# Discrete Time Signals and Switched Capacitor Circuits (rest of chapter 9 + 10.1, 10.2)

Tuesday 16th of February, 2010, 9:15 – 11:45

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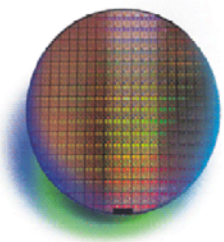
Last time – Tuesday 9th of February, and today, February the 16th:

- 8.5 Bandgap Voltage Reference Basics
- 8.6 Circuits for Bandgap References
- Chapter 9 Discrete-Time Signals
- 9.1 Overview of some signal spectra
- 9.2 Laplace Transforms of Discrete-Time Signals

- 9.2 -9.6
- 10.1-10.2 (10.3((?)))



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## 9.2 LAPLACE – TRANSFORM OF DISCRETE TIME SIGNALS

The sampled signal, $x_s(t)$ is related to the continuous-time signal, $x_c(t)$, as shown in Fig. 9.3.

Conceptual (See fig. 9.1)

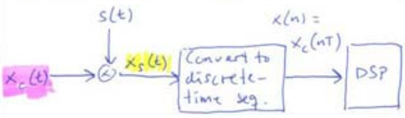
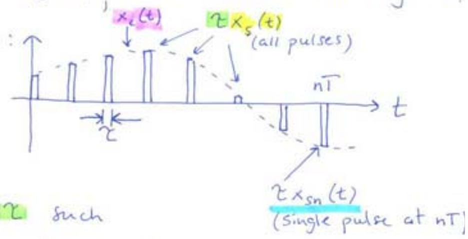


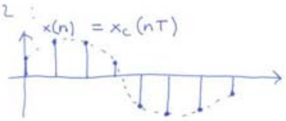

Fig. 9.3:




In Fig. 9.3 $x_s(t)$ is scaled by $\tau$ such that the area under the pulse equals the value of $x_c(nT)$.

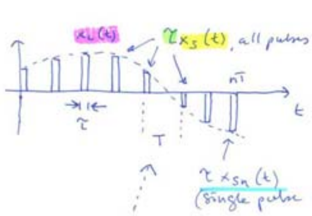
At $t = nT$ we then have $x_s(nT) = \frac{x_c(nT)}{\tau}$ such that the area under the pulse, $\tau x_s(nT)$, equals $x_c(nT)$

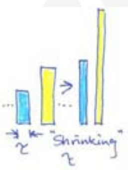
Fig. 9.2:





As $\tau \to 0$, the height of $x_s(t)$ at time nT goes to $\infty$




$\tau x_s(t)$ plotted

The single-pulse signal, $x_{sn}(t)$, can be written

$$x_{sn}(t) = \frac{x_c(nT)}{\tau}\left[\vartheta(t-nT) - \vartheta(t-nT-\tau)\right]$$

"SINGLE"

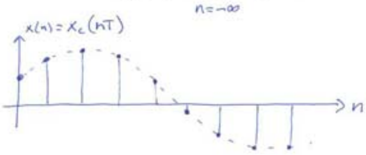
"ALL"

so that we can now write $x_s(t)$ as

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$$

$\vartheta(t)$ is defined to be the step function given by

$$\vartheta(t) = \begin{cases} 1 & (t \geq 0) \\ 0 & (t < 0) \end{cases}$$




$x_s(t)$ can be represented as a linear combination of a series of pulses, $x_{sn}(t)$, where $x_{sn}(t)$ is zero everywhere except for a single pulse at nT.

These signals are defined for all time so that the LAPLACE-transform may be found ~~in~~ for $x_s(t)$ in terms of $x_c(t)$

Laplace transform $X_{sn}(s)$ for $x_{sn}(t)$:

$$X_{sn}(s) = \frac{1}{\tau}\left(\frac{1-e^{-s\tau}}{s}\right) x_c(nT)\, e^{-snT}$$

Since $x_s(t)$ is a linear combination of $x_{sn}(t)$, we also have

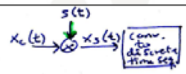
$$X_s(s) = \frac{1}{\tau}\left(\frac{1-e^{-s\tau}}{s}\right) \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-snT}$$

When $\tau \to 0$ the term before the summation goes to unity, so in this case:

(eq 9.7): $$X(s) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-snT}$$

pp. 376

# SPECTRA OF DISCRETE-TIME SIGNALS




⊗: convolution

eq. 9.7: $X_s(s) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-snT}$

The spectrum of the sampled signal, $x_s(t)$, can be found by replacing $s$ by $j\omega$ in (9.7).

A more intuitive approach is to recall that if $y(n) = h(n) \otimes x(n)$, then $Y(z) = H(z) \cdot X(z)$.

Using this fact, for $\tau \to 0$, $x_s(t)$ can be written as the product

$$x_s(t) = x_c(t)\, s(t) \quad (9.8)$$

where $s(t)$ is a periodic pulse train, or

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

where $\delta(t)$ is the impulse function (DIRAC DELTA FUNC.)

It is well known that the Fourier transform of a periodic impulse train is another periodic impulse train:

(9.10) $$S(j\omega) = \frac{2\pi}{t} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k \cdot \frac{2\pi}{T}\right)$$

(Spectrum of $s(t)$)

Writing (9.8) in the frequency d.:

(9.11): $$X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) \otimes S(j\omega)$$

$X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) \circledast S(j\omega)$

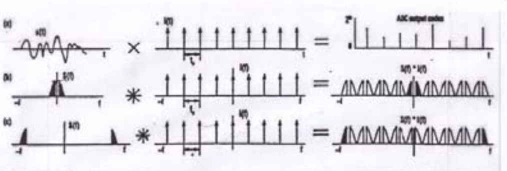
By performing this convolution either mathematically or graphically, the spectrum of $X_s(j\omega)$ can be seen to be given by

$$X_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(j\omega - \frac{jk2\pi}{T}\right) \quad (9.12)$$

or equivalently

$$X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j2\pi f - jk2\pi f_s) \quad (9.13)$$

9.12 and 9.13 show that the spectrum for the sampled signal, $x_s(t)$, equals a sum of shifted spectra of $x_c(t)$.
No aliasing occurs if $X_c(j\omega)$ is bandlimited to $\frac{f_s}{2}$



Figur 2.10: Grafisk fremstilling av sampling, i tids- og frekvensdomenet.

(9.13) confirms the example spectrum for $X_s(f)$, shown in Fig. 9.2.
Note that, for a discrete-time signal, $X_s(f) = X_s(f \pm kf_s)$, where k is an arbitrary integer as seen by substitution in (9.13).

9.3 Z - TRANSFORM pp 377 in "J&M"

(9.7): $X(s) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-snT} \quad \wedge \quad z \equiv e^{sT}$

(9.15) $X(z) \equiv \sum_{n=-\infty}^{\infty} x_c(nT)\, z^{-n}$ ; the z-transform of the samples $x_c(nT)$

TWO PROPERTIES, deduced from Laplace-tr. properties:

1) If $x(n) \leftrightarrow X(z)$ then $x(n-k) \leftrightarrow z^{-k} \cdot X(z)$
2) Conv. in the time domain equals mult. in the freq domain
Mult. — " — — " — Conv. — " —.
If $y(n) = h(n) \circledast x(n)$ then $Y(z) = H(z) \cdot X(z)$

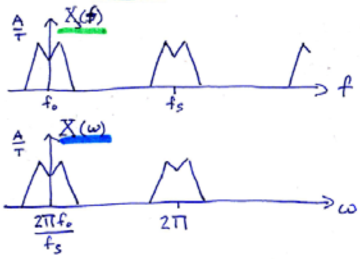
Note that $X(z)$ is not a function of the sampling rate but only to the numbers $x_c(nT)$.
The signal $x(n)$ is simply a series of numbers that may (or may not) have been obtained by sampling

"x(n) is simply a series of numbers..." (pp. 377)

One way of thinking about this series of numbers is that the original sample time, T, has been effectively normalized to 1.

The scaling justifies the spectral relation between $X(s)$ $(f)$ and $X(\omega)$ shown in Fig. 9.2

From fig. 9.2:




ω : radians/sample

Relationship between $X_s(f)$ and $X(\omega)$:

$$X_s(f) = X\left(\frac{2\pi f}{f_s}\right) \quad (9.16)$$

Alternatively:

$$\omega = \frac{2\pi f}{f_s}$$

At Nyquist rate:

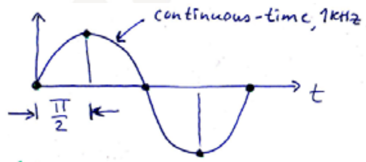
$$\omega = \frac{2\pi f}{f_s} = \frac{2\pi f}{2f} = \pi \left[\frac{\text{radians}}{\text{sample}}\right]$$

f : cycles/second (Hz)

ω : radians/sample

See fig. 9.4

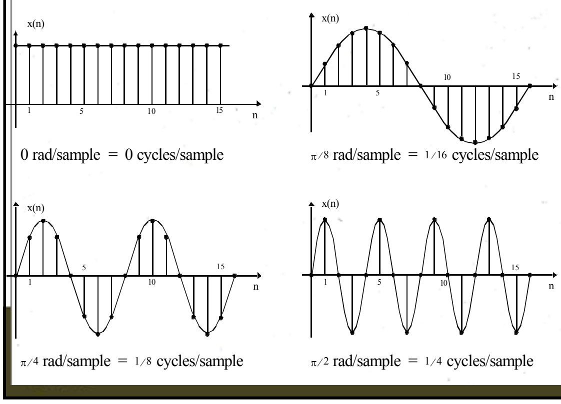
Normally discrete-time signals are defined to have frequency components only between $-\pi$ and $\pi$ $\frac{\text{rad.}}{\text{sample}}$




$f = 1\,kHz$, $f_s = 4\,kHz$

The signal changes $\frac{\pi}{2}$ radians between each sample

⇒ Such a discrete-time signal is defined to have a frequency of $\frac{\pi}{2}$ $\frac{\text{rad.}}{\text{sample}}$

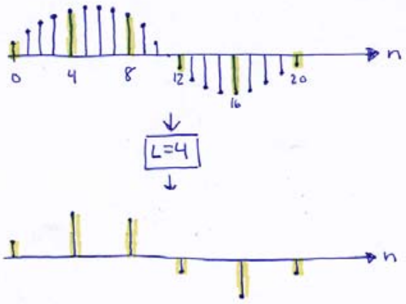



Note: Discrete-time signals are not unique since the addition of $2\pi$ results in the same signal.
(For example, a discrete-time signal having a freq. of $\frac{\pi}{4}$ $\frac{\text{rad.}}{\text{sam.}}$ is identical to that of $\frac{9\pi}{4}$ $\frac{\text{rad.}}{\text{sample}}$)

## 9.4 DOWNSAMPLING AND UPSAMPLING

DOWNSAMPLING To reduce the sample rate (without inform. loss)
UPSAMPLING to increase —— 11 ——.

DOWNSAMPLING: achieved by keeping every Lth sample and discarding the others.



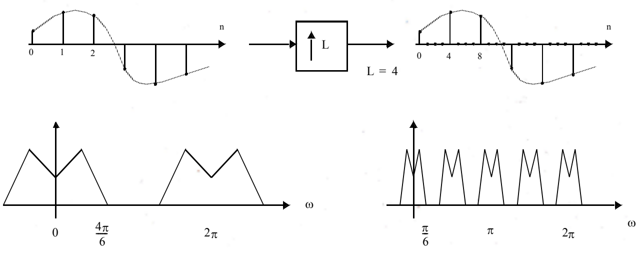

Noninteger rates can be achieved, but here L being integer is considered only.

FREQUENCY DOMAIN: Original spectra expanded by L: (See fig. 9.6 pp. 379)


SIGNAL MUST BE BAND LIMITED TO π/L BEFORE DOWNS. TO AVOID ALIASING

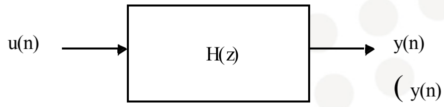
## UPSAMPLING – increasing the effective $f_s$ (pp 379)

Upsampling is accomplished by inserting L-1 zero values between samples (as shown in fig. 9.7)




- The spectra of the resulting upsampled signal are identical to the original signal but with a renormalization along the frequency axis.
- When a signal is upsampled by L, the frequency axis is scaled by L such that 2π now occurs where L2π occured in the original signal.

# 9.5 Discrete-Time Filters (pp. 382 in “J&M”)




( y(n) equals h(n) if u(n) is an impulse)

(Discrete-time filter)

- An input series of numbers is applied to a filter to create a modified output series of numbers
- Discrete-time filters are most often analyzed and visualized in terms of the z-transform
- In this figure (Fig. 9.9) the output signal is defined to be the impulse response, h(n), when the input, u(n), is an impulse (i.e. 1 for n = 0 and 0 otherwise. Transfer function; H(z) being the z-transform of the impulse response, h(n) .

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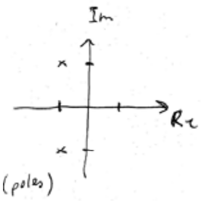
# Continuous time LP-filter

PP 382 "Johns & Martin"

The transfer function for discrete-time filters appear similar to those for continuous-time filters, except that, instead of polynomials in s, polynomials in z are obtained. For example, the transfer function of a Low-pass, continuous time filter, $H_c(s)$ might appear as




$$H_c(s) = \frac{4}{s^2 + 2s + 4}$$

$$ax^2 + bx + c \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{-2 \pm \sqrt{-3 \cdot 4}}{2} = \frac{-2 \pm 2\sqrt{-3}}{2}$$

$s = -1 \pm j\sqrt{3}$, roots of the denominator.

This LP-filter is also defined to have to zeros at $\infty$ since the denominator polynomial is two orders higher than the numerator polynomial. To find the frequency response of $H_c(s)$ the poles and zeros may be plotted (fig. 9.10 a))

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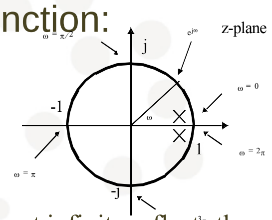
# Discrete-Time Transfer Function

- Assume the following (LP-) transfer function:

$$H(z) = \frac{0,05}{z^2 - 1,6z + 0,65}$$




- Poles: Complex conjugated at 0.8+/-0.1j
- Zeros: Two zeros at infinity (Defined). The number of zeros at infinity reflects the difference in order between denominator and nominator
- In the discrete time somain z=1 corresponds to the freq. response at both dc ($\omega$= 0) and $\omega = 2\pi$.
- The frequency respons need only be plotted for $0 \le \omega \le \pi$ (frequency response repeats every $2\pi$.
- The unit circle, $e^{j\omega}$ , is used to determine the frequency response of a system that has it's input and output as a series of numbers.
- (The magnitude is represented by the product of the lengths of the zero-vectors divided by the product of the lengths of the pole-vectors.
- The phase is calculated using addition and subtraction)

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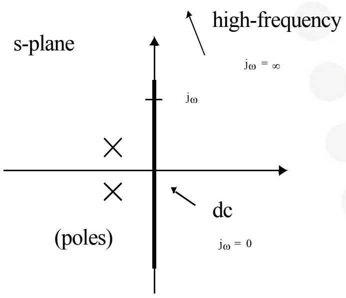
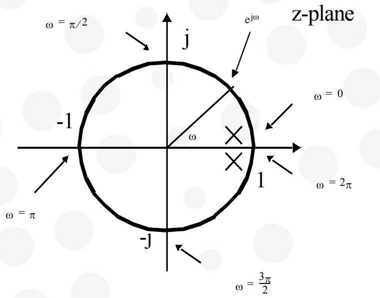
# Frequency response




- The frequency response of discrete-time filters are similar to the response of continuous-time filters. The poles and zeroes are located in the z-plane instead of the s-plane
- DC/$2\pi$ equals z=1, fs/2 equals z=-1
- The response is periodic with period $2\pi$

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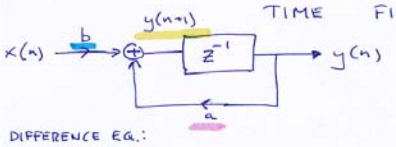
STABILITY OF DISCRETE TIME FILTERS (pp 385 in "J&M")




DIFFERENCE EQ.:

$y(n+1) = b\,x(n) + a\,y(n)$ (9.25)

Z-DOMAIN:

$z \cdot Y(z) = b\,X(z) + a\,Y(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{b}{z-a}$

pole on the real axis; $z=a$.

TEST FOR STABILITY:

We let the input be an impulse signal (i.e., 1 for $n=0$ and 0 otherwise)

16. We use equation (9.25)

If $x(n) \leftrightarrow X(z)$, then $x(n-k) \leftrightarrow z^{-k}X(z)$

continuous time filters : differential equations

difference equations : discrete-time filters

Use $y(0) = k$, where k is some arbitrary initial state

$y(n+1) = y(0) = k$

$y(n+1) = y(1) = b\,x(0) + a \cdot y(0) = b \cdot 1 + a \cdot k$

$y(n+1) = y(2) = b\,x(1) + a \cdot y(1) = b \cdot 0 + a(b+ak)$

$y(n+1) = y(3) = b\,x(2) + a \cdot y(2) = b \cdot 0 + a \cdot [a(b+ak)]$

$y(n+1) = y(4) = b\,x(3) + a \cdot y(3) = b \cdot 0 + a\{a[a(b+ak)]\}$

RESPONSE: $h(n) = \begin{cases} 0 & \text{for } n < 0 \\ k & \text{for } n = 0 \\ a^{n-1}b + a^n \cdot k & \text{for } n \geq 1 \end{cases}$

The response remains bounded only when $|a| < 1$, and unbounded otherwise.

ALL POLES MUST BE WITHIN THE UNIT CIRCLE FOR STABILITY. (Here: IIR)

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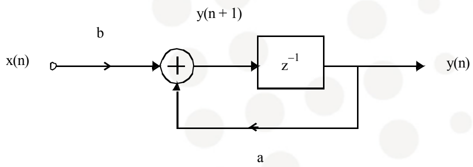
# Stability of Discrete-Time Filters




- The filters are described by finite difference equations $y(n+1) = bx(n) + ay(n)$
- In the z-domain: $zY(z) = bX(z) + aY(z)$
  $H(z) \equiv \frac{Y(z)}{X(z)} = \frac{b}{z-a}$
- H(z) has a pole in z=a. a<=1 to ensure stability
- In general a LTI system is stable if all the poles are located inside or on the unit circle

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## Test for stability

- Let the input, x(n) be an impulse signal (i.e. 1 for n=0, and 0 otherwise), which gives the following output signal, according to 9.25, y(0) = k, where k is some arbitrary initial state for y.
- $y(n+1)=bx(n) + ay(n)$
- $y(0+1) = b\,x(0) + a\,y(0) = b\,1 + ak = b + ak$,
- $y(2) = b\,x(1) + a\,y(1) = b\,0 + a\,(b + ak) = ab + a^2k$
- $Y(3) = b\,x(2) + a\,y(2) = b\,0 + a\,y(2) = a\,(ab + a^2k) = a^2b + a^3k$
- $Y(4) = a^3b + a^4k$
- Response, h(n) = 0 for (n < 0),
- k for (n=0)
- $(a^{n-1}b+a^nk)$ for n>=1
- This response remains bounded only when |a|<=1 for this 1st order filter, and unbounded otherwise.
- In general, an arbitrary, time invariant, discrete time filter, H(z), is stable if, and only if, all its poles are located within the unit circle.

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## IIR and FIR Filters

- Infinite Impulse Response (IIR) filters are discrete-time filters whose outputs remain non-zero when excited by an impulse:
  - Can be more efficient
  - Finite precision arithmetic may cause limit-cycle oscillations
- Finite Impulse Response (FIR) filters are discrete-time filters whose outputs goes precisely to zero after a finite delay:
  - Poles only in z=0
  - Always stable
  - Exact linear phase filters may be designed
  - High order often required

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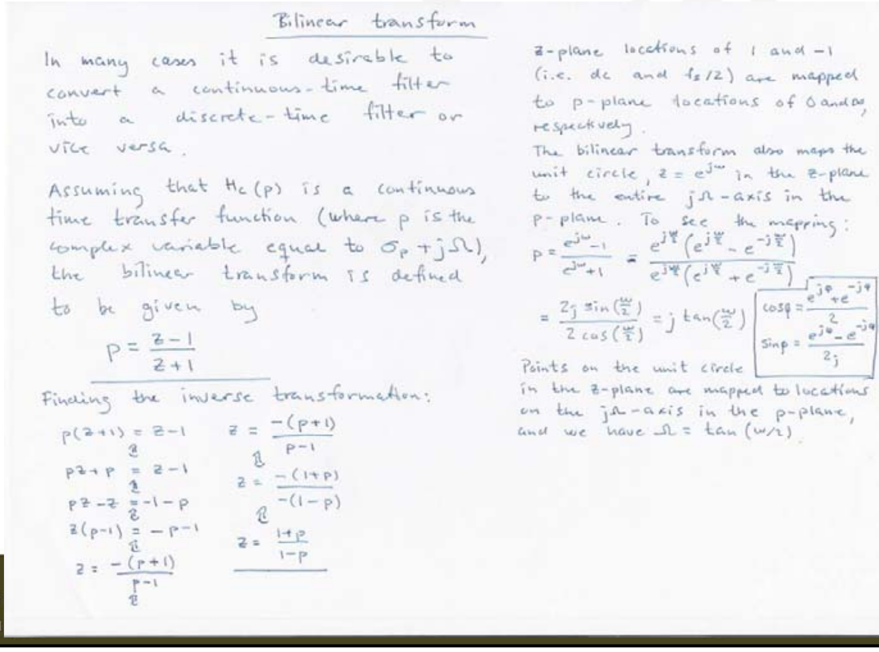
## Bilinear transform



Bilinear transform

In many cases it is desirable to convert a continuous-time filter into a discrete-time filter or vice versa.

Assuming that $H_c(p)$ is a continuous time transfer function (where p is the complex variable equal to $\sigma_p + j\Omega$), the bilinear transform is defined to be given by

$$p = \frac{z-1}{z+1}$$

Finding the inverse transformation:

$$p(z+1) = z-1 \Leftrightarrow pz + p = z-1 \Leftrightarrow pz - z = -1 - p \Leftrightarrow z(p-1) = -p-1 \Leftrightarrow z = \frac{-(p+1)}{p-1}$$

$$z = \frac{-(p+1)}{p-1} \Leftrightarrow z = \frac{-(1+p)}{-(1-p)} \Leftrightarrow z = \frac{1+p}{1-p}$$

z-plane locations of 1 and −1 (i.e. dc and $f_s/2$) are mapped to p-plane locations of 0 and ∞, respectively.

The bilinear transform also maps the unit circle, $z = e^{j\omega}$ in the z-plane to the entire $j\Omega$-axis in the p-plane. To see the mapping:

$$p = \frac{e^{j\omega}-1}{e^{j\omega}+1} = \frac{e^{j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}{e^{j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)} = \frac{2j\sin\left(\frac{\omega}{2}\right)}{2\cos\left(\frac{\omega}{2}\right)} = j\tan\left(\frac{\omega}{2}\right)$$

$$\cos\phi = \frac{e^{j\phi}+e^{-j\phi}}{2} \qquad \sin\phi = \frac{e^{j\phi}-e^{-j\phi}}{2j}$$

Points on the unit circle in the z-plane are mapped to locations on the $j\Omega$-axis in the p-plane, and we have $\Omega = \tan(\omega/2)$

## Bilinear Transform

- In many cases it is desirable to convert a continuous-time filter into a discrete-time filter or vice-versa.
- $H_c(p)$ is a CT transfer function with p = $\sigma_p + j\Omega$ .Then

$$p = \frac{z-1}{z+1} \qquad z = \frac{1+p}{1-p}$$

- The bilinear transforms map the z-plane locations of 1(DC) and -1(fs/2) to the p-plane locations 0 and $\infty$ .

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## Bilinear Transform

- The unit-circle $z = e^{j\omega}$ in the z-plane is mapped to the entire jΩ-axis in the p-plane:

$$p = \frac{e^{j\omega}-1}{e^{j\omega}+1} = \frac{e^{j(\omega/2)}(e^{j(\omega/2)} - e^{-j(\omega/2)})}{e^{j(\omega/2)}(e^{j(\omega/2)} + e^{-j(\omega/2)})}$$

$$= \frac{2j\sin(\omega/2)}{2\cos(\omega/2)} = j\tan(\omega/2)$$

- The following frequency mapping occurs:

$$\Omega = \tan(\omega/2)$$

- Then $H(z) \equiv H_c((z-1)/(z+1))$ and $H(e^{j\omega}) = H_c(j\tan(\omega/2))$

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## Sample-and-Hold Response (1/3)

- A sampled and held signal is related to the sampled continuous-time signal as follows:

$$x_{sh}(t) = \sum_{n=-\infty}^{\infty} x_c(nT)[\vartheta(t-nT) - \vartheta(t-nT-T)]$$

- Taking the Laplace-transform:

$$X_{sh}(s) = \frac{1-e^{-sT}}{s} \sum_{n=-\infty}^{\infty} x_c(nT)e^{-snT}$$

$$= \frac{1-e^{sT}}{s} X_s(s)$$

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## Sample-and-Hold Response (2/3)

- The hold transfer function $H_{sh}(s)$ is due to the previous result equal to:

$$H_{sh}(s) = \frac{1-e^{-sT}}{s}$$

- The spectrum is found by setting s=jω:

$$H_{sh}(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} = T \times e^{-\frac{j\omega T}{2}} \times \frac{\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}$$

- Finally the magnitude is given by:

$$|H_{sh}(j\omega)| = T\frac{\left|\sin\left(\frac{\omega T}{2}\right)\right|}{\left|\frac{\omega T}{2}\right|} \qquad |H_{sh}(f)| = T\frac{\left|\sin\left(\frac{\pi f}{f_s}\right)\right|}{\left|\frac{\pi f}{f_s}\right|}$$

- This response sin(x)/x is usually referred to as the *sinc-response*.

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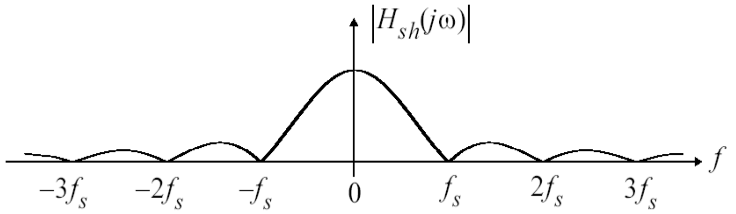
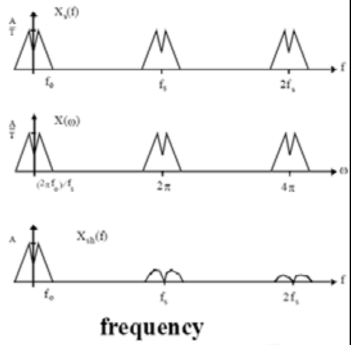
## Sample-and-Hold Response (3/3)




- Shaping only occurs for continuous-time signals, since a sampled signal will not be affected by the hold function.
- A S/H before an A/D converter does not reduce the demand of an anti-aliasing filter preceeding the A/D-converter, but simply allow the A/D to have a constant input value during the conversion.

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## Tuesday 16th of February:

- Discrete Time Signals (from chapter 9)

Today: as far as we get with:
Chapter10 Switched Capacitor Circuits

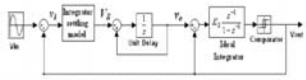
Figure 3. Second-order modulator model.

10.1 Basic building blocks (Opamps, Capacitors, Switches, Nonoverlappingg clocks)

10.2 Basic operation and analysis (Resistor equivalence of a Switched Capacitor, Parasitic Insensitive Integrators)

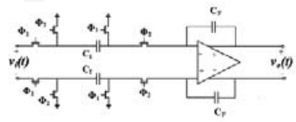
Figure 1. A typical fully differential SC integrator.

Effect of the Integrator Settling Behavior on SC ΣΔ Modulator Characteristics: a Theoretical Study

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## Properties of SC circuits

- Popular due to accurate frequency response, good linearity and dynamic range
- Easily analyzed with z-transform
- Typically require aliasing and smoothing filters
- Accuracy is obtained since filter coefficients are determined from capacitance ratios, and relative matching is good in CMOS
- The overall frequency response remains a function of the clock, and the frequency may be set very precisely through the use of a crystal oscillator
- SC-techniques may be used to realize other signal processing blocks like for example gain stages, voltage-controlled oscillators and modulators

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Basic building blocks in SC circuits; Opamps, capacitors, switches, clock generators (chapter 10.1)

- DC gain typically in the order of 40 to 80 dB (100 – 10000 x)
- Unity gain frequency should be > 5 x clock speed (rule of thumb)
- Phase margin > 70 degrees (according to Johns & Martin)
- Unity-gain and phase margin highly dependent on the load capacitance, in SC-circuits. In single stage opamps a doubling of the load capacitance halves the unity gain frequency and improve the phase margin
- The finite slew rate may limit the upper clock speed.
- Nonzero DC offset can result in a high output dc offset, depending on the topology chosen, especially if correlated double sampling is not used

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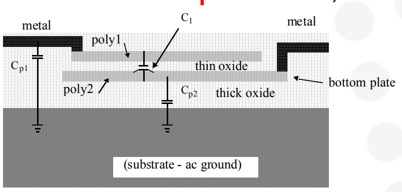
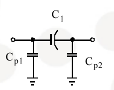
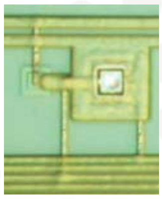
## Basic building blocks in SC circuits; Opamps, capacitors, switches, clock generators




cross-section view




equivalent circuit



- Typically constructed between two polysilicon layers
- Parasitics; Cp1, Cp2.
- Parasitic Cp2 may be as large as 20 % of the desired, C1
- Cp1 typically 1- 5 % of C1. Therefore, the equivalent model contain 3 capacitors

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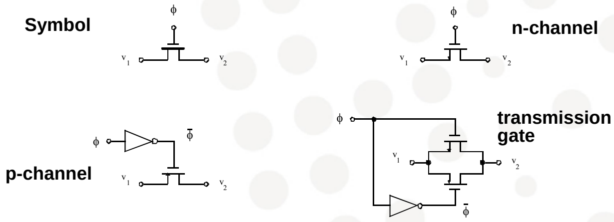
## Basic building blocks in SC circuits; Opamps, capacitors, switches, clock generators




- Desired: very high off-resistance (to avoid leakage), relatively low on-resistance (for fast settling), no offset
- Phi, the clock signal, switches between the power supply levels
- Convention: Phi is high means that the switch is on (shorted)
- Transmission gate switches may increase the signal range
- Some nonideal effects: nonlinear capacitance on each side of the switch, charge injection, capacitive coupling to each side

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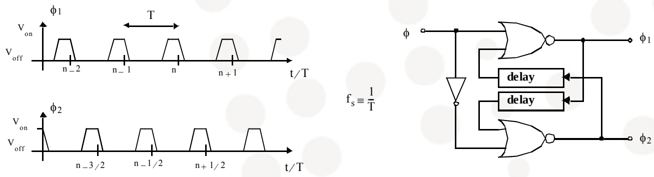
## Basic building blocks in SC circuits; Opamps, capacitors, switches, clock generators




- Must be nonoverlapping; at no time both signals can be high
- Convention in "Johns & Martin"; sampling numbers are integer values
- Location of clock edges need only be moderately controlled (assuming low-jitter sample-and-holds on input and output of the overall circuit)
- Delay elements above can be an even number of inverters or an RC network

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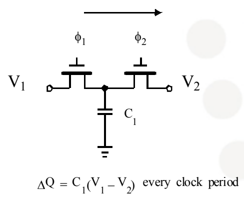
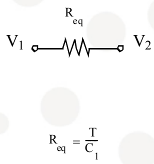
# SC Resistor Equivalent (1/2)




$$Q_x = C_x V_x$$

C1 is first charged to V1 and then charged to V2 during one clock cycle

$$\Delta Q_1 = C_1(V_1 - V_2)$$

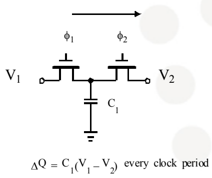
The average current is then given by the change in charge during one cycle

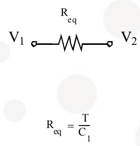
$$I_{avg} = \frac{C_1(V_1 - V_2)}{T}$$

Where T is the clock period (1/fs)



# SC Resistor Equivalent (2/2)




The current through an equivalent resistor is given by:

Combining the previous equation with I**avg**:

$$I_{eq} = \frac{V_1 - V_2}{R_{eq}}$$

The resistor equivalence is valid when fs is much larger than the signal frequency. In the case of higher signal frequencies, z-domain analysis is required :

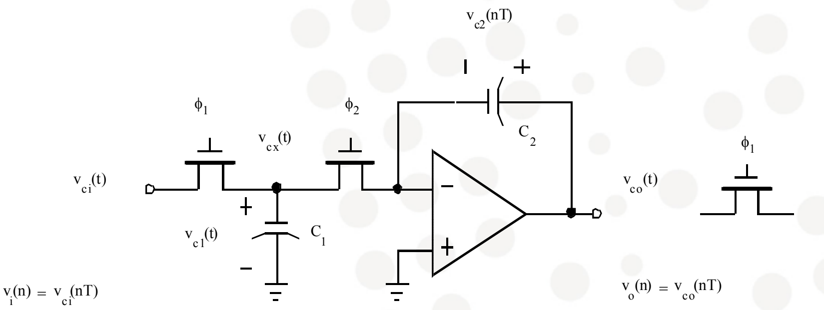
$$R_{eq} = \frac{T}{C_1} = \frac{1}{C_1 f_s}$$

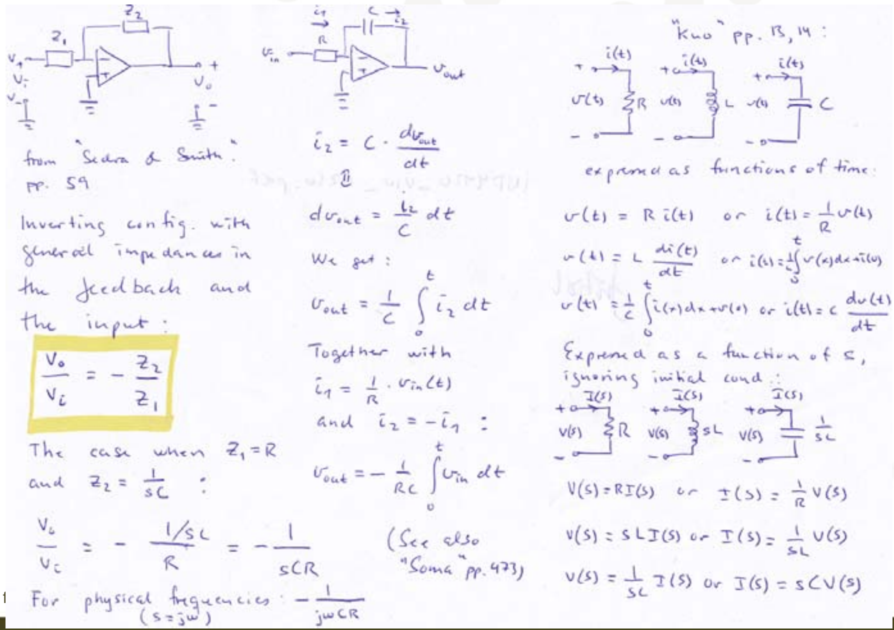
## Example of resistor implementation

- What is the resistance of a 5 pF capacitance sampled at a clock frequency of 100 kHz?
- Note the large resistance that can be implemented. Implemented in CMOS it would take a large area for a plain resistor of the same resistance

$$R_{eq} = \frac{1}{(5\times 10^{-12})(100\times 10^{3})} = 2M\Omega$$

## An inverting integrator






16.1

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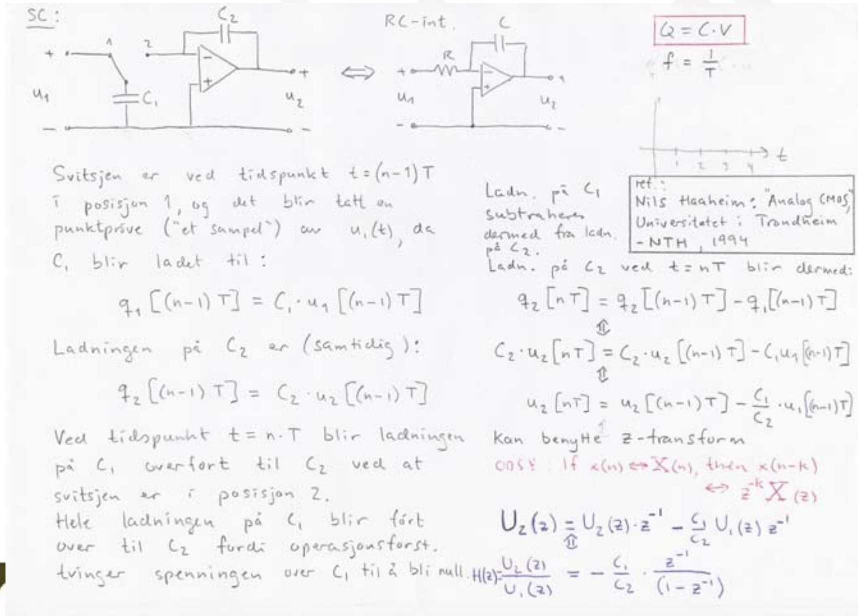
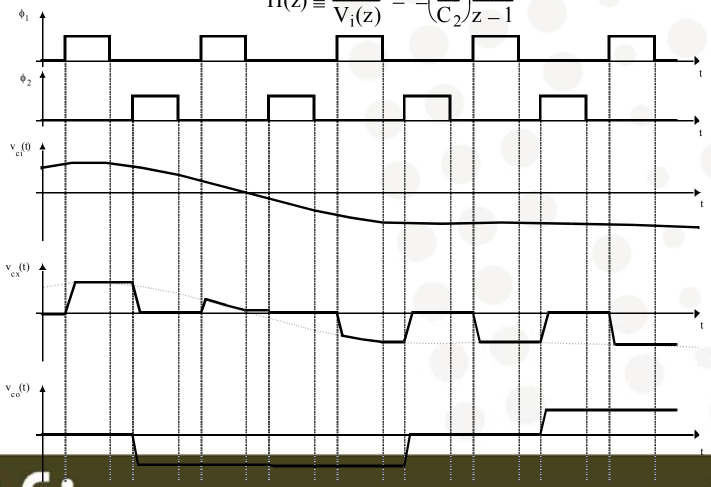
## Transfer function for simple discrete time integrator in chapter 10.2



Example waveforms. H(z) rewritten to eliminate terms of z having negative powers. Equation representative just before end of phi1 only

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\left(\frac{C_1}{C_2}\right)\frac{1}{z-1}$$






# Frequency response (Low frequency) (1/2)

$$H(z) = -\left(\frac{C_1}{C_2}\right)\frac{z^{-1/2}}{z^{1/2}-z^{-1/2}}$$

$$z = e^{j\omega T} = \cos(\omega T) + j\sin(\omega T)$$

$$z^{1/2} = \cos\left(\frac{\omega T}{2}\right) + j\sin\left(\frac{\omega T}{2}\right)$$

$$z^{-1/2} = \cos\left(\frac{\omega T}{2}\right) - j\sin\left(\frac{\omega T}{2}\right)$$

$$H(e^{j\omega T}) = -\left(\frac{C_1}{C_2}\right)\frac{\cos\left(\frac{\omega T}{2}\right) - j\sin\left(\frac{\omega T}{2}\right)}{j2\sin\left(\frac{\omega T}{2}\right)}$$

# Example 10.2 (2/2)

- Assuming low frequency i.e. $\omega T \ll 1$ :

- The gain-constant is depending only on the capacitor-ratio and clock frequency:

$$H(e^{j\omega T}) \cong -\left(\frac{C_1}{C_2}\right)\frac{1}{j\omega T}$$

$$K_I \cong \frac{C_1}{C_2}\frac{1}{T}$$

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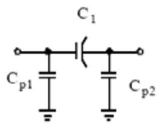
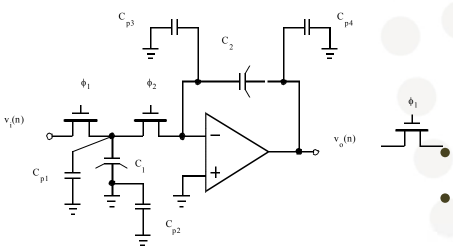
# Parasitics reducing accuracy and performance



$$H(z) = -\left(\frac{C_1 + C_{p1}}{C_2}\right)\frac{1}{z-1}$$

- Parasitics added
- $C_{p1}$ the one that is harmful, as accurate discrete-time frequency responses depends on precise matching of capacitors, (sometimes down to 0.1 percent)
- $C_{p1}$ 1-5 % of C1 (page 396)
- Gain coefficient related to $C_{p1}$ which is not well controlled and partly nonlinear→ larger area

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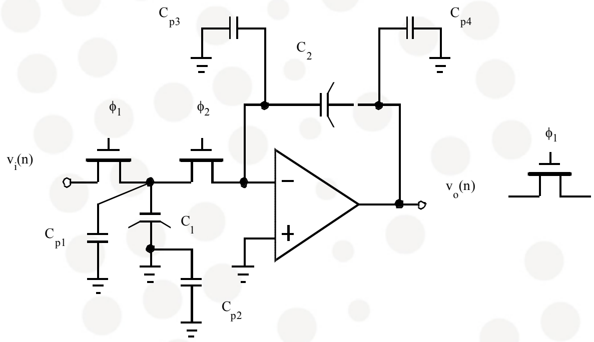
# Effect of parasitic capacitors

$$H(z) = -\left(\frac{C_1 + C_{p1}}{C_2}\right)\frac{1}{z-1}$$




- The gain coefficient depends on the parasitic and possibly non-linear capacitance

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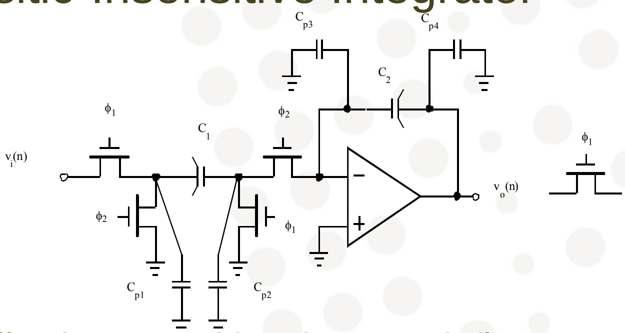
# Parasitic-Insensitive Integrator




- The following parasitics does not influence:
  - Cp2 is either connected to virtual ground or physical ground
  - Cp3 is connected to virtual ground
  - Cp4 is driven by the output
  - Cp1 is charged between vi(n) and gnd, and does not affect charge on $C_1$

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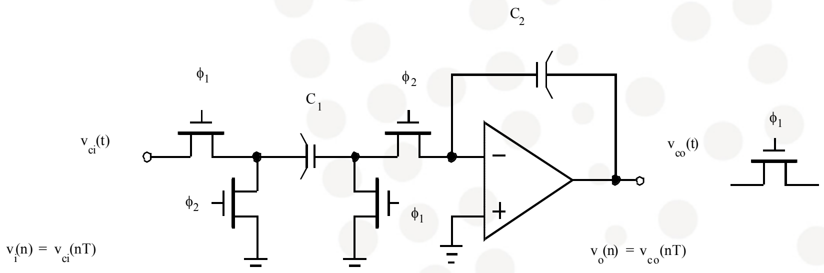
# Parasitic-Insensitive Integrator




- Two additional switches removes sensitivity to parasitics:
  - Improved linearity
  - More well-defined and accurate transfer-functions

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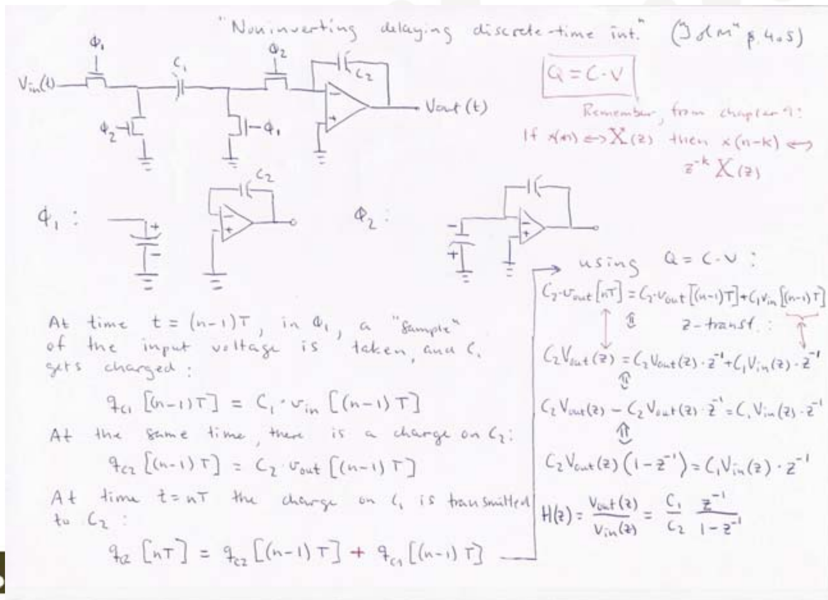
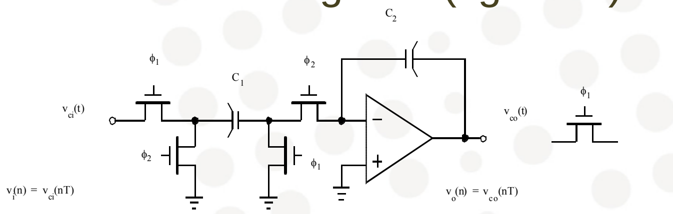
# Transfer function not dependent on Cp1:



# Parasitic-Insensitive Integrator (fig. 10.9)




$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = \left(\frac{C_1}{C_2}\right)\frac{1}{z-1}$$

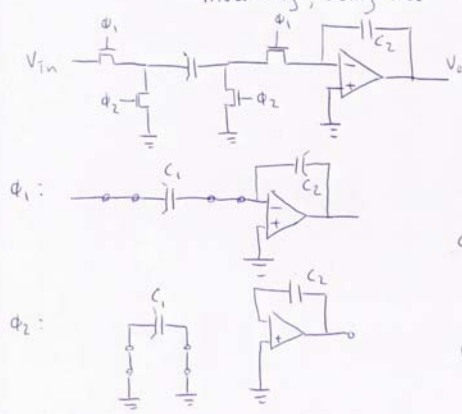
- Note that the integrator is now positive
- $C_1$ and $C_2$ no longer need to be much larger than parasitics
- A remaining limitation is the lateral stray capacitance between the lines leading to the electrodes of $C_1$ and $C_2$. This can be reduced by inserting a grounded line between the leads. In any case the minimum permissible $C_1$ and $C_2$ values are reduced by a factor 10 – 50 if the stray-insensitive configuration is used, hence reducing the area required by the capacitors is reduced by the same factor [GrTe86]. Price is proportional to area.
- While parasitics do not affect the discrete time difference equation (or H(z)), they may slow down settling time behaviour.



# H(z) for inverting, delay-free integrator

Inverting, delay-free integrator (fig 10.12 in "J&M")

$Q = C \cdot V$




Charge on $C_2$ at the end of $\phi_1$ is equal to it's old value minus the charge needed to charge $C_1$ to $v_{in}(nT)$:

$$C_2 V_{out}(nT) = C_2 v_{out}[(n-1)T] - C_1 v_{in}[nT]$$

Dividing by $C_2$ and switching to discrete-time variables:

$$V_{out}(n) = v_{out}(n-1) - \frac{C_1}{C_2} v_{in}(n)$$

z-transf.:

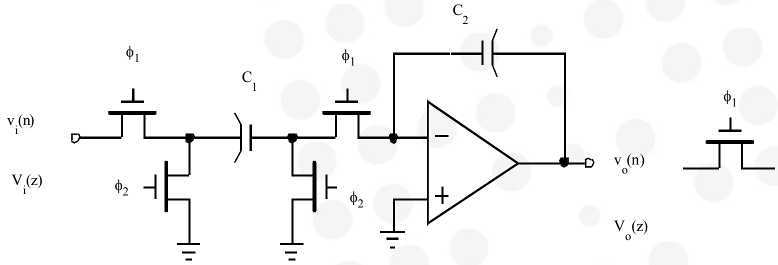
$$V_{out}(z) = v_{out}(z) \cdot z^{-1} - v_{in}(z) \cdot \frac{C_1}{C_2}$$

The charge on $C_2$ does not change when $\phi_2$ turns on (and $\phi_1$ is off).

$$H(z) = \frac{V_{out}(z)}{v_{in}(z)} = -\frac{C_1}{C_2}\frac{1}{1-z^{-1}}$$

$V_{in}(nT)$ occurs in the difference equation rather than $V_{in}[(n-1)T]$, since the charge on $C_2$ at the end of $\phi_1$ is related to $V_{in}(nT)$ at the same time → "DELAY-FREE"

## Inverting delay-free integrator (fig. 10.12)




- Equations similar to previous slide, but with clocking- and timing convention as in fig. 10.3:

$$C_2 v_{co}(nT - T/2) = C_2 v_{co}(nT - T)$$

$$C_2 v_{co}(nT) = C_2 v_{co}(nT - T/2) - C_1 v_{ci}(nT)$$

- H(z) having $z^{-1}$ removed: $H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\left(\frac{C_1}{C_2}\right)\frac{z}{z-1}$

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## Next time, Tuesday the 23rd

- Rest of chapter 10. (10.3, 10.4, 10.5, 10.7)
- Chapter 11, Data Converter Fundamentals

- Additional litterature (chapter 9 and 10):

- "Sedra & Smith"
- Franklin W. Kuo (FYS3220 (?))
- Nils Haaheim, Analog CMOS
- Basic Electrical Engineering, Schaum's outlines



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