UiO : Department of Informatics
University of Oslo

INF4420

Non-linearity

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Outline

Non-linearity and harmonic distortion

Differential circuits

Feedback

Improving linearity

Introduction

Linear distortion from filtering (not considered)

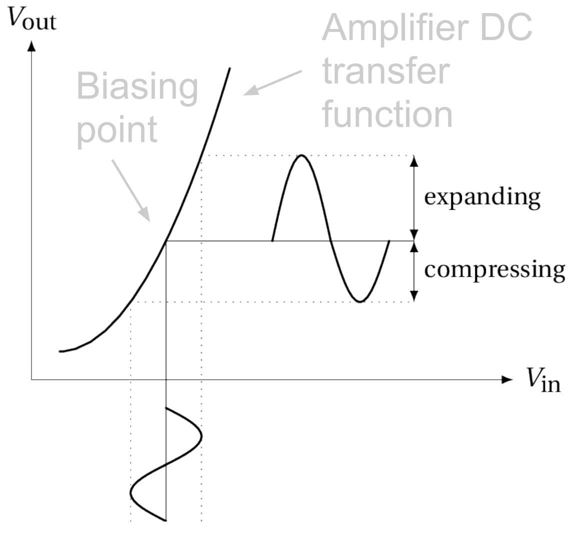
Soft non-linearity (expanding, compression)

Hard non-linearity (clipping)

Introduction

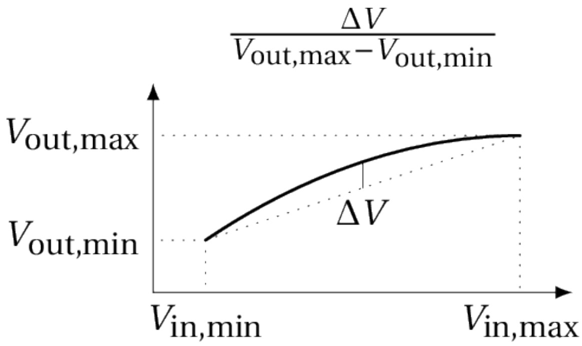
Amplification and non-linearity depends on the biasing point.

Soft non-linearity
Hard non-linearity (clipping)

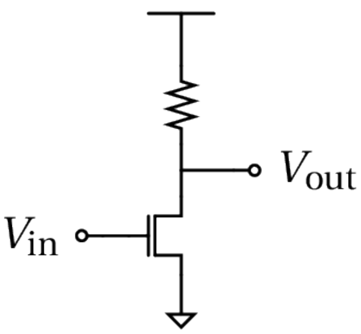


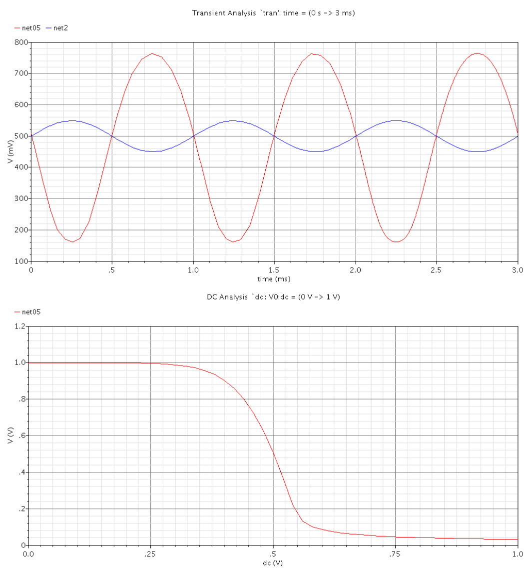
Introduction

Measure deviation from ideal straight line approximation



Common source amp





Taylor series

Using Taylor series allows studying distortion independent of the specific shape of the non-linearity.

Generic expression for total harmonic distortion (THD).

Taylor series

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \cdots$$

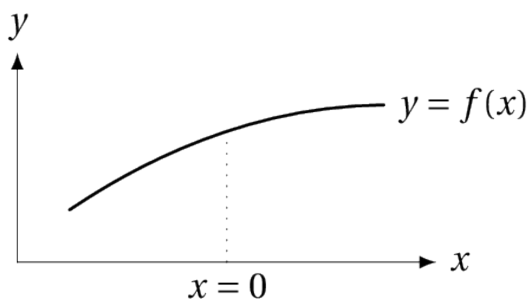
$$\alpha_0 = f(0)$$

$$\alpha_1 = \frac{df(0)}{dx}$$

$$\alpha_2 = \frac{1}{2}\frac{d^2 f(0)}{dx^2}$$

$$\alpha_3 = \frac{1}{6}\frac{d^3 f(0)}{dx^3}$$

$$\ldots$$



Harmonic distortion

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$x(t) = A\cos\omega t$$

$$\cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$$

$$\Rightarrow y(t) = \alpha_0 + \alpha_1 A\cos\omega t + \alpha_2 A\cos^2\omega t + \alpha_3 A^3\cos^3\omega t + \cdots$$

$$= \alpha_0 + \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}[1+\cos(2\omega t)] +$$

$$\frac{\alpha_3 A^3}{4}[3\cos\omega t + \cos(3\omega t)] + \cdots$$

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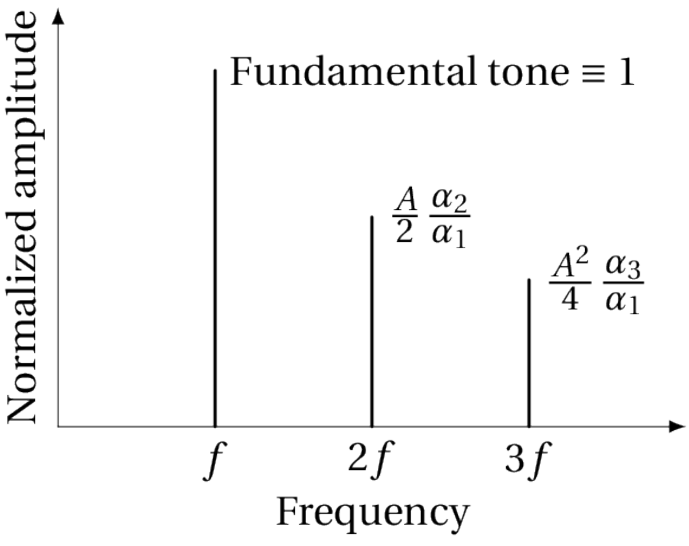
Harmonic distortion



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Harmonic distortion

$$\mathrm{THD} = \frac{\left(\frac{\alpha_2 A^2}{2}\right)^2 + \left(\frac{\alpha_3 A^3}{4}\right)^2}{\left(\alpha_1 A + \frac{3\alpha_1 A}{4}\right)^2}$$

$$\approx \frac{(2A\alpha_2 + A^2\alpha_3)^2}{16\alpha_1^2}$$

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Common source linearity

Large signal current:

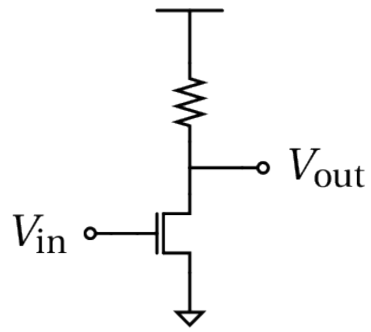
$$\frac{\beta}{2}(V_{\mathrm{GS}} - V_{\mathrm{TH}} + \underline{V_m \cos\omega t})^2 =$$

Input signal

$$\frac{\beta}{2}(V_{\mathrm{GS}} - V_{\mathrm{TH}})^2 +$$

$$\beta(V_{\mathrm{GS}} - V_{\mathrm{TH}})V_m \cos\omega t +$$

$$\frac{\beta}{4}V_m^2(1 + \cos 2\omega t)$$



Harmonic distortion:

$$\frac{V_m}{4(V_{\mathrm{GS}} - V_{\mathrm{TH}})}$$

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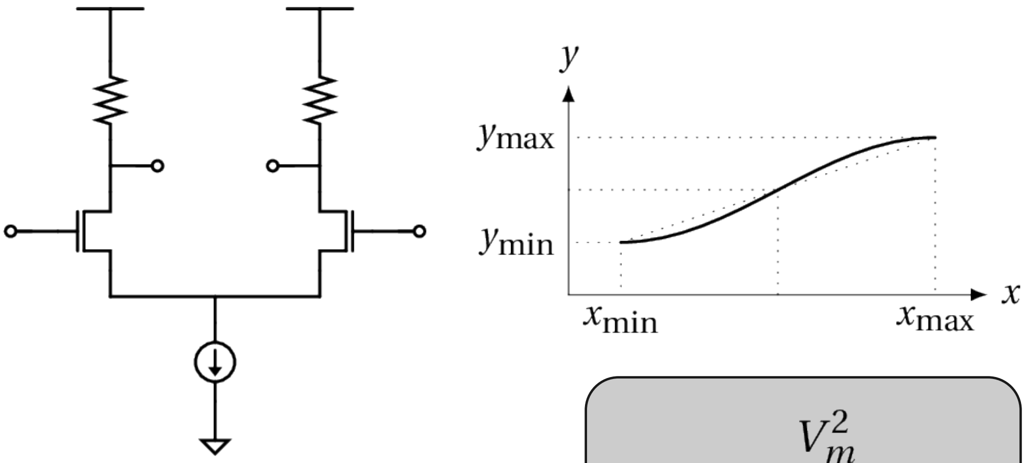
Differential pair



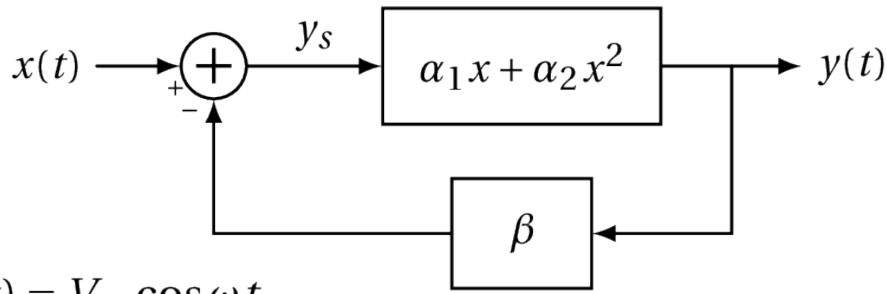
Harmonic distortion: $\approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}$

Differential pair

Differential circuits exhibit much less distortion (5 % vs 0.125 % for $V_m = 0.2(V_{GS} - V_{TH})$

Linearity is also better when accounting for 2x current.

Feedback

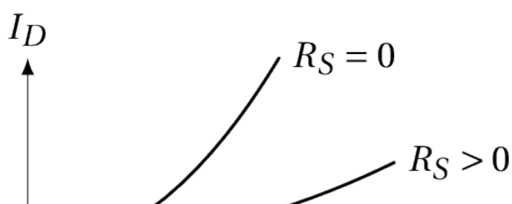


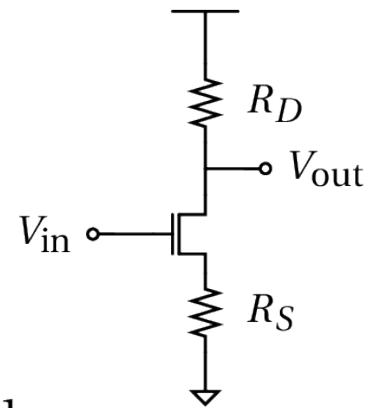
$$x(t) = V_m \cos \omega t$$

$$y(t) \approx a \cos \omega t + b \cos 2\omega t \qquad a = \frac{\alpha_1}{1+\beta\alpha_1} V_m$$

$$\frac{b}{a} = \frac{\alpha_2 V_m}{2} \frac{1}{\alpha_1} \frac{1}{(1+\beta\alpha_1)^2} \qquad b = \frac{\alpha_2 V_m^2}{2} \frac{1}{(1+\beta\alpha_1)^2}$$

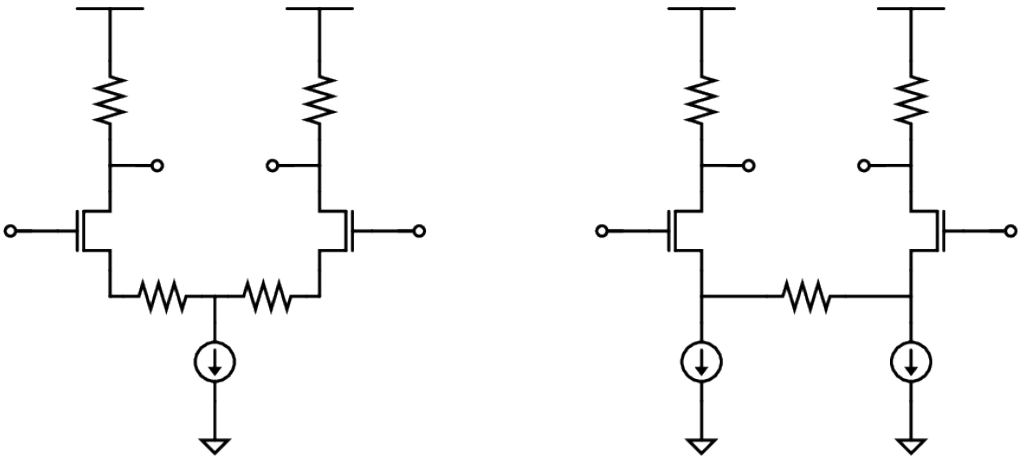
Resistive degeneration





$$G_m = \frac{g_m}{1+g_m R_S} \approx \frac{1}{R_S}, \; g_m R_s \gg 1$$

Resistive degeneration



Resistive degeneration also works for diff pairs

Post correction

Cascade non-linear stage with inverse non-linearity.

Ideally gives overall linear transfer funciton