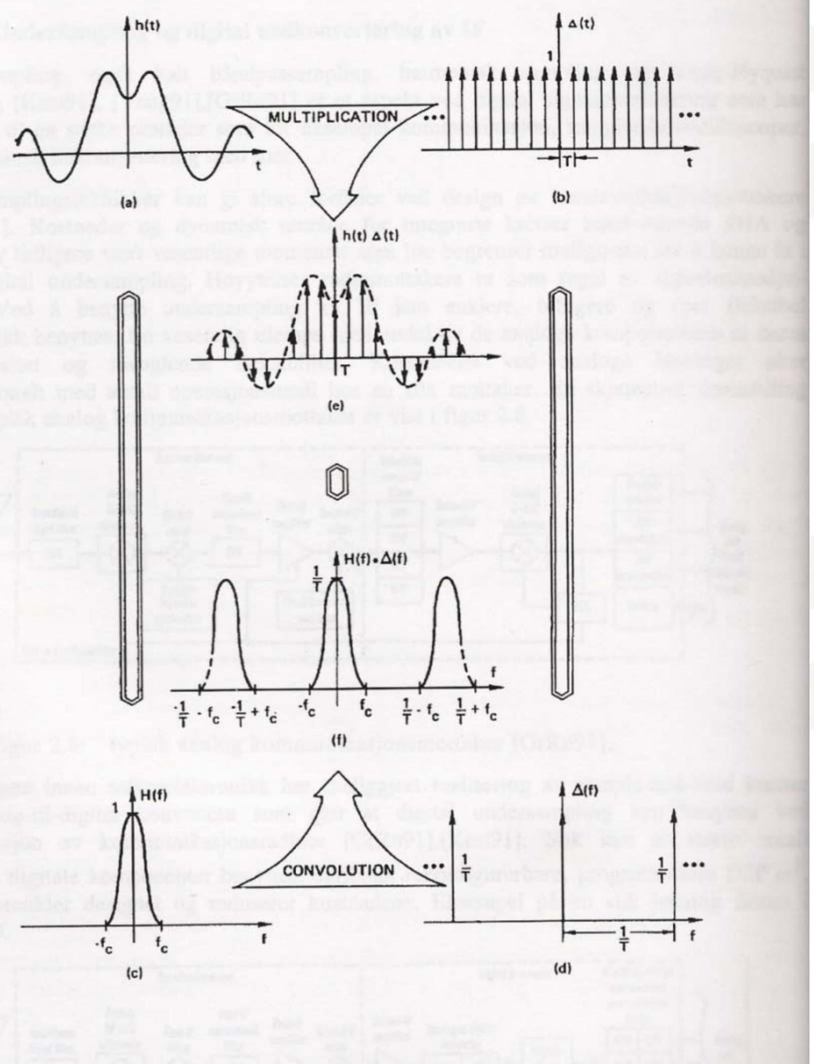
$$X_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j\omega - \frac{jk2\pi}{T})$$

which is periodic with period f_s (9.13:).

No aliasing occurs if f<fs/2

$$X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j2\pi f - jk2\pi f_s)$$

Multiplication in the time domain equals convolution in the frequency domain



Figur 2.7: Sammenhengen mellom sampling i tids- og frekvensdomenet. Produktet av h(t), figur 2.7 a), og Δ(t), i figur b), er lik den samplede kurveformen i figur c). Fouriertransformene av h(t) og Δ(t) er gitt i henholdsvis figur 2.7 c) og d). Frekvenskonvolusjonsteoremet er illustrert ved at Fouriertransformen av h(t)*Δ(t) er lik h(t)Δ(t) [Brig74].

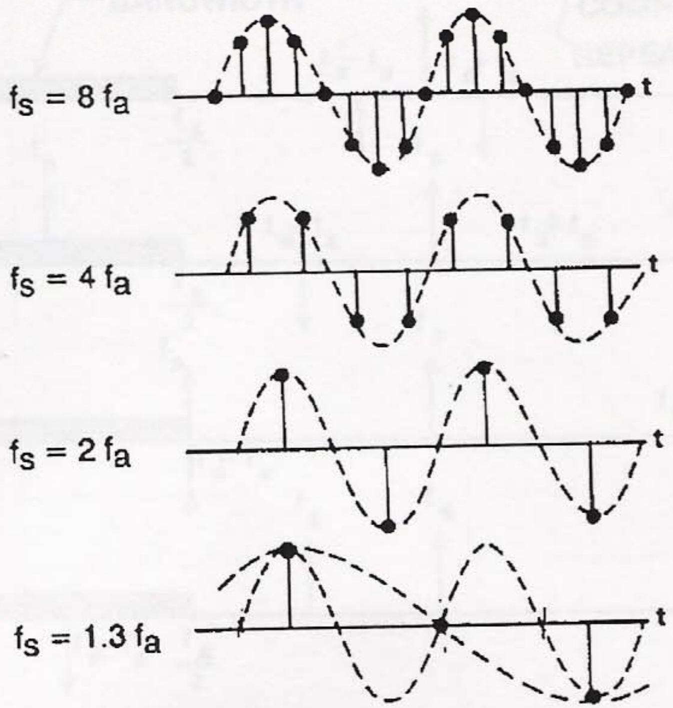
- Figure from E. O. Brigham: ”The Fast Fourier Transform”, Prentice Hall Inc., 1974., in S. Aunet: ”BiCMOS sample-and-hold for satellitt-kommunikasjon”, Cand. Scient. Thesis, University of Oslo, 1993.

- Wikipedia; Convolution:
- In mathematics and, in particular, functional analysis, **convolution** is a mathematical operation on two functions *f* and *g*, producing a third function that is typically viewed as a modified version of one of the original functions. Convolution is similar to cross-correlation.
- Computing the inverse of the convolution operation is known as deconvolution.

- In mathematics, the **Fourier transform** (often abbreviated **FT**) is an operation that transforms one complex-valued function of a real variable into another. In such applications as signal processing, the domain of the original function is typically time and is accordingly called the *time domain*. That of the new function is frequency, and so the Fourier transform is often called the *frequency domain representation* of the original function. It describes which frequencies are present in the original function.

Sampling at different frequencies

2.2 Signaler i tids- og frekvensdomenet, for ulike samplingsfrekvenser



Figur 2.4: Sampling ved ulike frekvenser, sett i tidsdomenet. f_s er samplingsfrekvensen, også kalt samplingsraten, mens f_a er frekvensen for det analoge signalet som samples.[Kest91].

Aliasing and potential degrading of signal / noise

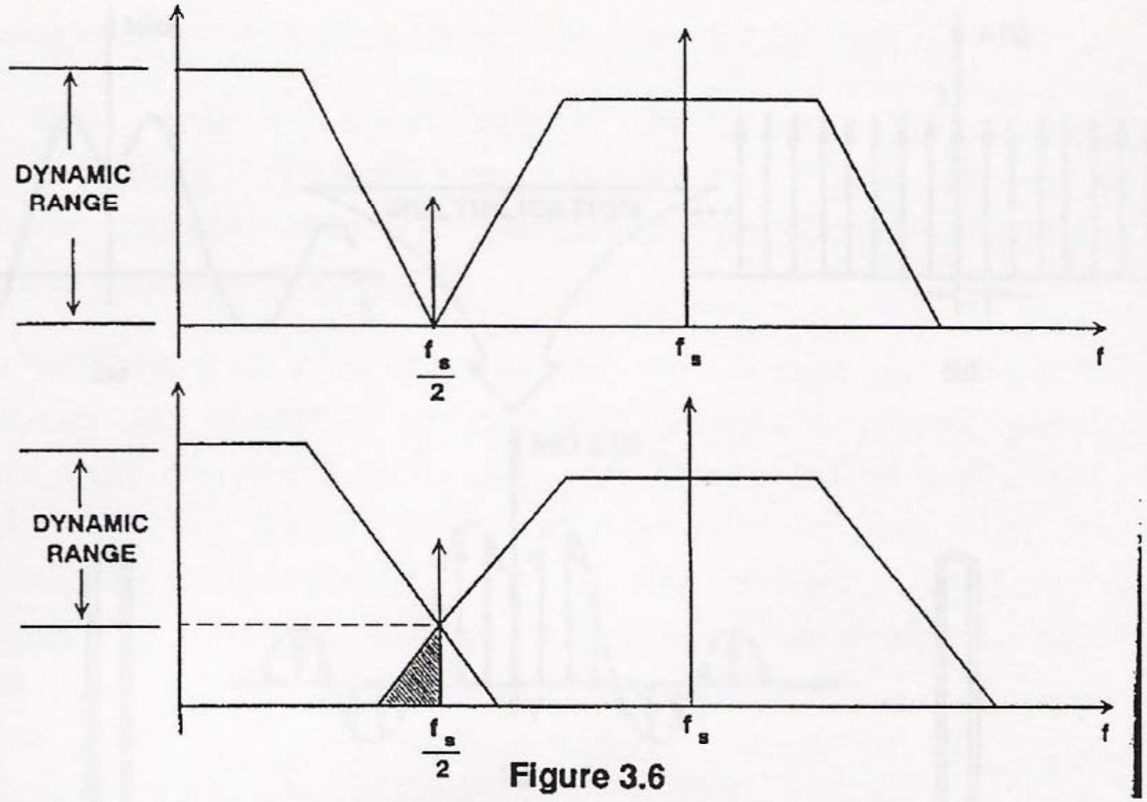


Figure 3.6

Figur 2.6: Aliasing og dynamisk område [Kest91] I det øverste tilfellet samples det slik at det dynamiske området beholdes. I det andre tilfellet overlapper frekvensspekterne slik at dynamisk område, eller signal/støy -forhold, reduseres.

Z-Transform

- Discrete-time systems are most often analyzed using the z-transform which is equivalent to the Laplace-transform with the following substitution:

- Then the z-transform is defined as :

$$z \equiv e^{sT}$$

$$X(z) \equiv \sum_{n=-\infty}^{\infty} x_c(nT) z^{-n}$$

Z-Transform

- Two important properties of the z-transform:
  - 1) If $x(n) \leftrightarrow X(z)$, then $x(n-k) \leftrightarrow z^{-k}X(z)$
  - 2) Convolution in the time-domain is equal to multiplication in the freq. domain (If y(n)=h(n) $\otimes$ x(n), then Y(z) = H(z)X(z). Similarly, multiplication in the time-domain equals convolution in the frequency domain
  - X(z) is only related to the sampled sequence of numbers, while $X_s(s)$ is the Laplace transform of $x_s(t)$ when τ -> 0
  - The frequency response of $X_s(f)$ is related to $X(\omega)$ the following way: $X_s(f) = X(\frac{2\pi f}{f_s})$
  - Thus, the following scaling has been applied:

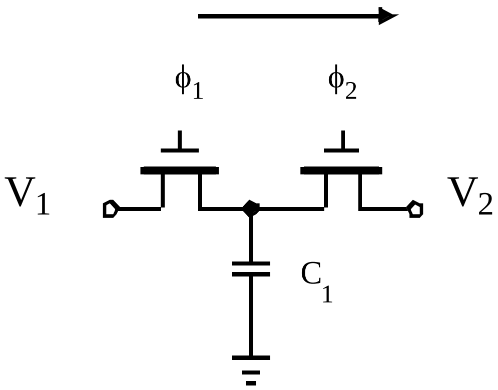
$$\omega = \frac{2\pi f}{f_s}$$

Z-Transform

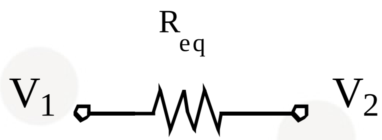
- Important observation:
  - Discrete-time signals have ω in units of radians/sample
  - The original continuous-time signal have frequency units of cycles/second (Hertz) or radians / second. (2 π Radians ~ 360 degrees)
- Example:
  - A continuous-time sinusoidal signal of 1kHz when sampled at 4 kHz will change by $\pi/2$ radians between each sample. In such case the discrete time signal is defined to have a frequency of $\pi/2$ radians per sample

Next time, Tuesday 16th of February

- Chapter 9; 9.4 – 9.6
- Chapter 10; Switched Capacitor Circuits



$\Delta Q = C_1(V_1 - V_2)$ every clock period



$R_{eq} = \frac{T}{C_1}$

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