



Solution to exercises

September 22, 2008



1)

```
Real division
var dividend, divisor, quotient : Real

pre divisor ≠ 0

post ( dividend = quotient' * divisor)
```

```
Real division
var dividend, divisor, quotient : Real

pre true

post
  if divisor ≠ 0 then
    ( dividend = quotient' * divisor )
  else quotient' = 0
```

```
Real division
var dividend, divisor, quotient : Real

pre divisor ≠ 0

post ( dividend = quotient' * divisor)
     dividend' = dividend &
     divisor' = divisor
```

2)

Integer division

var dividend, divisor, quotient, rest : Real

pre divisor $\neq 0$ & dividend \in Nat & divisor \in Nat

post (dividend = (quotient' * divisor) + rest') &
rest' < divisor & quotient' \in Nat

Integer division

var dividend, divisor, quotient, rest : Real

pre dividend \in Nat & divisor \in Nat

post

if divisor $\neq 0$ then

 (dividend = (quotient' * divisor) + rest') & rest' < divisor & quotient' \in Nat

else quotient' = 0

Integer division

var dividend, divisor, quotient, rest : Real

pre divisor $\neq 0$ & dividend \in Nat & divisor \in Nat

post (dividend = (quotient' * divisor) + rest') &
rest' < divisor & dividend' = dividend &
divisor' = divisor & quotient' \in Nat

3) The specifications in exercise 2) strengthen both the assumptions/pre-conditions and guarantees/post-conditions of the specifications in exercise 1) (by adding new conjuncts). They are therefore not refinements of the specifications in exercises 1).

4) The specifications in exercise 1) weaken both the assumptions/pre-conditions and the guarantees/post-conditions of the specifications in exercise 2). They are therefore not refinements of the specifications in exercise 2).

5) NOTE: In this exercise we only work with positive numbers

8-bit addition

var $x, y, z : \text{Nat}$

pre $x + y \leq 255$

post $z' = x + y$

8-bit subtraction

var $x, y, z : \text{Nat}$

pre $x \leq 255 \ \& \ y \leq x$

post $x = z' + y$

8-bit multiplication

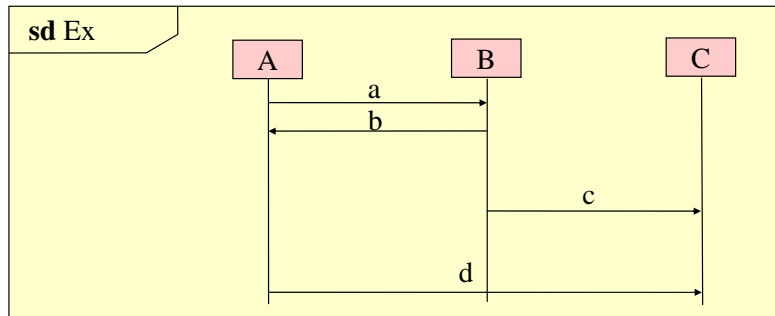
var $x, y, z : \text{Nat}$

pre $x \times y \leq 255$

post $z' = x \times y$



6)

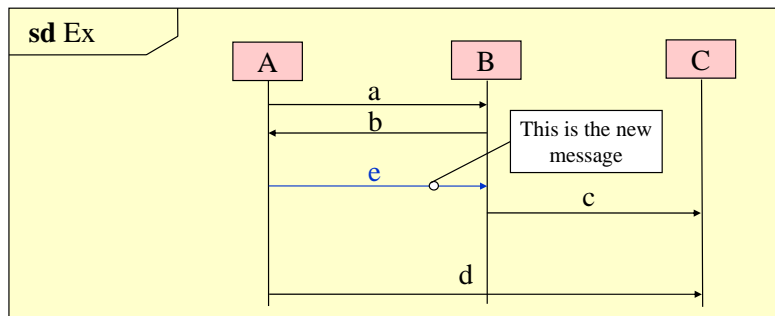


In the original diagram there are six possible traces:

- <!a, ?a, !b, ?b, !c, ?c, !d, ?d>
- <!a, ?a, !b, ?b, !c, !d, ?c, ?d>
- <!a, ?a, !b, ?b, !d, !c, ?c, ?d>
- <!a, ?a, !b, !c, ?b, ?c, !d, ?d>
- <!a, ?a, !b, !c, ?b, !d, ?c, ?d>
- <!a, ?a, !b, !c, ?c, ?b, !d, ?d>



6)

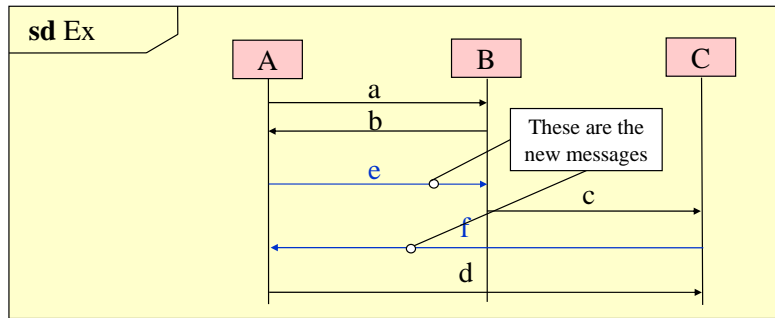


By adding one message we reduce the number of possible traces to four:

- <!a, ?a, !b, ?b, !e, ?e, !c, ?c, !d, ?d>
- <!a, ?a, !b, ?b, !e, ?e, !c, !d, ?c, ?d>
- <!a, ?a, !b, ?b, !e, !d, ?e, !c, ?c, ?d>
- <!a, ?a, !b, ?b, !e, ?e, !d, !c, ?c, ?d>



7)



By adding two messages we reduce the number of possible traces to one:

<!a, ?a, !b, ?b, !e, ?e, !c, ?c, !f, ?f, !d, ?d>



8) & 9) Traces

• Weak sequencing:

- events from the same lifeline are ordered in the trace in the same order as on the lifeline
- events on different lifelines from different operands may come in any order

