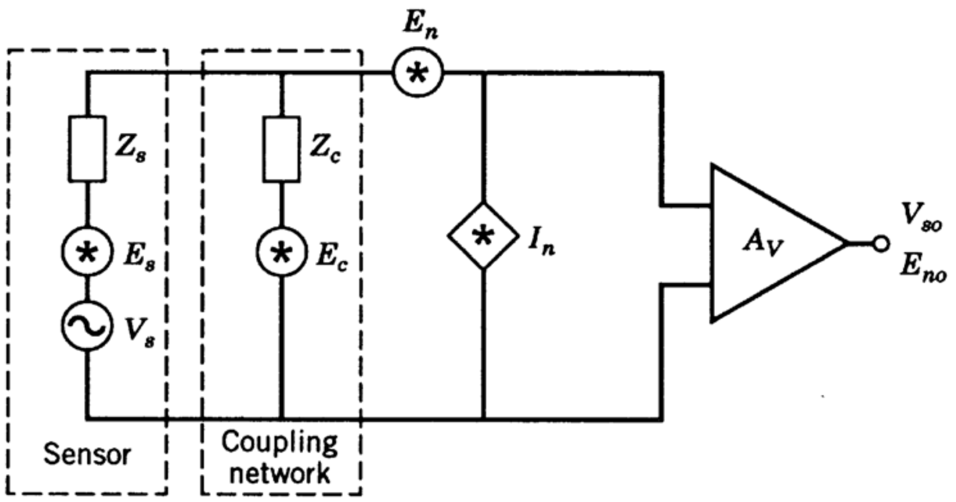
F10 (Mot 7): Modelling system noise

Modelling of noise must include:
• Sensors
• Bias and coupling network
• Amplifiers

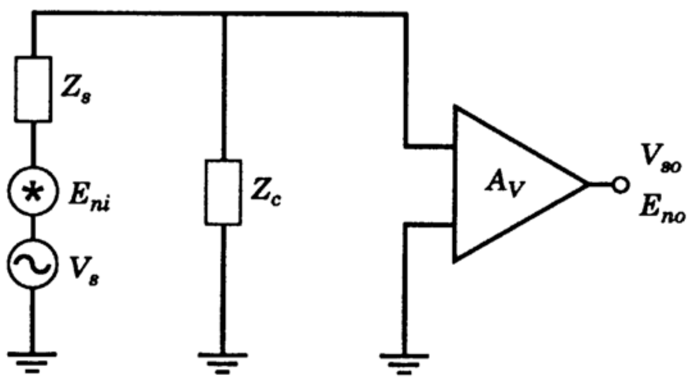
We use our standard method:
1. Determine the total noise at output: *Eno*
2. Determine the system gain: *Kt*
3. Divide *Eno* with K_t: $E_{ni}^2 = E_{no}^2 / K_t^2$

A general noise model



(a)

Equivalent noise voltage at the input:



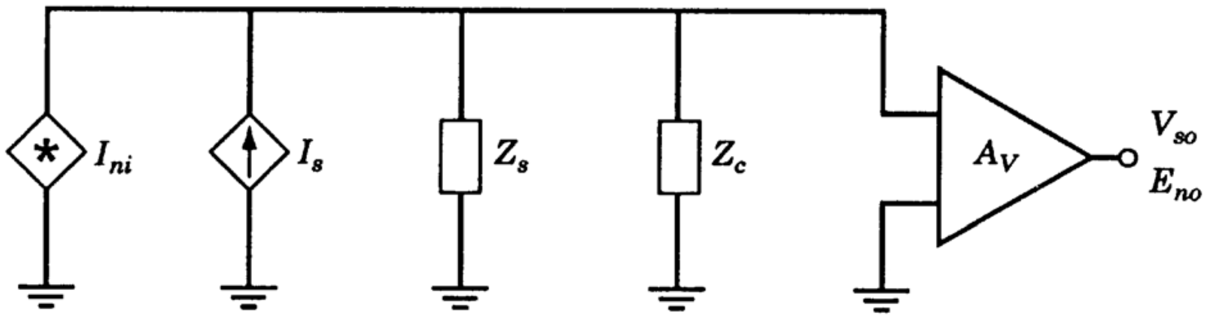
(b)

General expression:

$$E_{ni}^2 = A^2 E_S^2 + B^2 E_n^2 + C^2 I_n^2 Z_S^2 + D^2 E_C^2$$

A, B, C and D are functions of resistors, capacitors, coils etc and not functions of currents or voltages.

Equivalent noise current at the input:



(c)

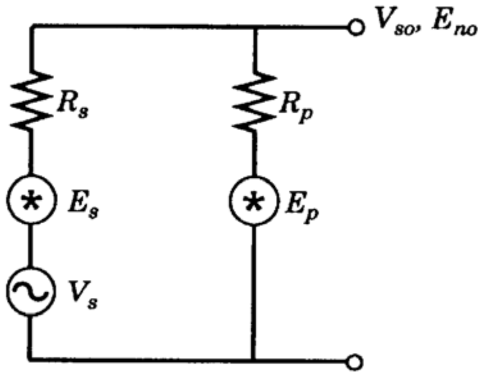
Figure 7-1 System noise model.

$$I_{ni}^2 = J^2 I_{ns}^2 + \frac{K^2 E_n^2}{Z_S^2} + L^2 I_n^2 + \frac{M^2 E_C^2}{Z_C^2}$$

Nor J, K, L and M are functions of voltage or current.

It is irrelevant whether one calculates the equivalent input noise voltage or current.

Effect of parallel load resistance



Input (i.e. without Rp):

$$\frac{S}{N} = \frac{V_{so}^2}{E_{no}^2} = \frac{V_S^2}{E_S^2}$$

Output (i.e. with Rp):

$$V_{so} = \frac{R_p}{R_S + R_P} V_S$$

$$E_{no}^2 = \left(\frac{R_P}{R_S + R_P} E_S \right)^2 + \left(\frac{R_S}{R_P + R_S} E_P \right)^2$$

$$\frac{S_{ut}}{N_{ut}} = \frac{V_{so}^2}{E_{no}^2} = \frac{\left(\frac{R_p}{R_s + R_p} \right)^2 V_s^2}{\left(\frac{R_p}{R_s + R_p} \right)^2 E_s^2 + \left(\frac{R_s}{R_p + R_s} \right)^2 E_p^2} = \frac{V_s^2}{E_s^2 + \left(\frac{R_s}{R_p} \right)^2 E_p^2}$$

$$\frac{S_{ut}}{N_{ut}} = \frac{V_{so}^2}{E_{no}^2} = \frac{\left(\frac{R_p}{R_s + R_p}\right)^2 V_s^2}{\left(\frac{R_p}{R_s + R_p}\right)^2 E_s^2 + \left(\frac{R_s}{R_p + R_s}\right)^2 E_p^2} = \frac{V_s^2}{E_s^2 + \left(\frac{R_s}{R_p}\right)^2 E_p^2}$$

When $R_s = R_p$ then $E_s = E_p$ and we get that $(S/N)_{ut} = \frac{1}{2}(V_s^2/E_s^2) = \frac{1}{2}(S/N)_{inn}$. R_p equally reduces Vs and Es but contribute in addition with its own noise. When $R_s >> R_p$ decreases $(S/N)_{ut}$ towards zero, while when $R_s << R_p$ will $(S/N)_{ut}$ increase towards $(S/N)_{inn}$ which is the best that can be achieved.

Calculation of amplifier noise

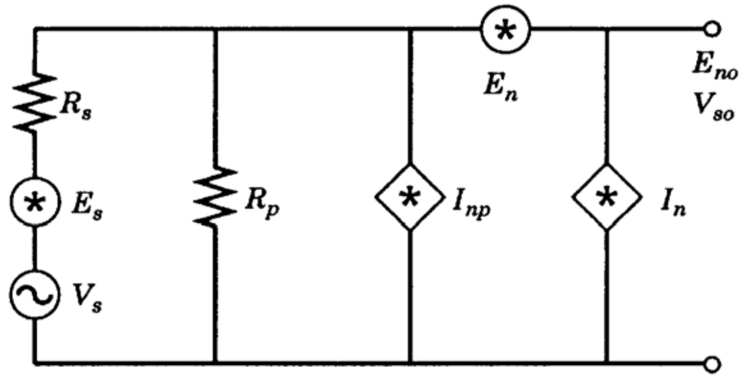


Figure 7-3 Amplifier and sensor models with shunt resistance.

1) Noise on output

$$E_{no}^2 = E_S^2\left(\frac{R_p}{R_S+R_p}\right)^2 + E_n^2 + I_n^2(R_p \parallel R_S)^2 + I_{np}^2(R_p \parallel R_S)^2$$

2) System gain

$$K_t = \frac{R_p}{R_S+R_p}$$

3) Equivalents input noise

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_S^2 + \left(\frac{R_S+R_p}{R_p}\right)^2 E_n^2 + \left(I_n^2 + I_{np}^2\right)R_S^2$$

We compare with our well-known equation:

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_s^2 + E_n^2 + I_n^2 R_S^2$$

- Terms in front of E_n: If $R_p << R_s$ we have that E_n will contribute a lot. If $R_S = R_p$ the contribution from E_n will be equal to $4E_n^2$. If $R_p >> R_s$ contributes E_n with only E_n^2
- $I_{np}^2 R_S^2$ is a new term. This is the thermal noise in R_p.

Countermeasures:

1)

Rp must be made as high as possible. If a certain voltage is required over R_S or a certain current through R_S we may change the bias voltage accordingly.

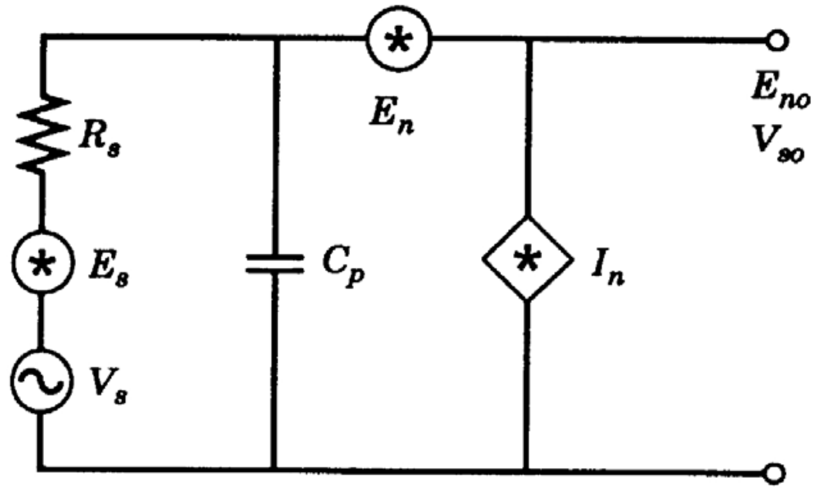
Example: R_S=1kΩ, *Rp*=4kΩ and *VB*=5V. If we change to new values *Rp*=49kΩ and *VB*=50V the voltage over the sensor and the current through the sensor remains the same but the noise is reduced.

2)

Alternatively, maybe it is possible to use a coil instead of Rp?

$$E_{ni}^2 = E_n^2 \left[\frac{R_S}{j\omega L} + 1 \right]^2 + I_n^2 R_S^2 + E_S^2$$

Effect of shunt capacitances



Here we have replaced the resistance Rp with a capacitor Cp.

1)

$$E_{no}^2 = E_S^2 \left(\frac{\frac{1}{j\omega C_p}}{R_S + \frac{1}{j\omega C_p}} \right)^2 + E_n^2 + I_n^2 \left(\frac{R_S \frac{1}{j\omega C_p}}{R_S + \frac{1}{j\omega C_p}} \right)^2 =$$

$$E_S^2 \left(\frac{1}{R_s^2 C_p^2 \omega^2 + 1} \right) + E_n^2 + I_n^2 \left(\frac{R_S^2}{R_S^2 C_p^2 \omega^2 + 1} \right)$$

2)

$$K_t^2 = \left(\frac{\frac{1}{j\omega C_p}}{R_S + \frac{1}{j\omega C_p}} \right) = \frac{1}{R_S^2 C_p^2 \omega^2 + 1}$$

3)

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_S^2 + E_n^2\left(R_S^2 C_p^2 \omega^2 + 1\right) + I_n^2 R_S^2$$

We compare with our well-known expression:

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_s^2 + E_n^2 + I_n^2 R_S^2$$

⇨ E_n^2 does not contribute as E_n^2 only but weighted as $E_n^2(R_S^2 C_p^2 \omega^2 + 1)$. *Note !* $R_S^2 C_p^2 \omega^2$ will often be substantially less than 1.

So: Just E_n^2 contribution increases.
NB! C_p is no noise source!
Cp is not the input capacitance of the amplifier.
This is included in E_n, I_n and K_t.

Noise in a resonant circuit

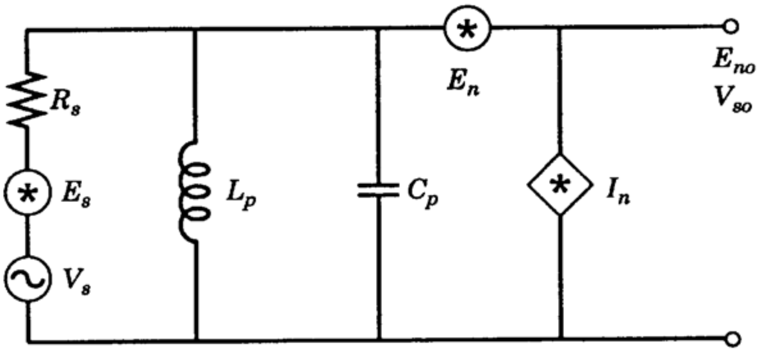


Figure 7-5 Resonant sensor equivalent circuit.

1)

$$E_{no}^2 = E_S^2\left(\frac{X_{L_P} \parallel X_{C_p}}{R_S + X_{L_P} \parallel X_{C_p}}\right)^2 + E_n^2 + I_n^2\left(R_S \parallel X_{L_p} \parallel X_{C_p}\right)^2$$

Calculation of parts:

$$\frac{X_{L_p} \parallel X_{C_p}}{R_S + X_{L_p} \parallel X_{C_p}} = \frac{1}{R_S\left(j\omega C_p + \frac{1}{j\omega L_p}\right)+1} = \frac{1}{R_S\left(\frac{1-\omega^2 C_p L_p}{j\omega L_p}\right)+1} = \frac{j\omega L_p}{j\omega L_p + R_s - \omega^2 L_p C_p R_S}$$

$$R_S \parallel X_{L_p} \parallel X_{C_p} = \frac{1}{\frac{1}{R_S} + \frac{1}{j\omega L_p} + j\omega C_p} = \frac{j\omega R_S L_p}{j\omega L_p + R_S - \omega^2 R_S C_p L_p}$$

$$E_{no}^2 = E_S^2\left(\frac{1}{R_S\left(j\omega C_p + \frac{1}{j\omega L_p}\right)+1}\right)^2 + E_n^2 + I_n^2\left(\frac{1}{\frac{1}{R_S}+\frac{1}{j\omega L_p}+j\omega C_p}\right)^2$$

2)

$$K_t^2 = \left(\frac{X_{L_p} \parallel X_{C_p}}{R_S + X_{L_p} \parallel X_{C_p}}\right)^2 = \left(\frac{1}{R_S\left(jC_p\omega + \frac{1}{jL_p\omega}\right)+1}\right)^2$$

3)

$$E_{ni}^2 = \frac{E_{no}^2}{K_t^2} = E_S^2 + E_n^2\left|1+\frac{R_S\left(1-\omega^2 C_p L_p\right)}{j\omega L_p}\right|^2 + I_n^2\left(\frac{R_S\left(j\omega C_p + \frac{1}{j\omega L_p}\right)+1}{\frac{1}{R_S}+\frac{1}{j\omega L_p}+j\omega C_p}\right)^2 =$$

$$E_S^2 + E_n^2\left|1+\frac{R_S\left(1-\omega^2 C_p L_p\right)}{j\omega L_p}\right|^2 + I_n^2 R_S^2$$

- ⇨ The *In*-coefficient is independent of frequency
- ⇨ The *En*-coefficient will have a weight larger than 1 except at resonance. At the resonance ($\omega^2 C_p L_p = 1$) is the reactance element 0 and the coefficient equal to 1. We will then end up with our well-known expression (without C_p and L_p).
- ⇨ L_p and C_p does not contribute itself but affects others.

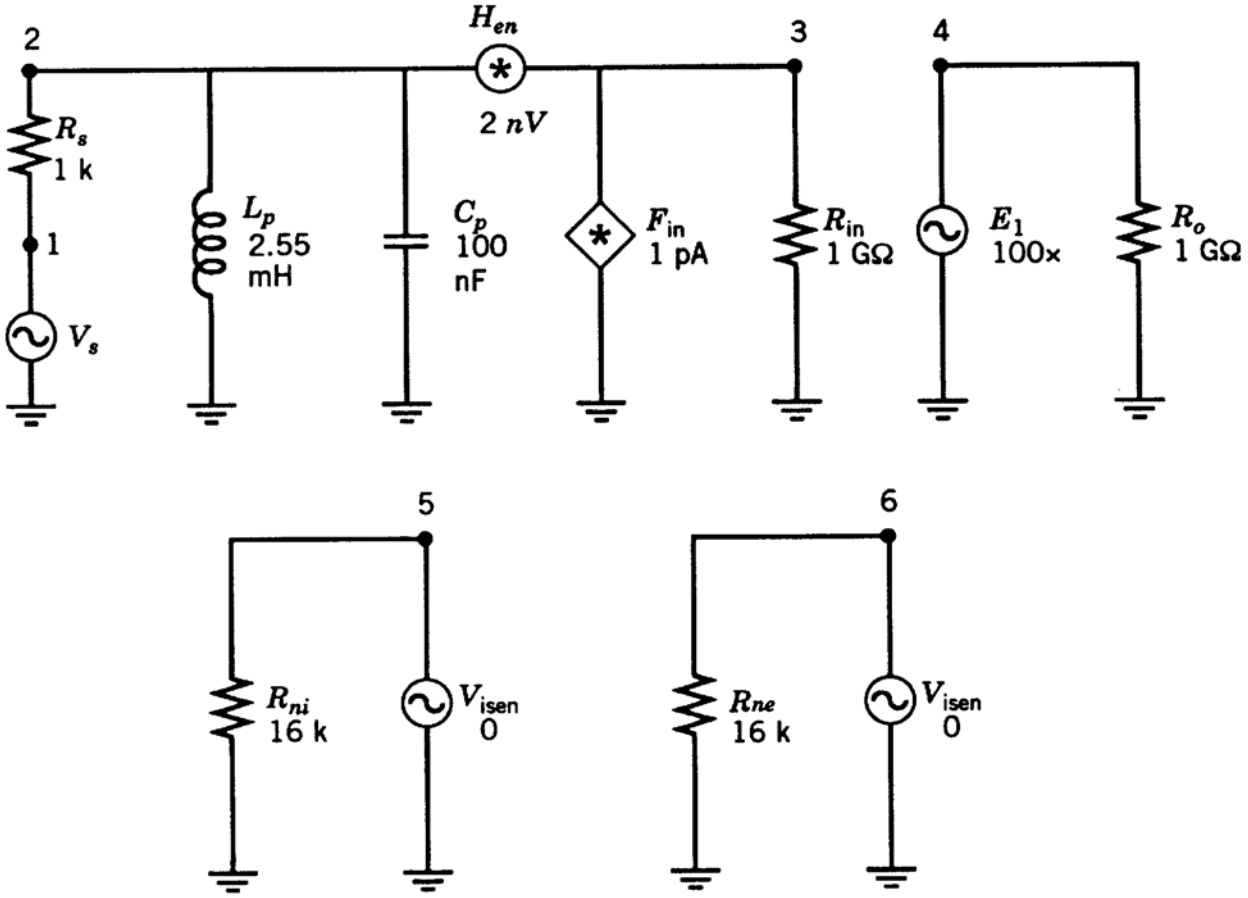


Figure 7-6 PSpice circuit for *RLC* example.

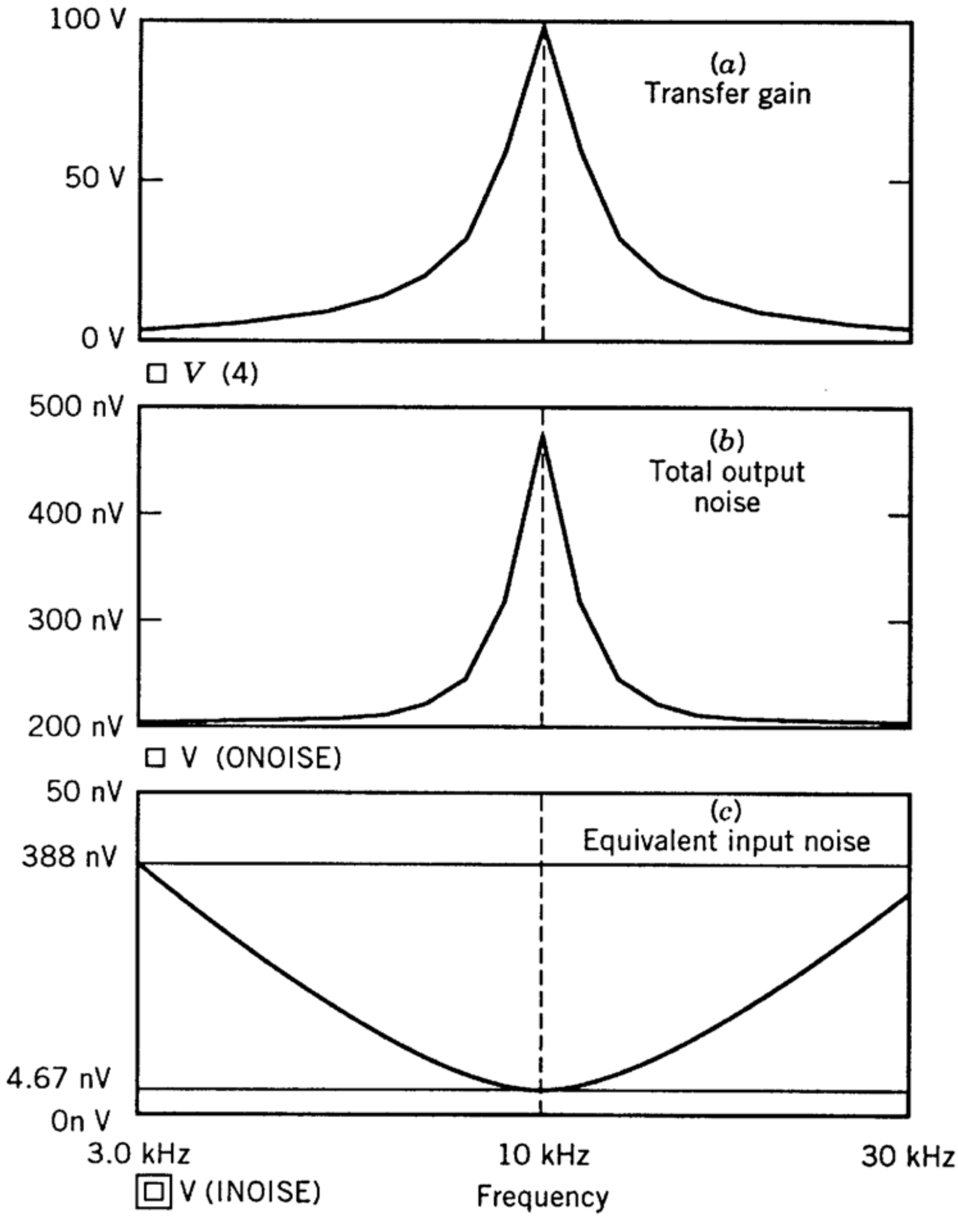


Figure 7-7 Plot of noise for *RLC* model.

The gain (top) is largest at resonance frequency. That is also the case for the noise output (in the middle). However the equivalent noise at the input (bottom) is lowest at the resonance. This means that the signal is amplified more than the increase in noise at resonance.

Resistance in parallel:

$$E_{ni}^2 = E_S^2 + \left(\frac{R_S}{R_P} + 1\right)^2 E_n^2 + \left(I_n^2 + I_{np}^2\right) R_S^2$$

Capacitor in parallel:

$$E_{ni}^2 = E_S^2 + \left(R_S^2 C_p^2 \omega^2 + 1\right) E_n^2 + I_n^2 R_S^2$$

Coil in parallel:

$$E_{ni}^2 = E_S^2 + \left(\frac{R_S^2}{\omega^2 L^2} + 1\right) E_n^2 + I_n^2 R_S^2$$

Xs in the series and *Xp* in parallel:

$$E_{ni}^2 = E_S^2 + \left(\frac{R_S + X_S}{X_p} + 1\right)^2 E_n^2 + \left(R_S + X_S\right)^2 I_n^2$$

Xs and *Xp* are combinations of capacitances and coils in series and/or parallel.

X_{LC} is a coil and a capacitor in parallel:

$$X_{LC} = \frac{j\omega L \dfrac{1}{j\omega C}}{j\omega L + \dfrac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$