Refinement – basic concepts and ideas

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Objectives for the lectures on refinement

- **The two lectures on refinement aim to**
  - to motivate and explain a basic apparatus to define and relate the notions of refinement
    - this includes
      - representing executions by traces
      - explaining the significance of a notion of observation
      - outlining the assumption-guarantee paradigm
  - introduce and related the following notions of refinement
    - supplementing
    - narrowing
    - detailing
    - property refinement
    - interface refinement
  - Illustrate the use of these notions of refinement
    - the interplay between specification and refinement

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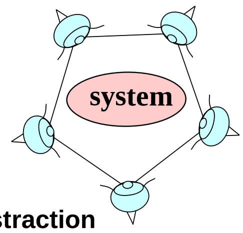
The role of refinement

- System development makes use of refinement as a means to check and document incremental steps aiming to
  - reduce the set of legal implementations
  - introduce error handling
  - introduce time constraints
  - introduce finer granularity of interaction and execution
  - introduce implementation dependent data types
  - introduce implementation oriented communication protocols
  - introduce constraints on unlimited resources
  - extend the input domain

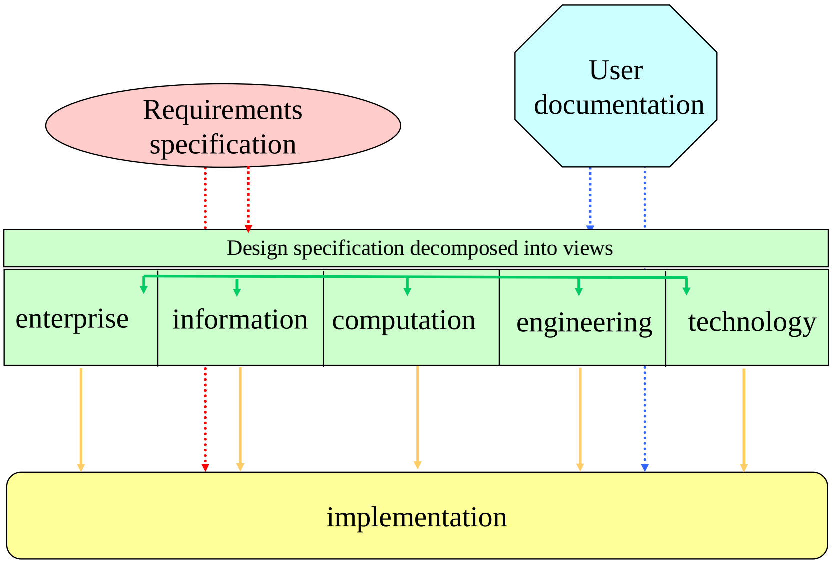
Why refinement is important

- Systems of today are large and complex – abstraction is a necessary means to
  - explain what the systems do
  - explain how the systems are built
  - distinguish the essentials from the inessential
  - decompose large and complicated aspects into small more easily understandable entities
  - extract specialized system views



- Formal documentation gives new possibilities
- Refinement
  - relates system descriptions at different levels of abstraction
  - connects and relates different system views
  - provides a foundation for verifications and validations

Why refinement must be documented



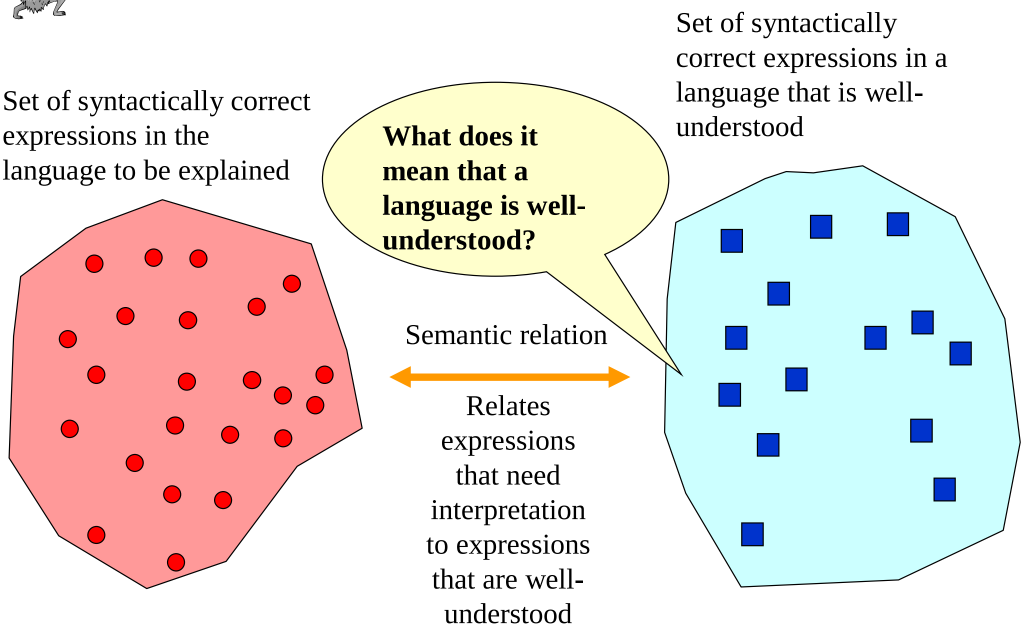
Documenting refinement

- Precision is just as important when we document refinements as when we write specifications
- Refinements can be documented using standard specification languages
  - in INF 5150 we will use UML for this purpose
- Formal documentation of refinements facilitates integrated analysis, validation, testing and verification

Three main concepts of language theory

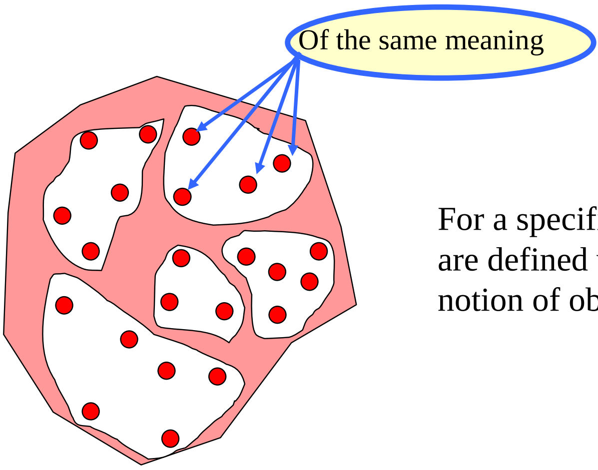
- **Syntax**
  - The relationship between symbols or groups of symbols independent of content, usage and interpretation
- **Semantics**
  - The rules and conventions that are necessary to interpret and understand the content of language constructs
- **Pragmatics**
  - The study of the relationship between symbols or groups of symbols and their interpretation and usage

Semantic relation



The need for a notion of observation

- A semantic relation will define an equivalence relation on the language that should be understood



For a specification language these are defined with respect to a notion of observation

Definition of a notion of observation

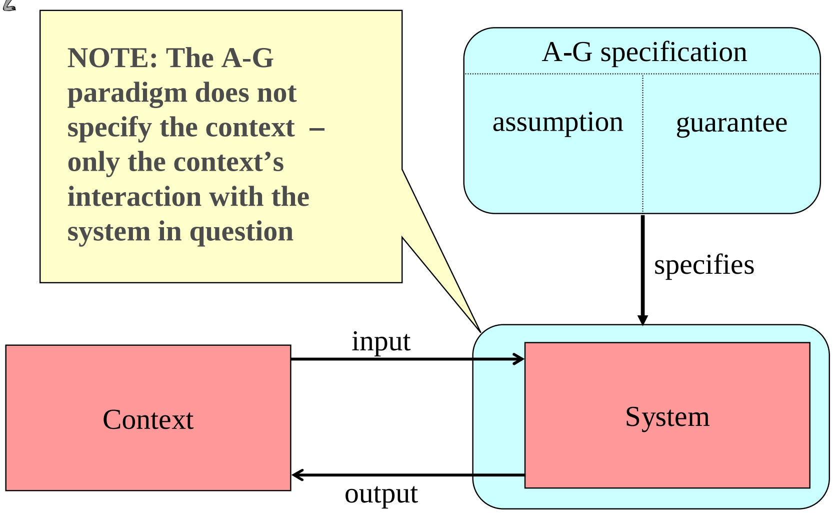
- May observe only external behavior
- May observe any potential behavior
- May observe time with respect to a global clock
- May observe safety properties
  - Always falsified by a partial execution
- May observe liveness
  - Falsified only by complete executions

May our notion of observation be implemented by a human being?

Assumption-guarantee paradigm

- **Well-known specification technique to facilitate modularity**
  - appeared first with pre-post specifications in the 60ies
  - since then taken further and adapted in many directions
  - referred to as: pre-post, rely-guarantee, assumption-commitment, assumption-guarantee, contracts, goal-means-task
- **Motivation:**
  - The behavior of a system component depends on the context it is executed in
  - Not all contexts are equally interesting
- **The assumption describes expected input**
  - The input that can be produced by the relevant contexts
- **The guarantee describes the output the specified component is obligated to produce as long as the context behaves in accordance with the assumption**

Graphical illustration of the A-G paradigm



Pre-post specifications

Pre-post specifications are based on the assumption-guarantee paradigm

Integer division

var dividend, divisor, quotient, rest : Nat

pre divisor $\neq$ 0

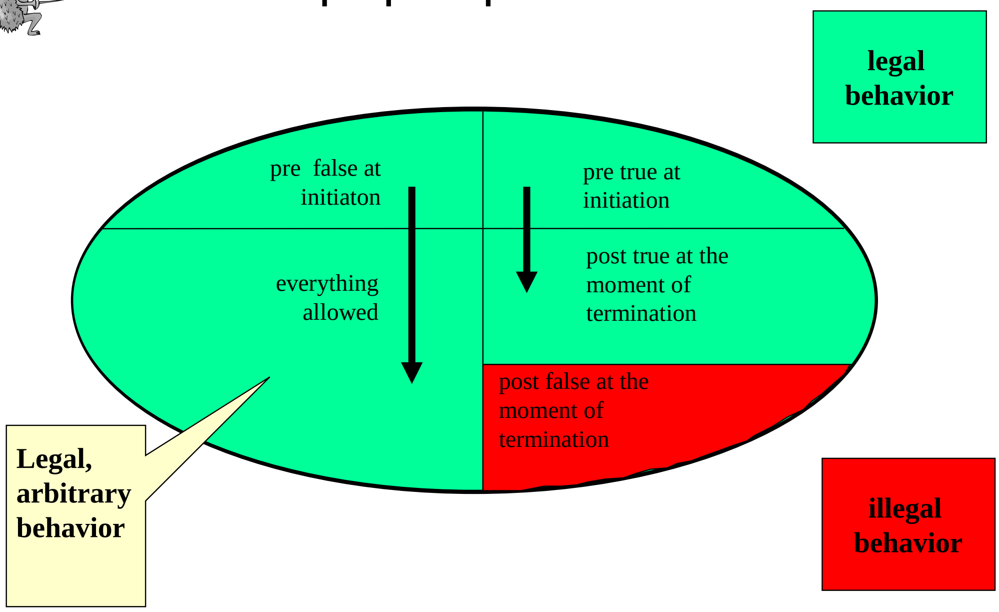
Assumption about the state at the moment the execution is initiated

post (dividend = (quotient’ * divisor) + rest’) &

rest’ < divisor

Guarantee with respect to the state at the moment of termination

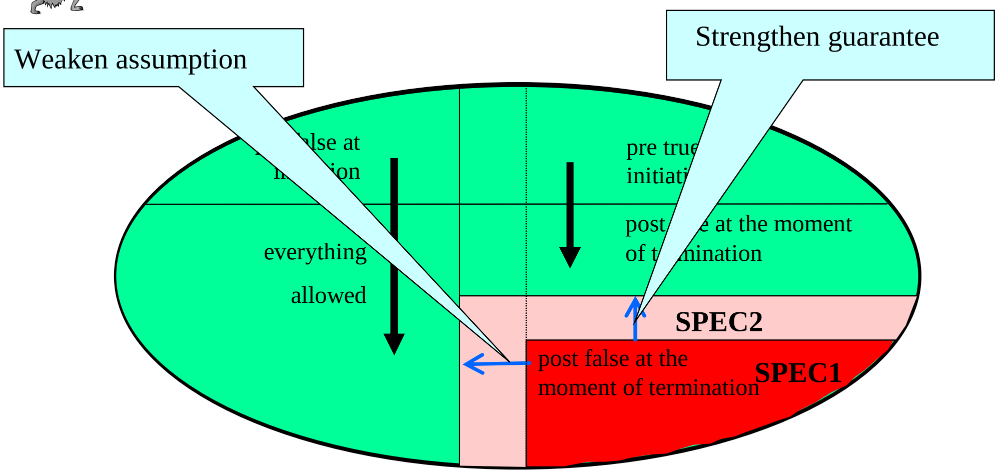
Semantics for pre-post specification



Semantics for pre-post specifications

- A state is a function from the set of variable names to type correct values
  - e.g.,
    - state(dividend)=600
    - state(divisor)=6
    - state(quotient)=100
- A state S satisfies a pre-condition if the condition evaluates to true when for any variable v
  - S(v) is substituted for each occurrence of v in the condition
- A pair of states (S,S’) satisfies a post-condition if the condition evaluates to true when for any variable v
  - S(v) is substituted for each occurrence of v in the condition
  - S’(v) is substituted for each occurrence of v’ in the condition
- The semantics of a pre-post specification is the set of all pairs of states (S,S’) such that
  - S satisfies pre and (S,S’) satisfies post, or
  - S does not satisfy pre
  - In other words: pre(S) => post(S,S’)
- We use [SPEC] to denote the semantics of the pre-post specification SPEC

Property refinement for pre-post specifications



SPEC2 is a property refinement of SPEK1
if [SPEC2] is contained in [SPEC1]
This corresponds to logical implication

Weakening the pre-condition (assumption)

Integer division

var dividend, divisor, quotient, rest : Nat

pre **true**

post

if divisor $\neq$ 0 then

(dividend = (quotient' * divisor) + rest') & rest' < divisor

else quotient' = 0

Strengthening the post-condition (guarantee)

Integer division

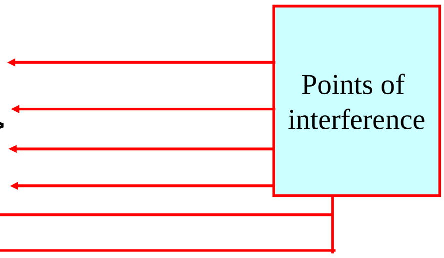
var dividend, divisor, quotient, rest : Nat

pre divisor $\neq$ 0

post (dividend = (quotient' * divisor) + rest') &
rest' < divisor & dividend' = dividend &
divisor' = divisor

The shortcomings of pre-post specifications

- Pre-condition describes only what the context may do before the operation is started up – not what the context may do during the execution of the operation
  - pre { divisor $\neq$ 0 }
  - <quotient := 0>
  - while <dividend > divisor> do
    - <dividend := dividend - divisor>
    - <quotient := quotient + 1>
  - od
  - <rest:=dividend>
  - post { (dividend' = (quotient * divisor') + rest) & rest < divisor' }



- "<Statement>" denotes that "statement" is atomic (in the meaning that the context cannot interfere with its execution)

Traces

- Traces are used to represent system runs matematically
- In the literature there are many different kinds of traces
- INF 5150 traces are sequences of events

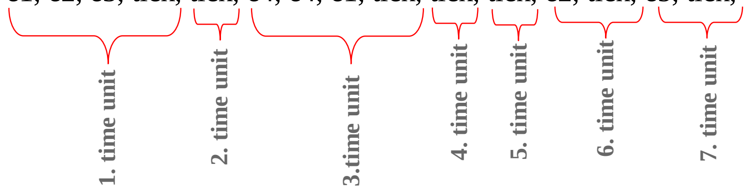
<e1, e2, e3, e4, e4, e1, e2, e5, ……………>

- Events are instantaneous
- The number of events in a trace may be finite
  - may be caused by: termination, deadlock, infinite waiting, system crash
- The number of events in a trace may be infinite
  - May be cause by: nontermination, livelock, nontermination by purpose

Traces with time ticks

- Traces are infinite sequences of events and time ticks

<e1, e2, e3, tick, tick, e4, e4, e1, tick, tick, tick, e2, tick, e5, tick, ……………>



- Events and time ticks are instantaneous
- Each trace contains infinitely time ticks
  – this reflects that time never halts
- The number of events in a trace may be finite

Traces with time stamps

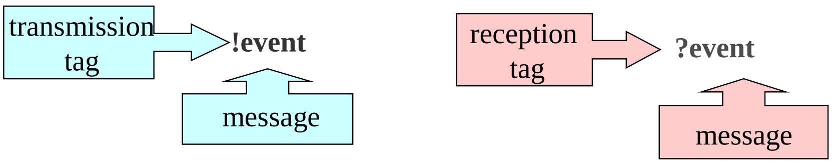
- Each element of the trace is a pair of an event and a time stamp

<e1:t1, e2:t2, e3:t3, e4:t4, e4:t5, e1:t6, e2:t7, e5:t8, ……………>

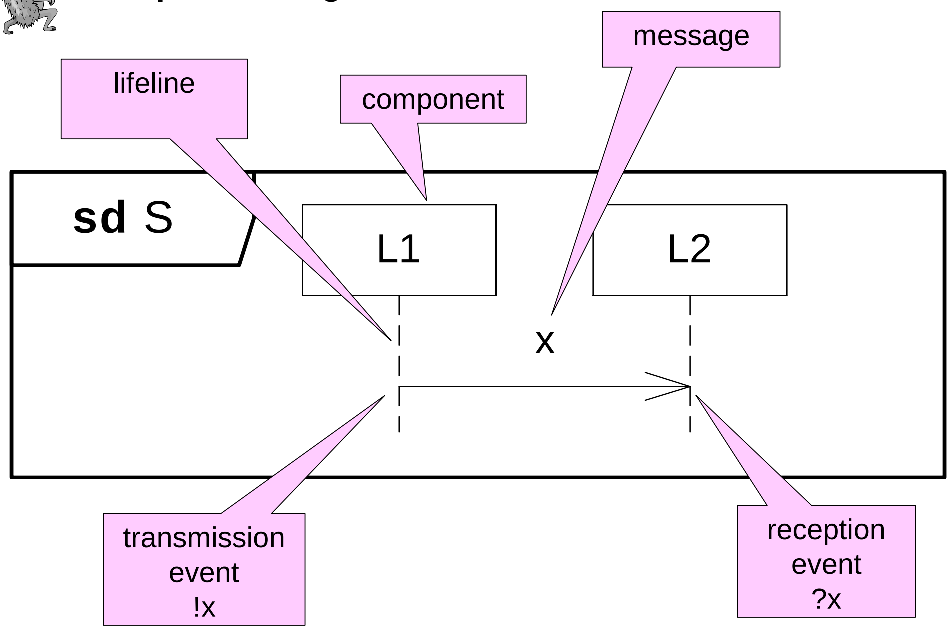
- The elements are ordered according to their time stamps
  – (t1<=t2<=t3 ….)
- Events are instantaneous
- A trace is either finite or there is for every point in time k an element n:t with time stamp t such that k< t
  – this is necessary to avoid Zenon's paradox

Traces for sequence diagrams

- **Two kinds of events:**
  - **transmission events**
  - **reception events**



Sequence diagram

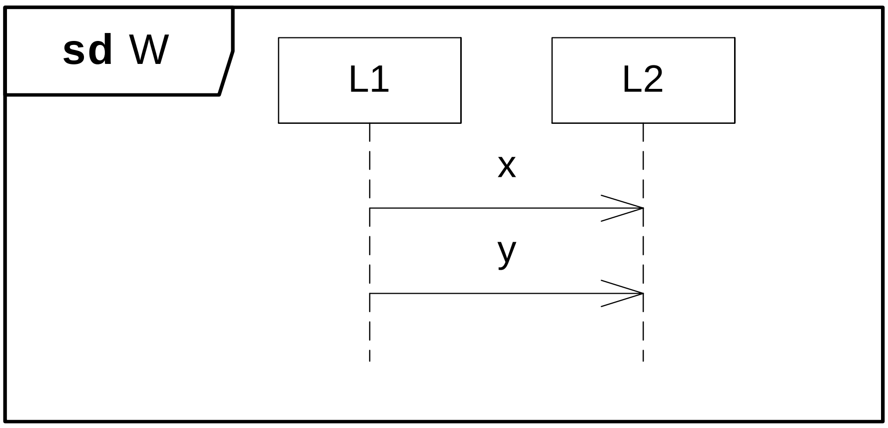


Causality and weak sequencing

- **Causality:**
  - a message can never be received before it has been transmitted
  - the transmission event for a message is therefore always ordered before the reception event for the same message
- **Weak sequencing:**
  - events from the same lifeline are ordered in the trace in the same order as on the lifeline

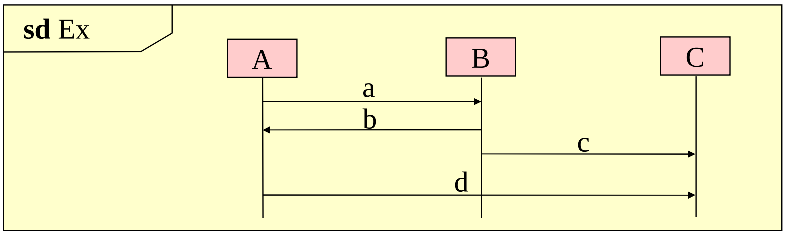
- **NOTE: A sequence diagram will normally be represented by more than one trace, and in some cases by infinitely many traces**

Weak sequencing



<!x,?x,!y,?y>
<!x,!y,?x,?y>

Example



There are six possible traces if time information is ignored:

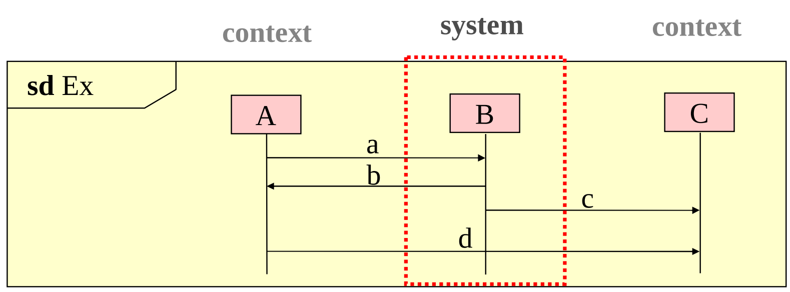
<!a, ?a, !b, ?b, !c, ?c, !d, ?d>
<!a, ?a, !b, ?b, !c, !d, ?c, ?d>
<!a, ?a, !b, ?b, !d, !c, ?c, ?d>
<!a, ?a, !b, !c, ?b, ?c, !d, ?d>
<!a, ?a, !b, !c, ?b, !d, ?c, ?d>
<!a, ?a, !b, !c, ?c, ?b, !d, ?d>

Each of these corresponds to infinitely many traces with time information

External behavior

- **Property refinement in the classical sense takes only external behavior into consideration**
- **We therefore need a well-defined interface between**
  - **the component to be refined, and**
  - **its context**

Projection on B



System has one possible external trace:

<?a, !b, !c>

This trace is an abstraction of infinitely many traces with time information

– STAIRS –
Steps to Analyze Sequence Diagrams with Refinement Semantics

Motivation

- Make use of classical refinement theory in a practical UML setting
  - From theory to practice, and not the other way around
- We aim to explain how classical theory of refinement can be used to refine specifications expressed with the help
- Sequence diagrams can be used to explain other kinds of UML diagrams
- By defining refinement for sequence diagrams we implicitly define refinement for the UML as a whole

Requirements to STAIRS

- Should support specification of potential behavior
  - Means to abstraction
- Should support specification of mandatory behavior
  - Important within the security domain
- Should support specification of negative behavior in addition to positive behavior
- Should support classical refinement theory
- Should formalize incremental system development
- Should facilitate modular analysis, verification and testing

Next lecture on refinement – September 29

- Example based introduction to STAIRS
- Semantics of sequence diagrams
- Refinement in STAIRS
  - Supplementing
  - Narrowing
  - Detailing
- Relation to pre-post