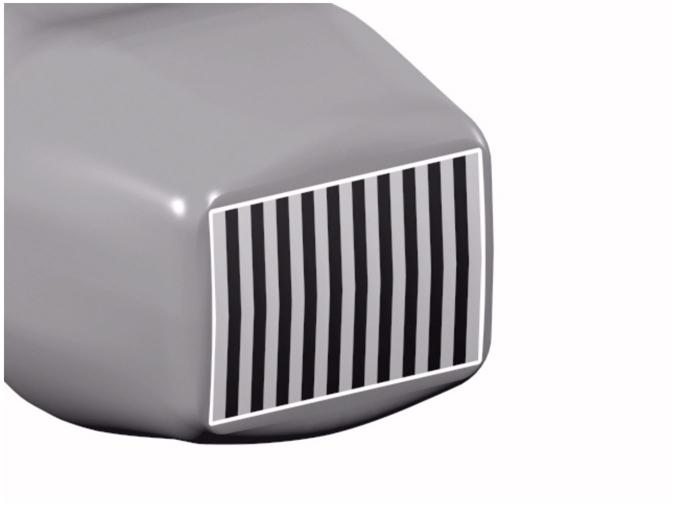
Experience with sparse arrays at the University of Oslo

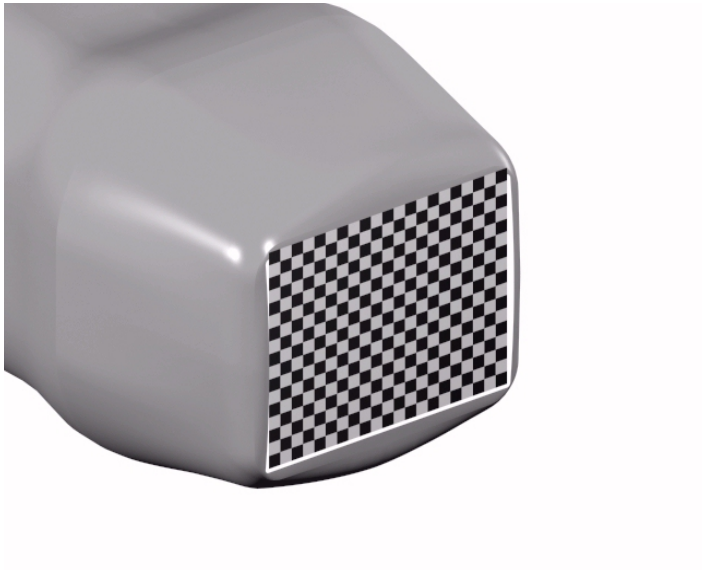
Sverre Holm,

Andreas Austeng,

Jan Egil Kirkebø

29th ESA Antenna Workshop on
Multiple Beams and Reconfigurable Antennas
18 - 20 April 2007





Overview

- Building blocks in thinned arrays
  - Random arrays
  - Binned arrays
  - Periodic arrays
- Optimization of one-way beampattern
  - Linear Programming
  - Genetic algorithms
  - Simulated annealing
- Optimization of two-way beampattern
  - Different tx and rx layouts
- Non-flat arrays

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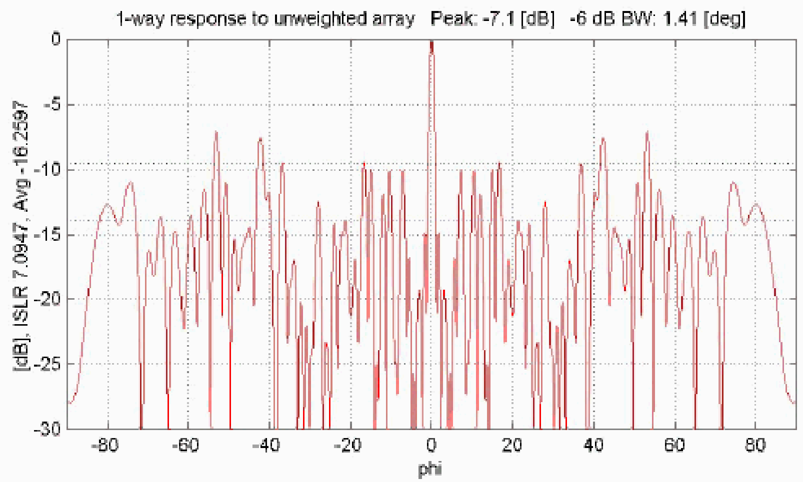
Random array

- Average array pattern = aperture weighted with pdf of thinning => Determines main lobe
- Normalized average sidelobe level

  1/K
- Variance is about K for angles larger than

  $|\sin \phi| > \lambda/L$

  L is the aperture.
- Peak sidelobe level



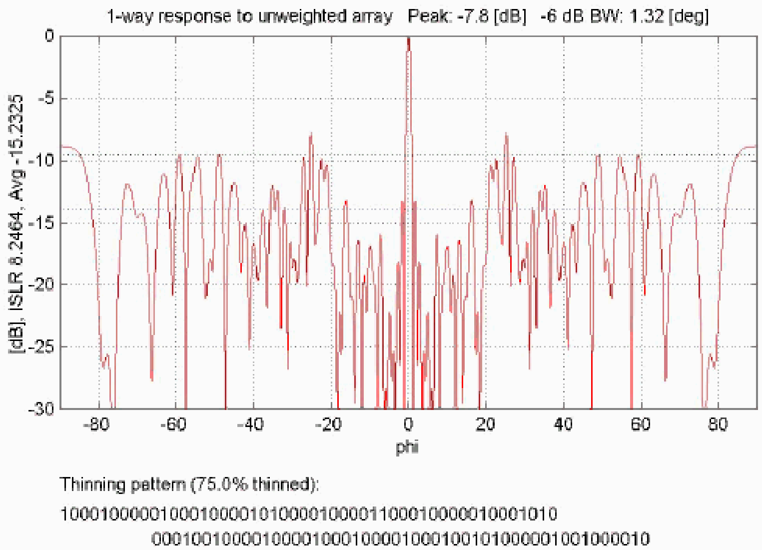
Thinning pattern (75.2% thinned):
10000000000000101000001010000000100101000000000100100 1
0000010001011000011000000011010010000000100100000101

- Ex: K=25, M=101, uniform pdf

Binned array / nearest neighbor restriction

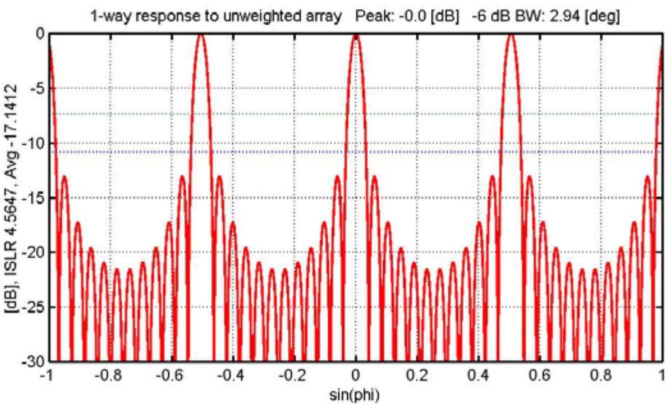
- Divide array into K equal bins
- Select 1 element at random in each bin
- Max two neighbors
- Resembles random array, but variance does not reach full value until

  $|\sin \phi| > K$ ¢ λ/L
- Ex: K=25, M=100, bin size N=4,
- i.e. low sidelobes for $|\sin \phi| < 0.25$



Periodic arrays

- 1001001001, 101010101, 11001100 ...
- Grating lobes: position and size are easily predicted.
- Useful in imaging systems with different periodicities for tx ands rx.
- Two-way beampattern = product of transmitter and receiver beampatterns.
- Grating lobes from tx are suppressed by the rx and vice versa.
- Special case: Vernier array which has periodicities p and p-1.



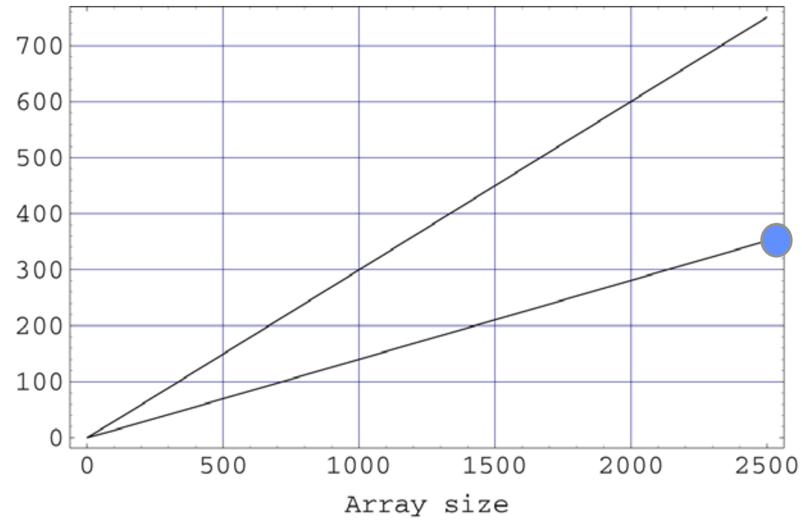
Thinning pattern (50.0% thinned):
101010101010 c 101010101010

Array thinning and full search

- Array with *M* elements and *K* active elements:

$$\binom{M}{K} = \frac{M!}{K!(M-K)}$$

- 10 $\log_{10}$ # combinations with 10% and 50% thinning
- Example: ~10^{358} for 250 elements out of 2500
- Number of electrons in universe: ~10^{80}



Optimization, reduction of search space

- 1-D array: end-elements always on to maintain aperture?
- Symmetry?
- Binned array
- Elements on a fixed grid or free element positions

Linear programming

- Guaranteed optimal solution
- Our implementation: only symmetric arrays
- In practice
  - OK for 1D combinatorial problems
  - OK for 2D weighting problems

Simulated annealing

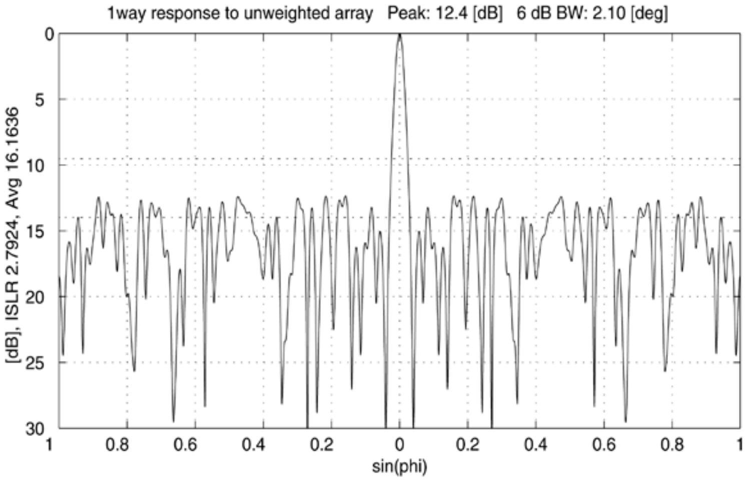
- Speed improved with a method which is faster than FFT for beam pattern evaluation
- Only perturbs a single element at a time
- Subtract the contribution of the moved element and add the contribution of the new one
- All contributions from all elements at all angles are precomputed and stored in memory
- N=256 point evaluation: 6.7 times faster than FFT
- For larger N's, the speed increase is even larger

Genetic algorithm

- Improved initialization over uniform probability distributions (first sidelobes around -13 dB)
- Cross-over does not significantly alter the pdf => pdf is still close to uniform.
- Too little randomness introduced by mutation.
- Better: Initialize search with density functions that already have desirable sidelobe properties.
- Improves convergence time and makes convergence to a good solution possible.
- Speed can be enhanced as simulated annealing, but requires much more memory as the responses of the whole population needs to be stored in memory.

1D array; simulated annealing thinning

- 25 elements from 101, classical thinning example
- Fixed end elements
- Lower CW peak than any published result, -12.36 dB
- MSc of J. F. Hopperstad
- Try it yourself – applet, folk.uio.no/sverre, hi-score list



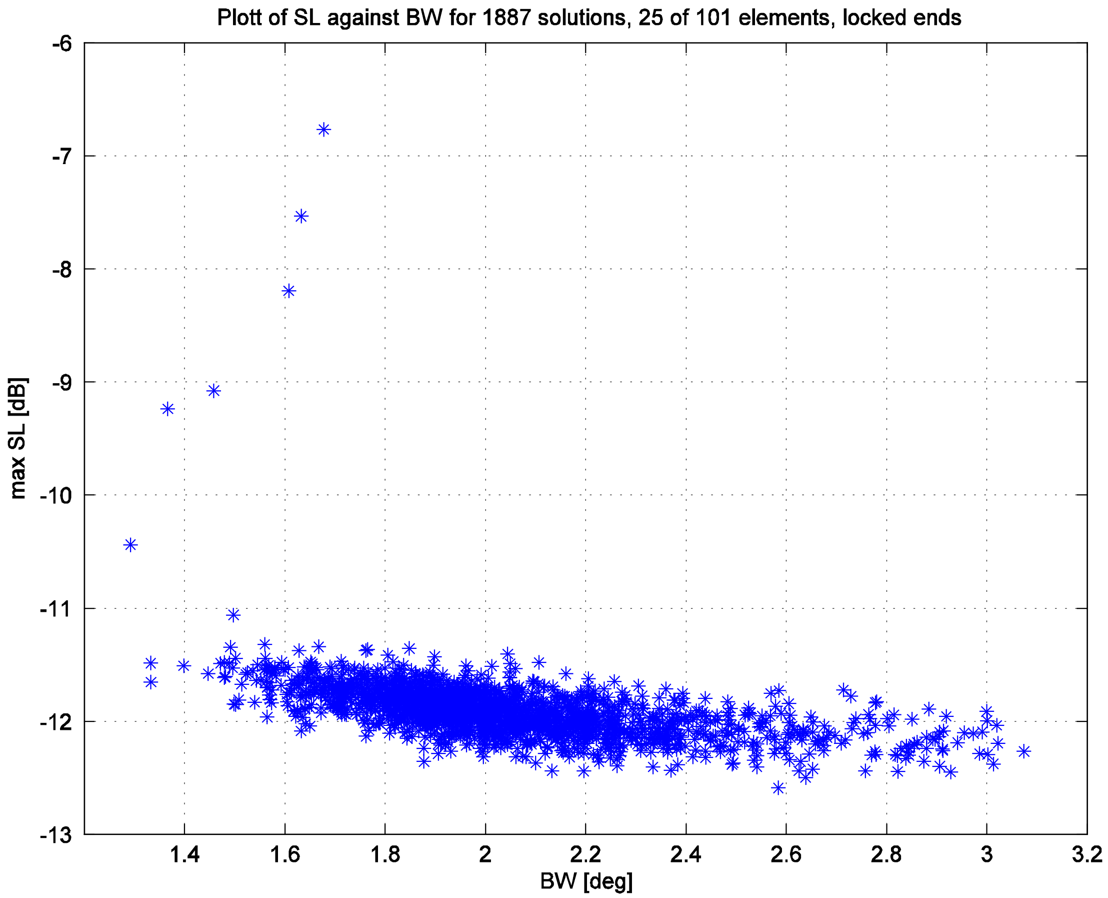
Thinning pattern (75.2% thinned):

1000000100000000000000011110101000101101000010110011 0
001100100011000001001000000000000000000000000000001

25 elements from 101, $\lambda/2$, no weights

Min sidelobe	Method	Reference
-8.8 dB	Dynamic programming	Skolnik et al. *IEEE Trans. Ant. Prop.*, Jan. 1964.
-8.9 dB	Space-tapering	Lo & Lee, *IEEE Trans. Ant. Prop.*, Jan. 1966.
-10.14 dB	Dynamic prog.	Arora et al. *IEEE Trans. Ant. Prop.*, July 1968.
-12.07 dB	Simulated annealing	Murino, Trucco, Regazzoni, *IEEE T. Sign. Proc, J*an. 1996.
-12.36 dB	Simulated annealing, fixed ends	Hopperstad, MSc thesis, Univ. Oslo, May 1998.
-12.42 dB	Simulated annealing, no conditions on ends	Austeng, Holm, IEEE NORSIG,Oct. 2002.
-12.54 dB	Simulated annealing, Java applet	Steinar H. Gunderson, NTNU, 2007
-14.09 dB	Simulated annealing, Arbitrary el's, lim. steering	Austeng, Holm, IEEE NORSIG,Oct. 2002.

Peak sidelobe vs. beamwidth

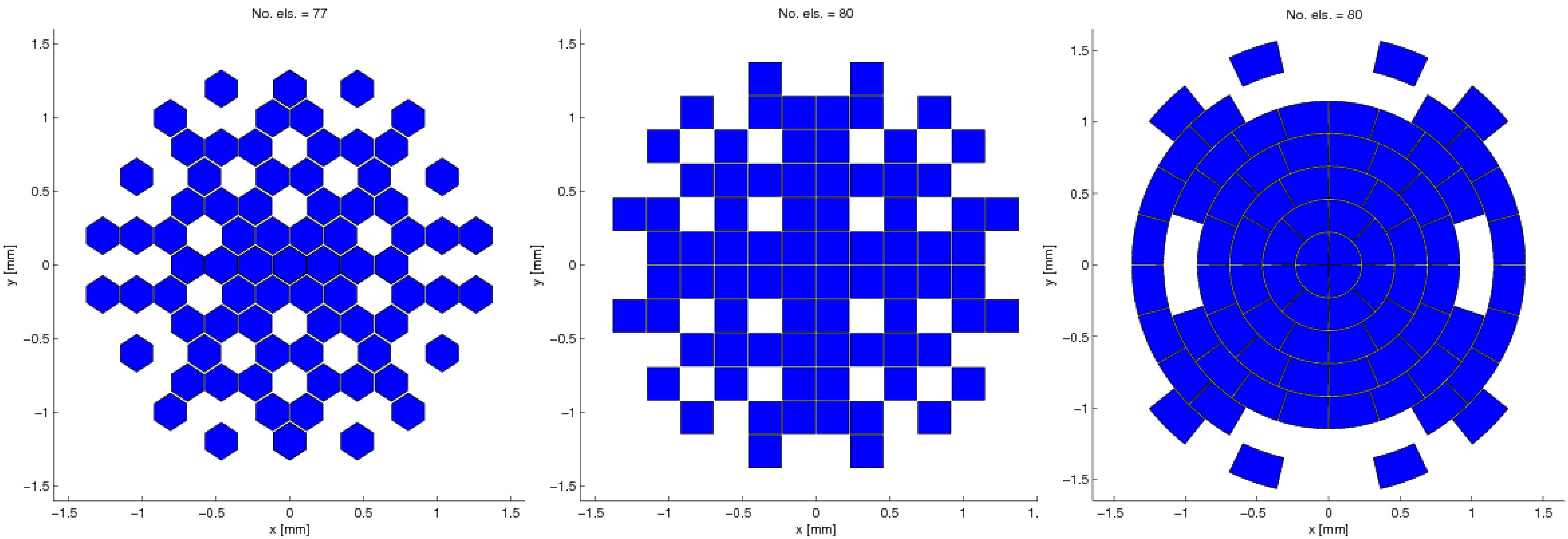


Parallel optimizations

- 25 ... 69 elements out of 121, i.e. 45 different config's
- Purpose: find lower limit on peak sidelobe level
- For each: 2000 indep. sim. annealing combinatorial optimizations, each with 800 000 beam patterns
- 45 * 2000 * 800 000 = 7.2 e10 beam patterns

- Condor: High Throughput Computing (HTC) on large collections of distributed computing resources.
- Up to 240 desktop computers, in practice 100 – 200 in use at the same time
- Throughput: about 1000 optimizations per hour
- Svein Bøe

Sim. anneal for arbitrary layouts



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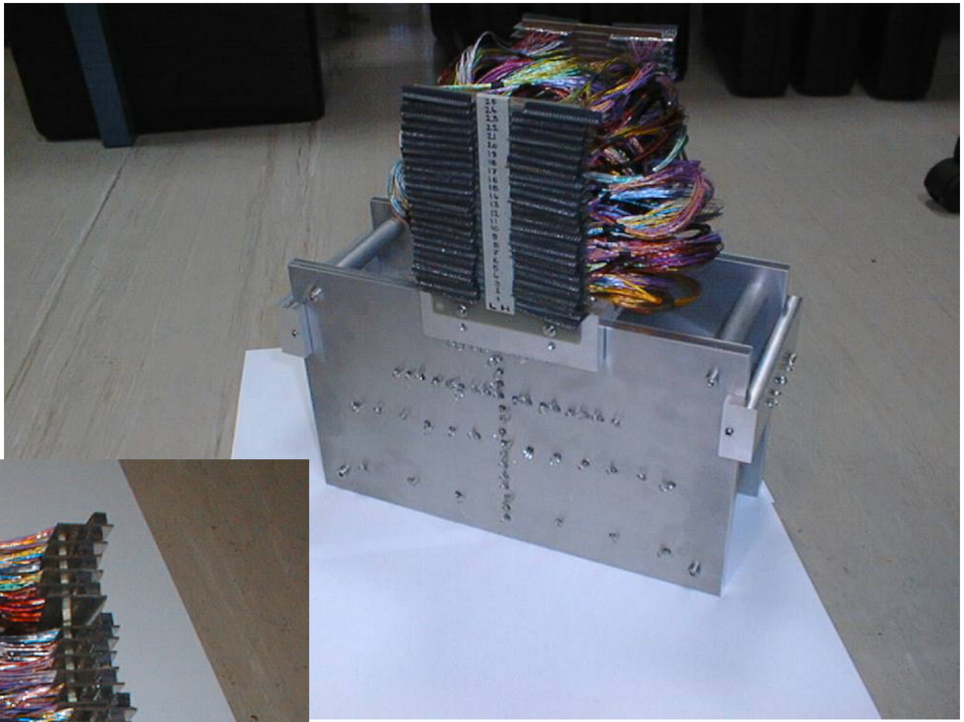
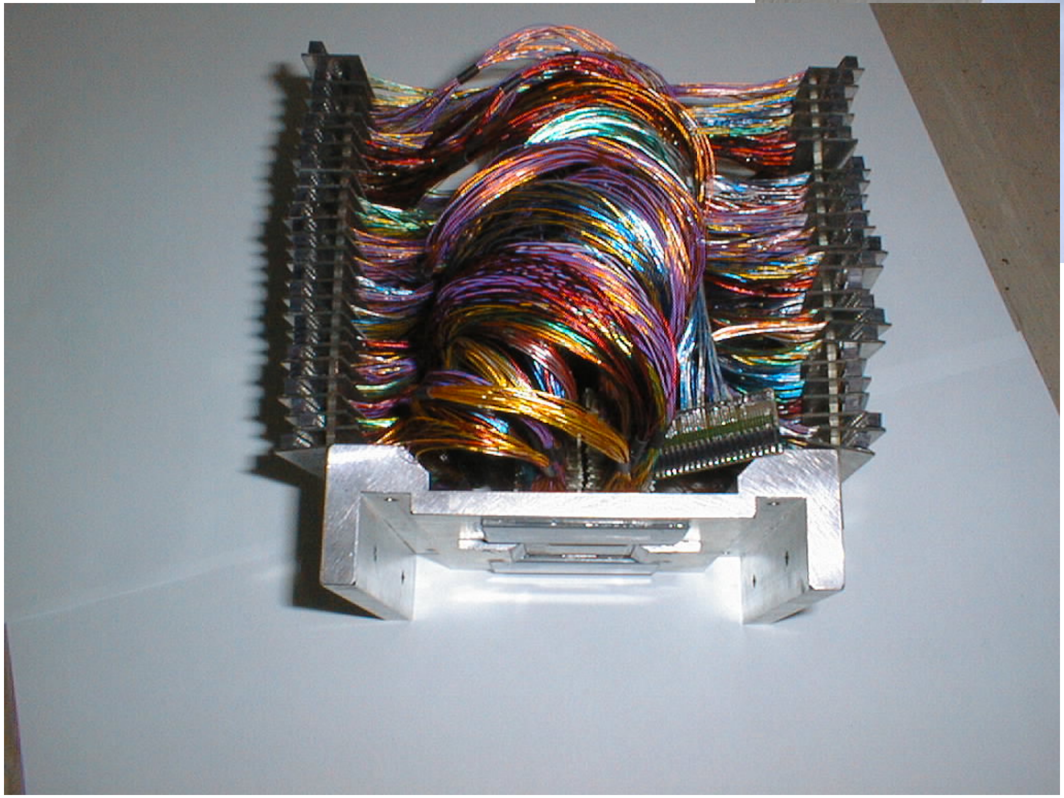
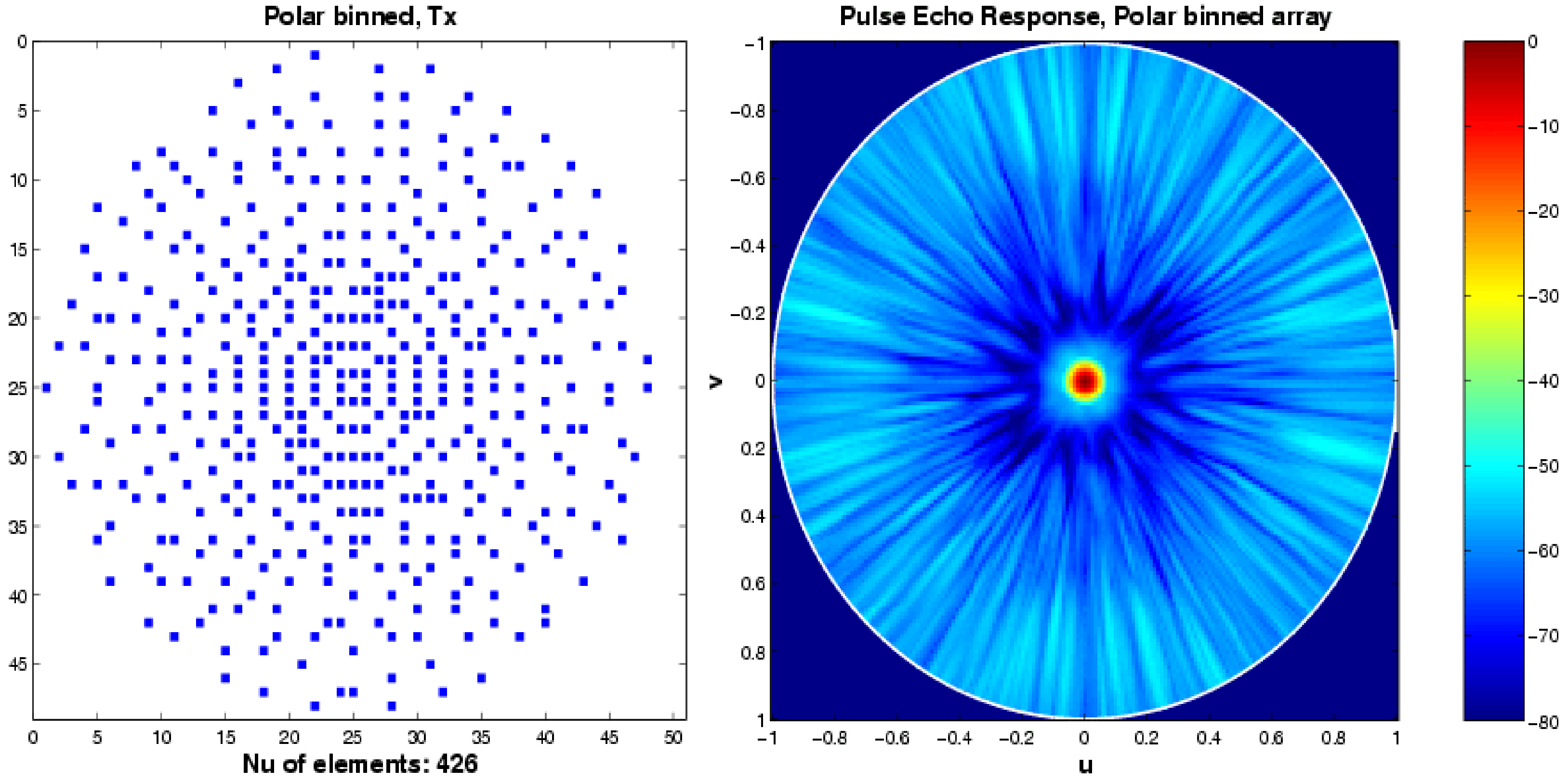




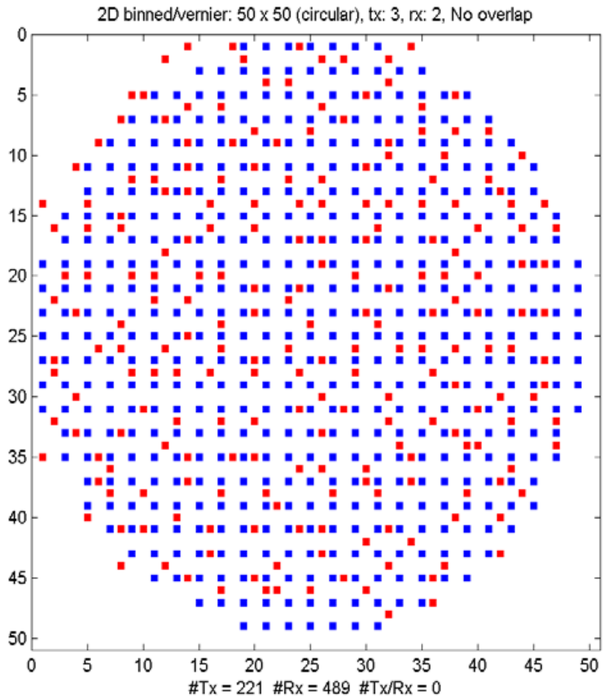
50 x 50 element array from Thomson Micro-sonics (IEEE Ultrason. Symposium, 1998)

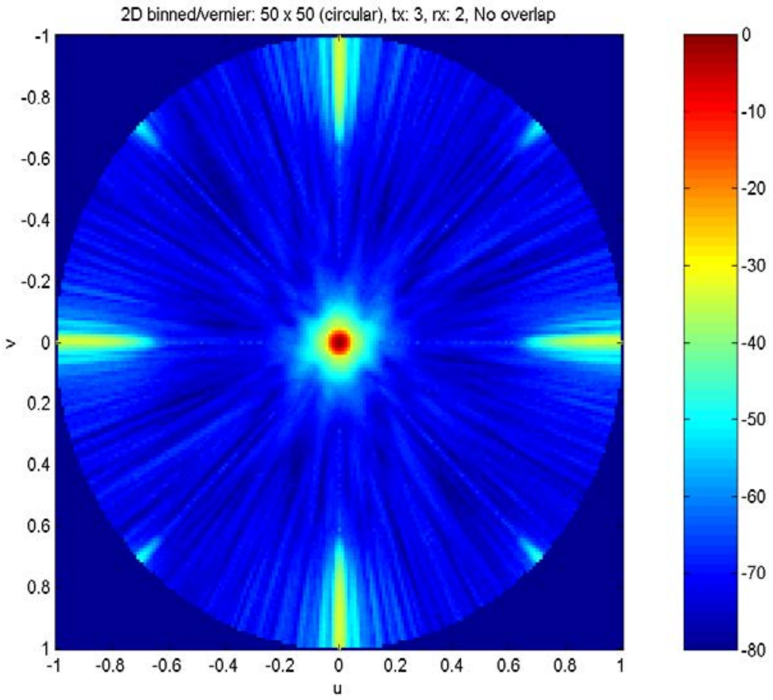
Binning in a polar pattern (Rx=Tx)

Tested



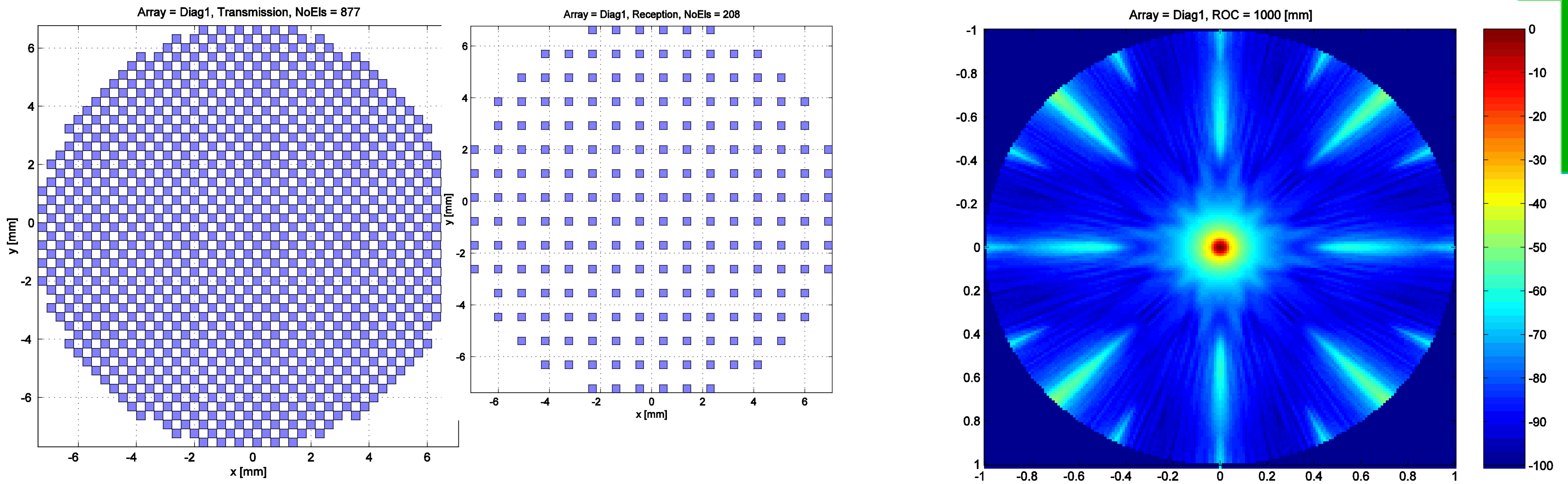
Periodic + binned
489 Rx, 221 Tx : no overlap





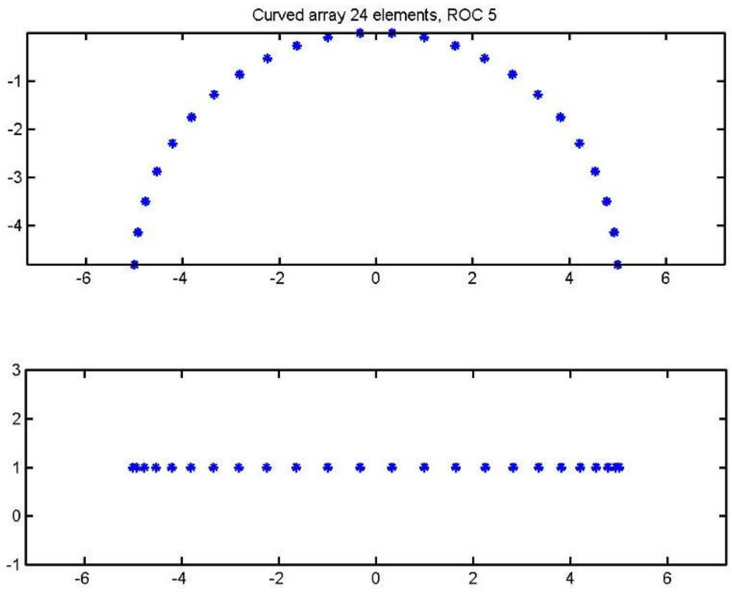
Diagonal periodic tx, periodic rx

Diag1 array, Tx = 877 els., Rx = 208 els., max. SL. = -50 dB



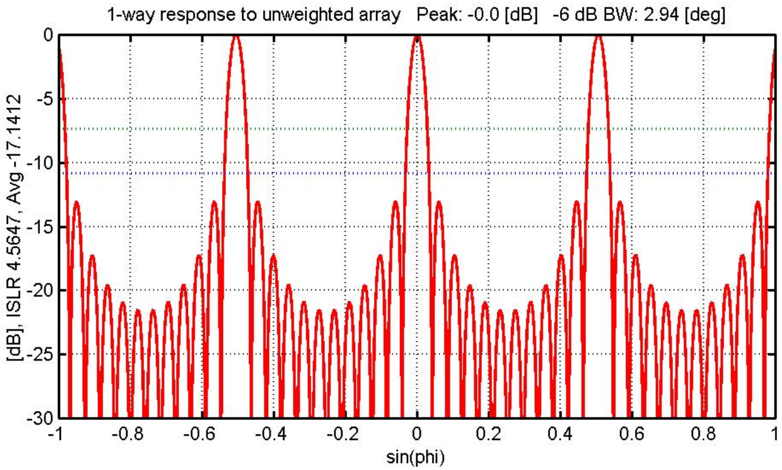
Austeng, Holm,
T. UFFC, 2002

Element shadowing property

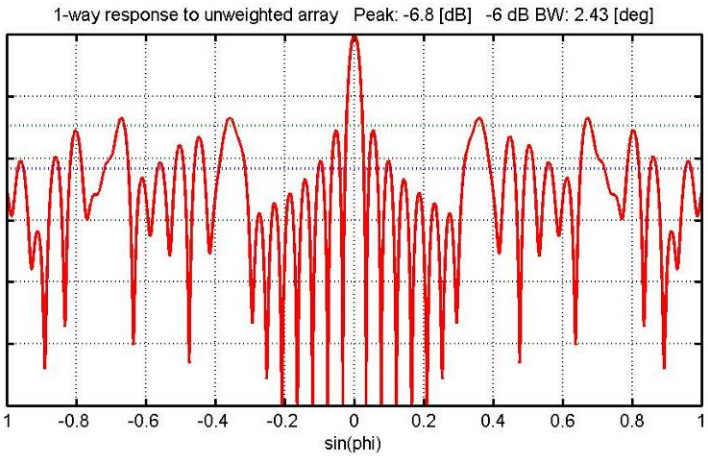


- Beampattern is given by Fourier transform of projection
- Curving breaks the periodicity

Grating lobes: Curved vs. Linear



Thinning pattern (50.0% thinned):
101010101010 c 101010101010



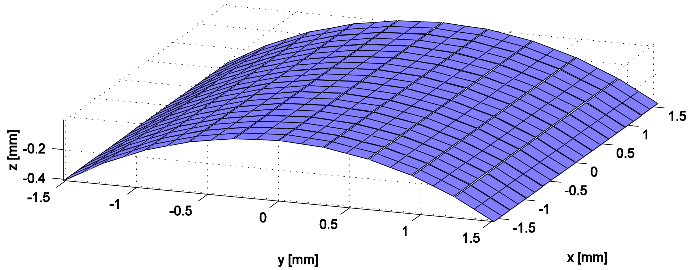
Thinning pattern (50.0% thinned):
101010101010 c 101010101010

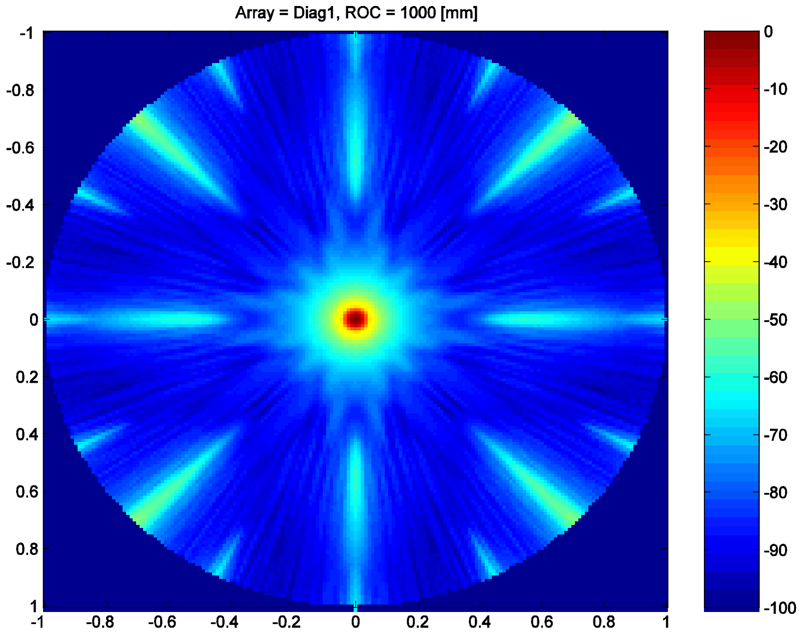
24 elements over 23.73 lambda aperture/chord, 50% periodic thinning

Periodic arrays suitable for curving

•When curved in one direction, grating lobes at angles other than those orthogonal to the curving are suppressed.

•Curving along y-axis
=> Periodicity along diagonals.





Conclusions

- Building blocks: Random, random binned and periodic arrays
- Linear programming, genetic methods and simulated annealing.
- Preference for simulated annealing method
- Examples:
  - ULA, 2D uniform flat array
  - Optimized arrays on non-Cartesian grids such as polar and hex.
  - Non-planar arrays (curved and cylindrical) have also been optimized.
- Are working on establishing an empirical lower limit on peak sidelobe level for a given array with a given percentage of thinning.

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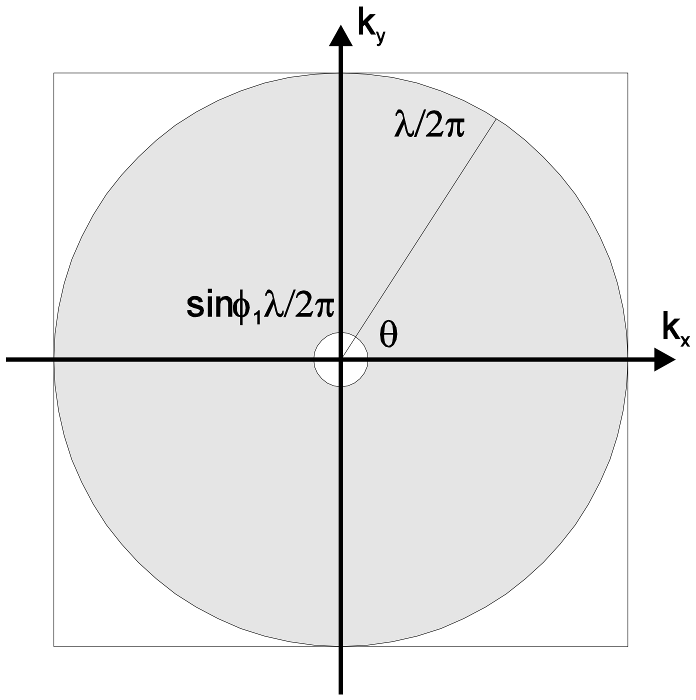
Steered vs. unsteered arrays

- Unsteered:
  - Optimization => the sidelobe level should be minimized over all visible angles.
  - Annular region since (kx, ky) = 2p/λ*(sinϕ cosθ, sinϕ sinθ)
  - Sampling => beampattern will be repeated for argument of kx and ky larger than 2p/λ => annulus will repeat along the kx-axis and the ky-axis.
  - With element distance λ/2, the circles will exactly touch.
- Steering:
  - The visible region will have its center at the steering direction, while the optimized region from the array is still centered at the origin.
  - May no longer have full overlap between the optimized region and the visible region.
  - One must optimize a larger region
  - With element distance λ/2, and for all possible steering angles, one must optimize over the square region.



Optimization criterion

- Criterion 1: Minimum max sidelobe
  - Weighting: Dolph-Chebyshev
  - Good for discrete targets in non-reflecting background
  - Restriction on maximum mainlobe width
- Criterion 2: Minimum sidelobe energy
  - Weighting: Prolate-spheroid ¼ Kaiser-Bessel
  - Related to image contrast
  - Restriction on peak sidelobe and/or mainlobe

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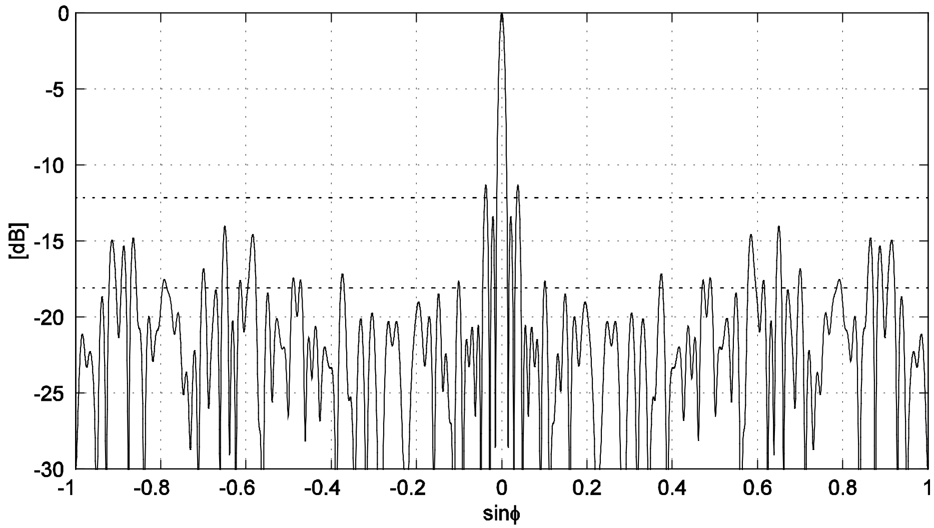
Random array

- Average array pattern = aperture weighted with pdf of thinning.
- Determines the main lobe
- Uniform thinning:
- Average sidelobe power to main lobe power is 1/K.
- Variance is about K for $|u| = |\sin\phi| > \lambda/L$, L is the aperture.
- Relative peak level of a 1D random array is $\sqrt{K} \ln K$
- Ex: K=64 selected from M=128, uniform density

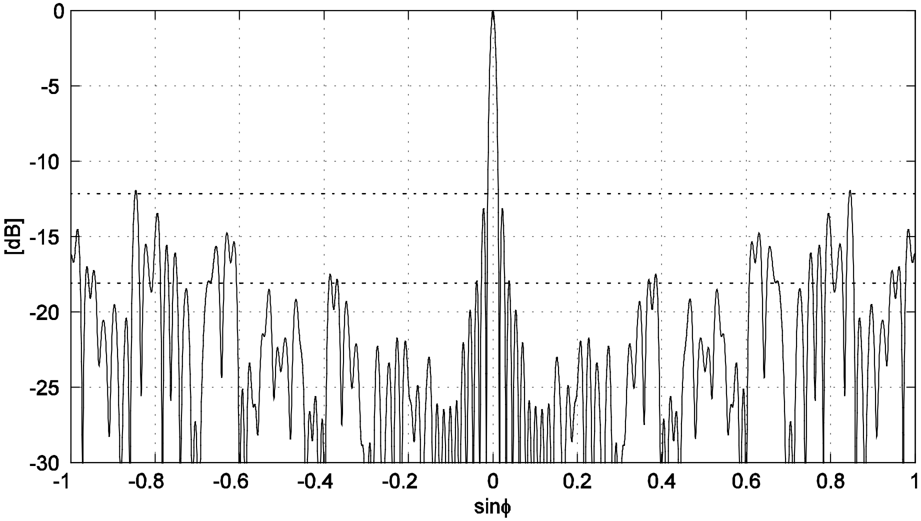


Thinning pattern (50.0% thinned):

1101110101100100111101100010111010000100011000101010110100111111
0010110010011001010110100010000010010110001101010110101011011001

Binned array

- Aperture is divided into K equal size bins and one element is chosen at random in each bin.
- No more than two neighbor elements in a 1D binned array.
- Resembles random array, but variance does not reach K until $|u| = |\sin \phi| > K \lambda/L$.
- Close-in sidelobes are much lower than the random array
- Ex: K=64 selected from M=128, bin size 2, uniform



Thinning pattern (50.0% thinned):

01101010100110101010101010010110100101011010101010011010011001101001011001100101011010010101011010011001011001101001011010100110