Conventional beamforming

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Slide 2: Beamforming

Chapter 4: Conventional Beamforming

- Delay-and-sum beamforming
- Space-time filtering
- Filter-and-sum beamforming
- Frequency domain beamforming
- Resolution

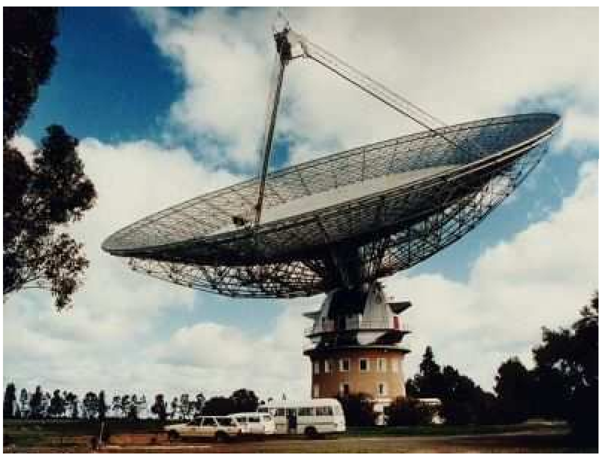
Chapter 7: Adaptive beamforming / Direction of Arrival (DOA) estimation

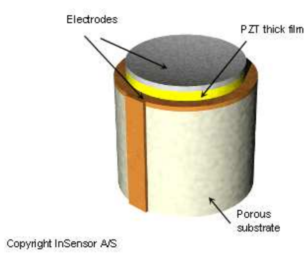
- Minimum-variance (Capon) beamformer
- Eigenvector method, MUSIC (Multiple signal classification), linear prediction

Slide 3: Norwegian Terminology / *Norsk terminologi*

- Beamforming: *Stråleforming*
- Beampattern: *Strålingsdiagram*
- Delay-and-sum: *Forsinkelse-og-sum*

Slide 4: Focusing w/ single & directional sensor





Slide 5: Single-sensor characteristics

- Geometrical pre-focusing, *e.g.* spherical curving or lens

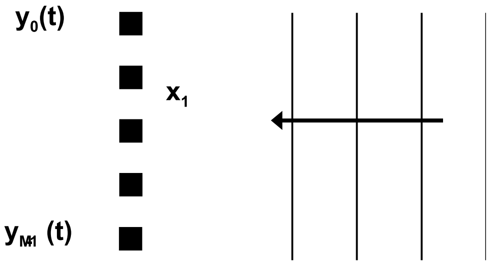
- Ability to distinguish between sources (lateral resolution): governed by physical size
- Simple: processing not required for focusing
- Inflexible: focusing depth fixed

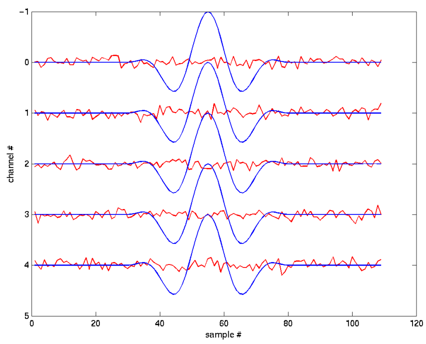
Slide 6: Array beamforming

- [Old French: *areer* = "to put in order"]

- Physical elements: apply delays & sometimes amplitude weights
- Flexible: May change focusing without altering of physical array
- Requires processing of recorded signals
- Allows for adaptive methods (chapter 7)
- On receive: Possible to aim @ more than one source "simultaneously"

Slide 7: Array gain example





$$y_m(t) = s(t) + n_m(t)$$

Slide 8: Array gain example, *continued*

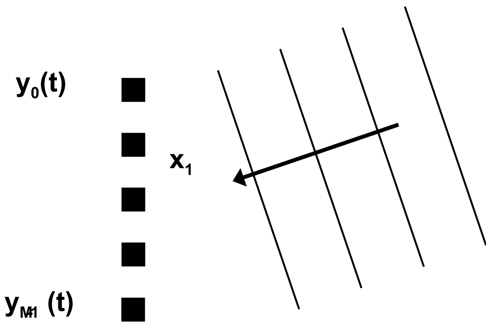
$$y_m(t) = s(t) + n_m(t)$$

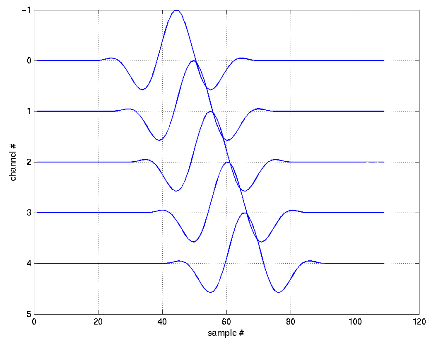
□

$$\dots$$

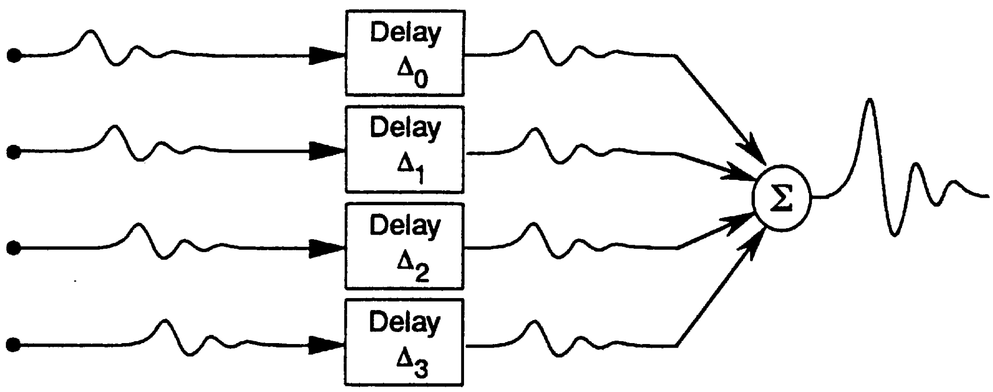
$$\mathrm{SNR}_m = \frac{\sigma_s^2}{\sigma_n^2}, \quad \mathrm{SNR} = M\frac{\sigma_s^2}{\sigma_n^2}$$

Slide 9: Non-zero angle of arrival



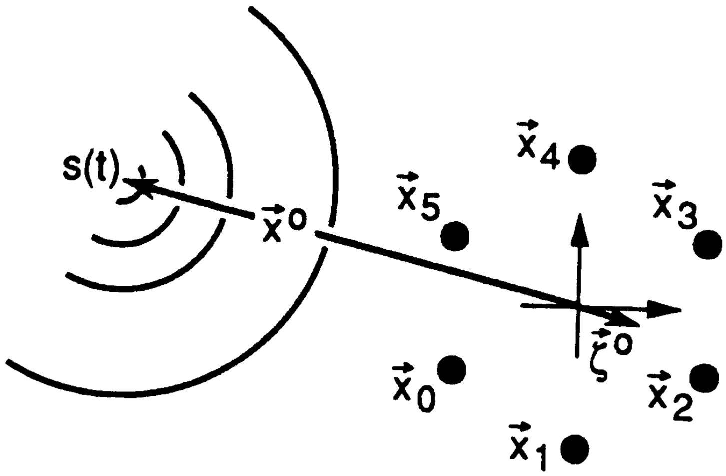


Slide 10: Delay-and-sum



stacking $\triangleq$ adjustment of $\Delta_0 \dots \Delta_{M-1}$

Slide 11: Phase center



□

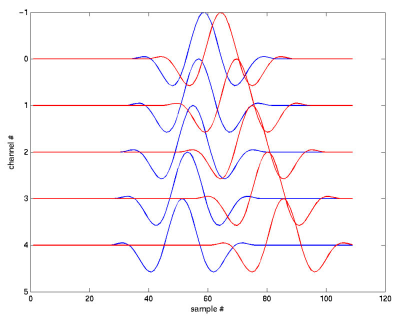
Slide 12: Delay-and-sum, definition

$$z(t) = \sum_{m=0}^{M-1} w_m y_m(t - \Delta_m)$$

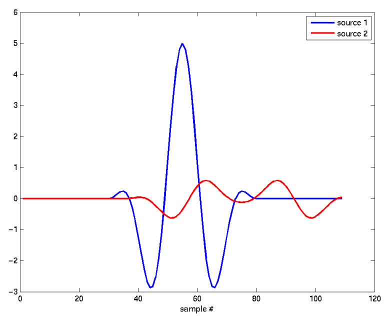
- $z(t)$: beamformer output
- m: element #
- M: # of elements
- w_m: amplitude weight $\#m$
- y_m: signal @ sensor m
- Δ_m: delay of signal @ sensor m

Slide 13: Delay-and-sum example

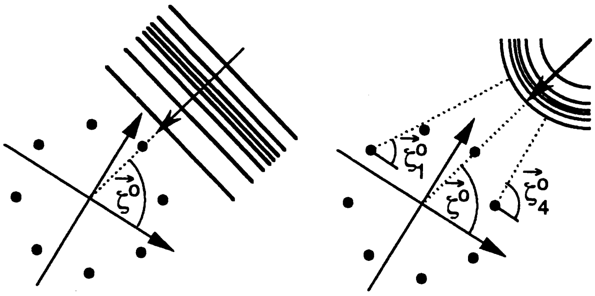
Steering ("listening") towards source 1



$\Rightarrow$



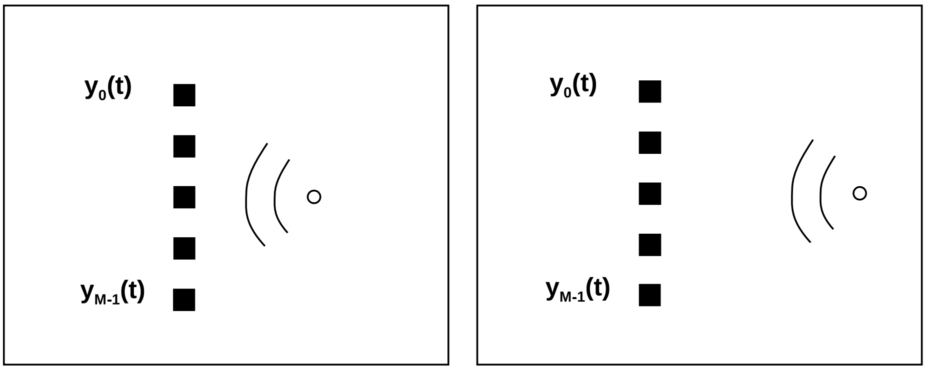
Slide 14: Near-field / far-field sources



- *Plane wave approaching* (left): [source in far-field] $\forall$ sensors: Propagation direction ($\triangleq \vec{\zeta}^\circ$): same relative to $\forall \vec{x}_m$
- *Spherical wave approaching* (right): [source in near-field] Propagation direction relative to sensor m ($\triangleq \vec{\zeta}^\circ_m$) differs.

  $\angle(\vec{\zeta}^\circ - \vec{\zeta}^\circ_m)$: discrepancy between far- & near-field @ sensor m

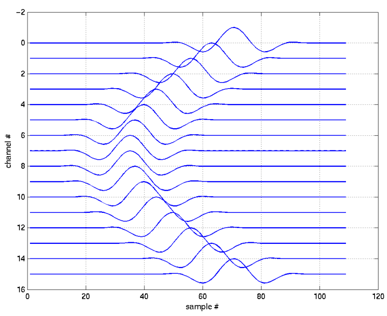
Slide 15: Near-field sources



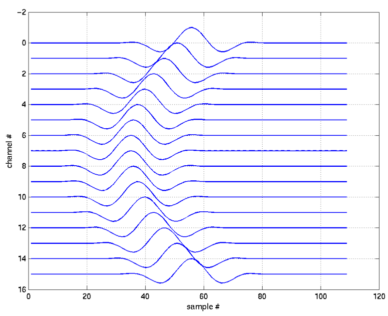
- Wavefront propagation direction $\vec{\zeta}^\circ_m$ differs with distance to source

Slide 16: Near-field sources, *continued*

Example received signals per sensor:



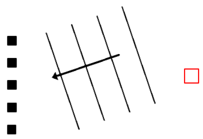
Source *close* to sensors



Source further away

Slide 17: Beamforming for plane waves

Source in the far-field



- Wavefield @ aperture:

$$f(\vec{x}, t) = s(t - \vec{\alpha}^\circ \cdot \vec{x}), \qquad \vec{\alpha}^\circ \triangleq \vec{\zeta}^\circ / c\text{: slowness}$$

- Delay @ element: $\Delta_m = -\vec{\alpha}^\circ \cdot \vec{x}_m \Rightarrow$

$$z(t) = \sum_{m=0}^{M-1} w_m s(t - \Delta_m - \vec{\alpha}^\circ \cdot \vec{x}_m) = s(t) \sum_{m=0}^{M-1} w_m$$

- Steering direction $\vec{\zeta} \Leftrightarrow \left(\vec{\alpha} = \vec{\zeta}/c\right)$: $\Delta_m = -\vec{\alpha} \cdot \vec{x}_m \Rightarrow$

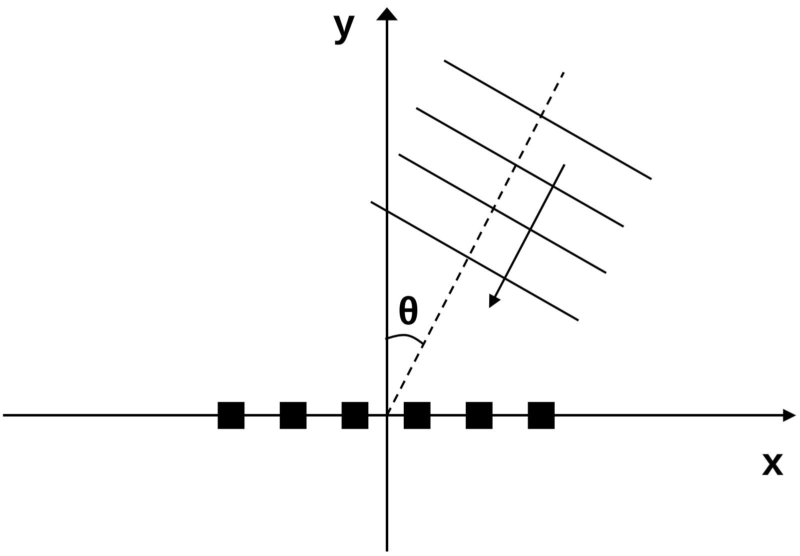
$$z(t) = \sum_{m=0}^{M-1} w_m s\left(t - (\vec{\alpha}^\circ - \vec{\alpha}) \cdot \vec{x}_m\right)$$

Slide 18: Beamforming for plane waves, *continued*

$$z(t) = \sum_{m=0}^{M-1} w_m s\left(t - (\vec{\alpha}^\circ - \vec{\alpha}) \cdot \vec{x}_m\right)$$

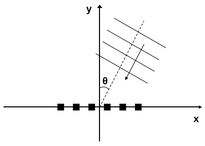
- If $\vec{\alpha} \neq \vec{\alpha}^\circ \;\Rightarrow$ mismatch $\Rightarrow y_m$ not summed constructively $\forall$ sensors m
- Mismatch sources for incorrect $\vec{\alpha}^\circ \triangleq \vec{\zeta}^\circ / c$:
  - Assumption about $\vec{\zeta}^\circ$
  - Assumption about c
- If $\vec{\alpha}^\circ$ or c is known: Get the other by variation of c or $\vec{\alpha}$, until max energy output in $z(t)$, *e.g.*: $\max_c \int_{-\infty}^{\infty} |z|^2 \mathrm{d}t$

Slide 19: Linear array example



Slide 20: Linear array example, *continued*

- Monochromatic, plane wave: $s(t-\vec{\alpha}^\circ \cdot \vec{x}) = e^{j(\omega^\circ t - \vec{k}^\circ \cdot \vec{x})}$



□

- Steering delays: $\Delta_m = -\frac{\vec{k}}{\omega^\circ} \cdot \vec{x}_m$, & $\vec{x}_m = [x_m, 0, 0]^{\mathrm{T}} \quad \Rightarrow$

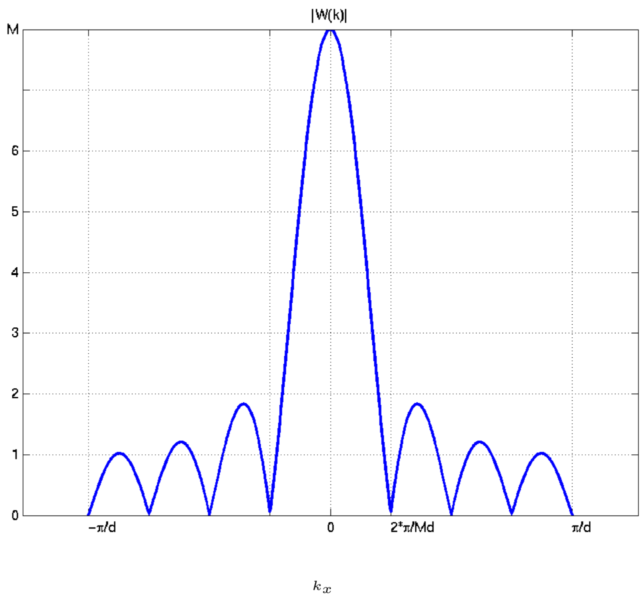
$$z(t) = e^{j\omega^\circ t} W(k_x - k_x^\circ),$$

w/ Array pattern def.: $$W(\vec{k}) \triangleq \sum_{m=0}^{M-1} w_m e^{j\vec{k}\cdot\vec{x}_m}$$

- Uniform weights, $w_m = 1, \quad \forall m \qquad \Rightarrow$

$$W(k_x - k_x^\circ) = \frac{\sin\left[\frac{M}{2}(k_x - k_x^\circ)d\right]}{\sin\left[\frac{1}{2}(k_x - k_x^\circ)d\right]}$$

Slide 21: Array pattern amplitude $|W(k_x)|$

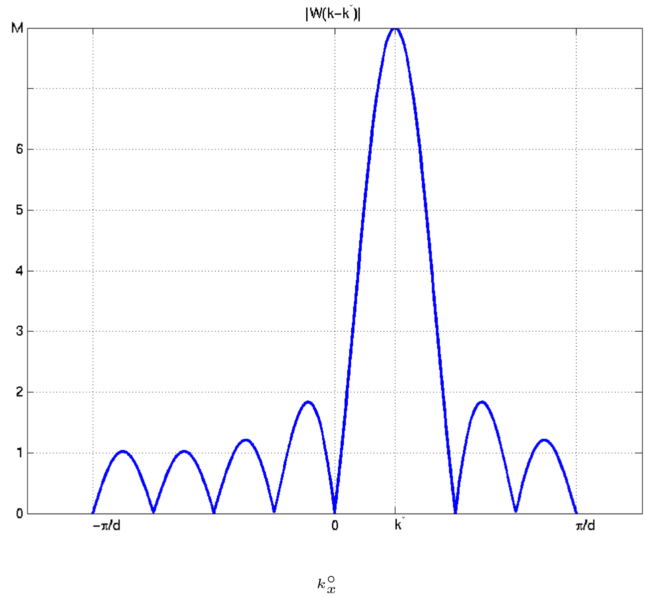


Example: $|W(k_x)|$, for $x_m = [x_m, 0, 0]^{\mathrm{T}}$

□

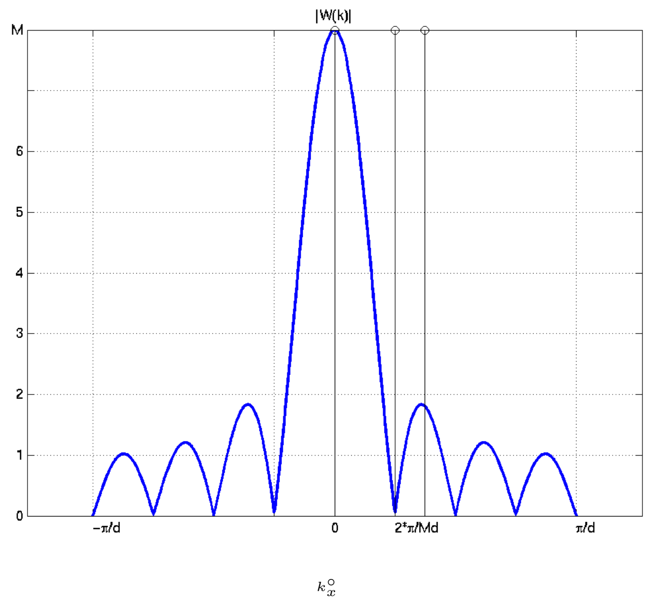
Slide 22: Beampattern amplitude $|W(\vec{k} - \vec{k}^\circ)|$

Array steered in fixed $\vec{k} = \omega^\circ \vec{\alpha}^\circ$. Plotting $|W(\vec{k} - \vec{k}^\circ)|$ for different $\vec{k}^\circ$.



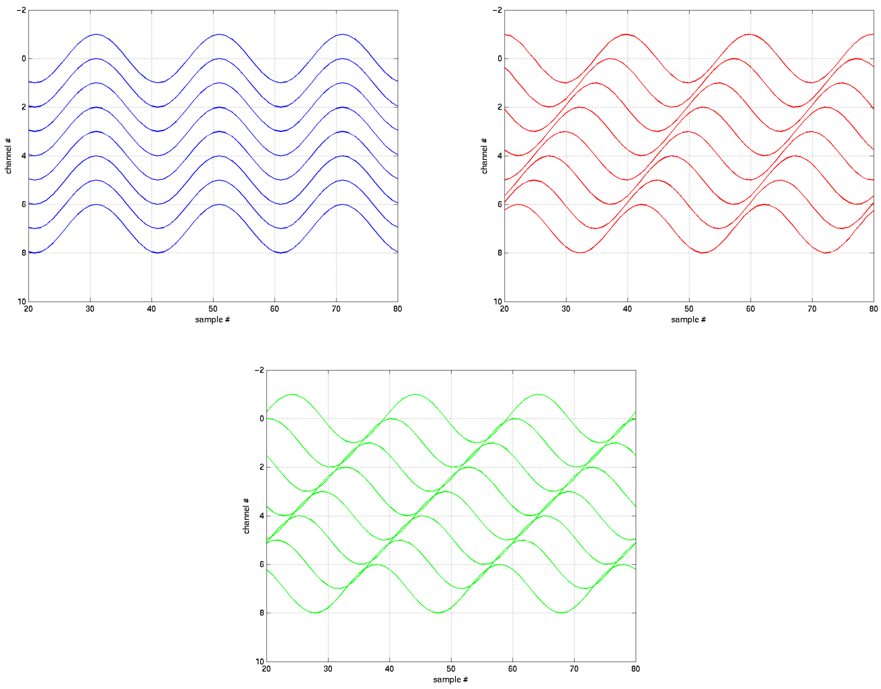
Slide 23: Example: 3 sources in different directions

Steering: $k_x = 0$



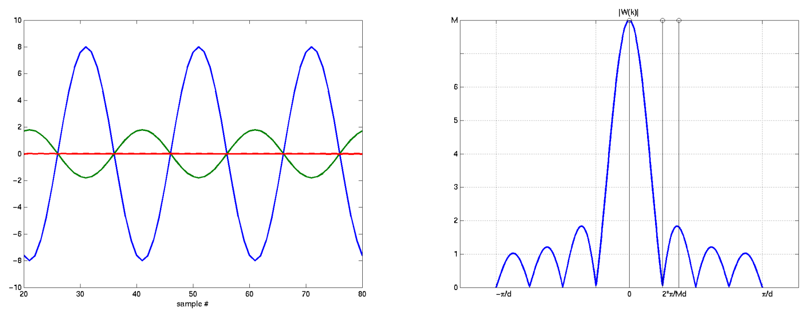
Slide 24: Example, *continued*

Recorded sensor signals

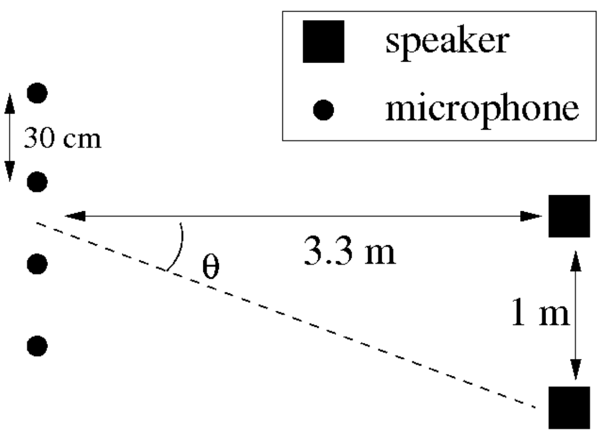


Slide 25: Example, *continued*

Delay-and-sum beamforming of each of the 3 signals. Steering: $k_x = 0$



Slide 26: Example: Microphone array



Questions

1. For what frequencies is the wavefield properly sampled? Assume $c = 340$ m/s.
2. Up to what frequency are the speakers considered to be in the array far-field?

□

Slide 27: Sound examples

- Single microphone
  [follow link]
- Delay-and-sum, steering to speaker 1
  [follow link]
- Delay-and-sum, steering to speaker 2
  [follow link]

  http://johanfr.at.ifi.uio.no/lydlab/

Slide 28: Beamforming for spherical waves

Single-source in array near-field:

- Maximum beamformer output $\leftrightarrow$ a *spatial location* [far-field case: $\leftrightarrow$ a *propagation direction*]
- $\Delta_m = f(\vec{x}_{\text{source}})$
- Adjustments of Δ_m $\Rightarrow$ may focus to $\vec{x}$ in near-field

□

Delays: $\Delta_m = (r^\circ - r_m^\circ)/c$

r°: distance from origin to source $\Rightarrow$ delay-and-sum output:

$$\begin{aligned} z(t) &= \sum_{m=0}^{M-1} w_m \frac{s\left(t - r_m^\circ/c - [r^\circ - r_m^\circ]/c\right)}{r_m^\circ} \\ &= s\left(t - r^\circ/c\right) \sum_{m=0}^{M-1} w_m \frac{1}{r_m^\circ} \\ &= \underbrace{\frac{1}{r^\circ} s\left(t - r^\circ/c\right)}_{\substack{\text{spherically spreading wave} \\ \text{received @ phase center}}} \cdot \underbrace{\sum_{m=0}^{M-1} w_m \frac{r^\circ}{r_m^\circ}}_{\substack{\text{weighted sum of} \\ \text{sensor weights}}} \end{aligned}$$