

## INF5150 Suggested solutions to exercises 2008-10-06

1. As usual we let  $\mathcal{H}$  denote the set of all well-formed traces, and  $\emptyset$  denote the empty set.  $\setminus$  is the symbol for set-minus, so  $S_1 \setminus S_2$  denotes the set containing all elements that are in  $S_1$  but not in  $S_2$ .

To compute the traces of Ex 1, Ex2 and Ex3 we need the definitions of seq, refuse, veto and assert:

### Weak sequencing of trace sets:

$s_1 \succcurlyeq s_2$  denotes the set of all traces that may be constructed by selecting one trace  $t_1$  from  $s_1$  and one trace  $t_2$  from  $s_2$  and combining them in such a way that for each lifeline, the events from  $t_1$  comes before the events from  $t_2$ .

Formally:

$$s_1 \succcurlyeq s_2 \stackrel{\text{def}}{=} \{h \in \mathcal{H} \mid \exists h_1 \in s_1, h_2 \in s_2 : \forall l \in \mathcal{L} : h \upharpoonright l = h_1 \upharpoonright l \circ h_2 \upharpoonright l\}$$

Note: if  $s_1$  or  $s_2$  is empty then  $s_1 \succcurlyeq s_2$  is also empty

### Weak sequencing of interaction obligations:

$$(p_1, n_1) \succcurlyeq (p_2, n_2) \stackrel{\text{def}}{=} (p_1 \succcurlyeq p_2, (n_1 \succcurlyeq p_2) \cup (n_1 \succcurlyeq n_2) \cup (p_1 \succcurlyeq n_2))$$

### seq:

$$[[d_1 \text{ seq } d_2]] \stackrel{\text{def}}{=} \{o_1 \succcurlyeq o_2 \mid o_1 \in [[d_1]] \wedge o_2 \in [[d_2]]\}$$

### refuse:

$$[[\text{refuse } d]] \stackrel{\text{def}}{=} \{(\emptyset, p \cup n) \mid (p, n) \in [[d]]\}$$

### veto:

$$[[\text{veto } d]] = \{(\langle \rangle, p \cup n) \mid (p \cup n) \in [[d]]\}$$

### assert:

$$[[\text{assert } d]] \stackrel{\text{def}}{=} \{(p, n \cup (\mathcal{H} \setminus p)) \mid (p, n) \in [[d]]\}$$

$$\text{Let } t_1 = \langle !e, ?e, !f, ?f \rangle \quad t_2 = \langle !e, !f, ?e, ?f \rangle \quad t_3 = \langle !e, ?e \rangle$$

$$\begin{aligned} [[\text{Ex1}]] &= \{ (\langle !e, ?e \rangle, \emptyset) \succcurlyeq (\emptyset, \langle !f, ?f \rangle) \} \\ &= \{ (\langle !e, ?e \rangle \succcurlyeq \emptyset, (\emptyset \succcurlyeq \emptyset) \cup (\emptyset \succcurlyeq \langle !f, ?f \rangle) \cup (\langle !e, ?e \rangle \succcurlyeq \langle !f, ?f \rangle)) \} \\ &= \{ (\emptyset, \{t_1, t_2\}) \} \end{aligned}$$

$$\begin{aligned} [[\text{Ex2}]] &= \{ (\langle !e, ?e \rangle, \emptyset) \succcurlyeq (\langle \rangle, \langle !f, ?f \rangle) \} \\ &= \{ (\{t_3\}, \{t_1, t_2\}) \} \end{aligned}$$

$$\begin{aligned} [[\text{Ex3}]] &= \{ (\langle !e, ?e \rangle, \emptyset) \succcurlyeq (\langle !f, ?f \rangle, \mathcal{H} \setminus \langle !f, ?f \rangle) \} \\ &= \{ (\{t_1, t_2\}, n) \}, \text{ where} \\ & n = \{t \in \mathcal{H} \mid \text{the first event on lifeline } y \text{ is } !e \text{ and the first event on lifeline } x \text{ is } ?e\} \setminus \end{aligned}$$

$\{ \langle !e, ?e, !f, ?f \rangle, \langle !e, !f, ?e, ?f \rangle \}$ .

This means that the set  $n$  contains all traces where  $!e$  is the first event on lifeline  $y$  and  $?e$  is the first event on lifeline  $x$ , except from the traces  $\langle !e, ?e, !f, ?f \rangle$  and  $\langle !e, !f, ?e, ?f \rangle$ .

To compute  $Ex_4$  we also need the definition of  $alt$ :

**alt:**

$[[d_1 \text{ alt } d_2]] \stackrel{\text{def}}{=} \{ o_1 \uplus o_2 \mid o_1 \in [[d_1]] \wedge o_2 \in [[d_2]] \}$ , where

$(p_1, n_1) \uplus (p_2, n_2) \stackrel{\text{def}}{=} (p_1 \cup p_2, n_1 \cup n_2)$

Let

$t_4 = \langle !a, ?a, !b, ?b \rangle$

$t_5 = \langle !a, !b, ?a, ?b \rangle$

$t_6 = \langle !c, ?c \rangle$

$t_7 = \langle !a, ?a, !b, ?b, !e, ?e \rangle$

$t_8 = \langle !a, !b, ?a, ?b, !e, ?e \rangle$

$t_9 = \langle !a, ?a, !b, ?b, !e, ?e, !f, ?f \rangle$

$t_{10} = \langle !a, !b, ?a, ?b, !e, ?e, !f, ?f \rangle$

$t_{11} = \langle !a, ?a, !b, ?b, !e, !f, ?e, ?f \rangle$

$t_{12} = \langle !a, !b, ?a, ?b, !e, !f, ?e, ?f \rangle$

$t_{13} = \langle !c, ?c, !e, ?e \rangle$

$t_{14} = \langle !c, ?c, !e, ?e, !f, ?f \rangle$

$t_{15} = \langle !c, ?c, !e, !f, ?e, ?f \rangle$

$[[Ex_4]] = \{ (\{t_4, t_5\}, \emptyset) \uplus (\emptyset, \{t_6\}) \}$   
 $= \{ (\{t_4, t_5\}, \{t_6\}) \}$

$[[Ex_5]] = [[Ex_4 \text{ seq } Ex_2]]$

$= \{ (\{t_4, t_5\} \succ \{t_3\}, (\{t_6\} \succ \{t_3\}) \cup (\{t_6\} \succ \{t_1, t_2\})) \cup (\{t_4, t_5\} \succ \{t_1, t_2\}) \}$   
 $= \{ (\{t_7, t_8\}, \{t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14}, t_{15}\}) \}$

To compute  $Ex_6$  we need the definitions of  $xalt$  and constraints:

**xalt:**

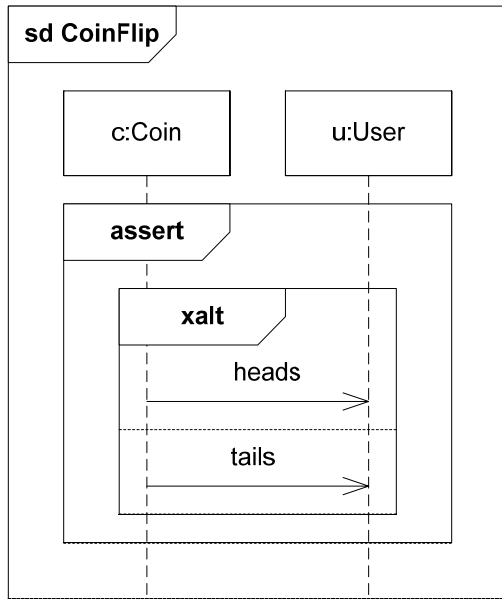
$[[d_1 \text{ xalt } d_2]] \stackrel{\text{def}}{=} [[d_1]] \cup [[d_2]]$

**Constraints (i.e., guards):**

$[[ \text{constr}(c) ]] \stackrel{\text{def}}{=} \{ \langle \langle \text{check}(\sigma) \rangle \mid c(\sigma) \rangle, \langle \langle \text{check}(\sigma) \rangle \mid \neg c(\sigma) \rangle \}$

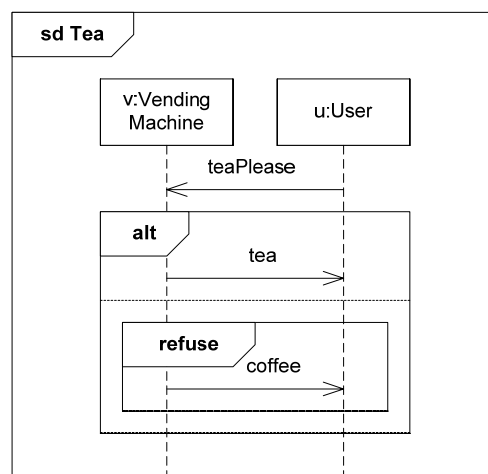
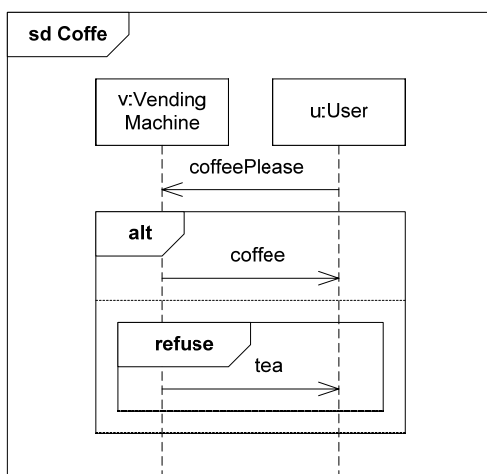
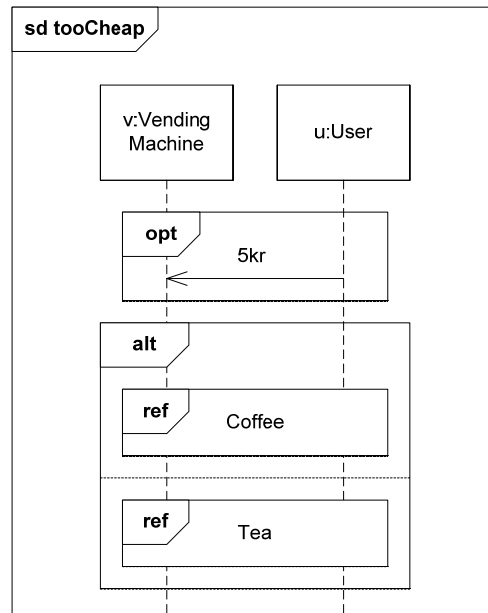
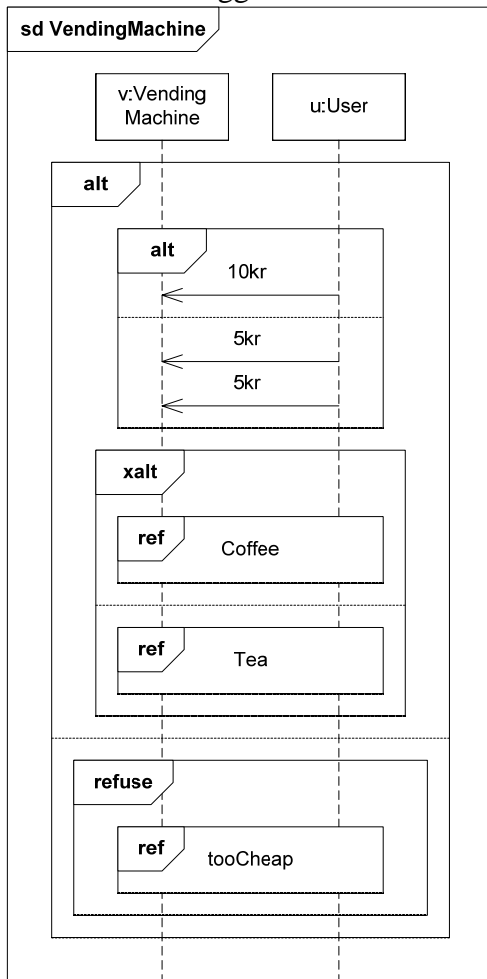
$[[Ex_6]] = \{ (\langle \langle \text{chk}(att=7), !a, ?a \rangle \rangle, \langle \langle \text{chk}(att \neq 7), !a, ?a \rangle \rangle) \} \cup$   
 $\{ (\langle \langle \text{chk}(att \neq 7), !b, ?b, !c, ?c \rangle \rangle, \langle \langle \text{chk}(att=7), !b, ?b, !c, ?c \rangle \rangle) \}$   
 $= \{ (\langle \langle \text{chk}(att=7), !a, ?a \rangle \rangle, \langle \langle \text{chk}(att \neq 7), !a, ?a \rangle \rangle),$   
 $(\langle \langle \text{chk}(att \neq 7), !b, ?b, !c, ?c \rangle \rangle, \langle \langle \text{chk}(att=7), !b, ?b, !c, ?c \rangle \rangle) \}$

2.



We choose to use limited refinement. This ensures that no new interaction obligations representing other outcomes are introduced.

3. Here is one suggestion:



Note that all traces from the “tooCheap” specification become negative in both interaction obligations in the VendingMachine specification.

We choose to use general refinement, in order to leave open the possibility that the final implementation also offers other drinks.

4. The mapping L is given by

$$L = \{ p:\text{PaymentHandler} \mapsto v:\text{VendingMachine}, d:\text{drinkPreparator} \mapsto v:\text{VendingMachine}, u:\text{User} \mapsto u:\text{User} \}$$

Notice that we have broken a UML principle since it is not clear from the diagrams that  $d:\text{drinkPreparator}$  and  $p:\text{PaymentHandler}$  are parts of  $v:\text{VendingMachine}$

