Estimates of mutual inductance

Biot-Savarts law:

$$B = \frac{\mu I}{2\pi r}$$

B is the magnetic flux density at a distance *r* from a long conductor carrying a current *I*.

- *B* increases with increasing *I*
- *B* decreases with increasing *r*.

Example:

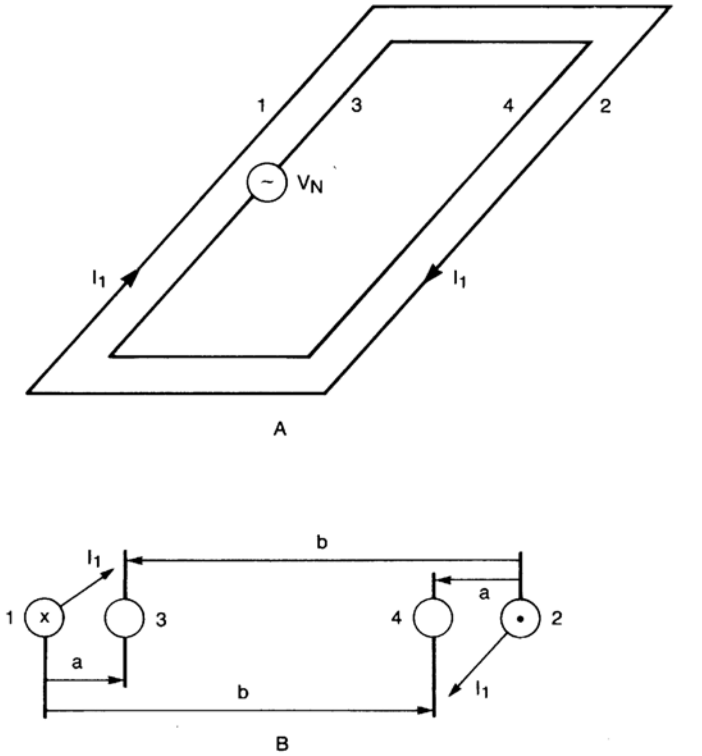

Figure 2-10. (*A*) *Nested coplanar loops*; (*B*) *cross-sectional view of A*.

Assume the long edges are significantly much larger than the sort edges.

I_1: Current in outer loop (1 and 2).

V_N: Induced voltage in the inner loop (3 and 4).

a: distance between outer and inner loop.

b: distance between outer loop and inner loop on the opposite side. The flux in the loop consisting of 3 and 4 due to the current in 1:

$$\theta_{12} = \int_a^b \frac{\mu I_1}{2\pi r} dr = \frac{\mu I_1}{2\pi} \ln\left(\frac{b}{a}\right)$$

Conductor 2 generates a similar flux in 3 and 4 in the same direction. Thus the flux is doubled.

Total flux in 3 and 4 due to 1 and 2:

$$\theta_{12} = \left[\frac{\mu}{\pi}\ln\left(\frac{b}{a}\right)\right]I_1$$

We use the following expression introduced earlier:

$$M_{12} = \frac{\theta_{12}}{I_1}$$

replaces and achieves:

$$M = 4\times10^{-7}\ln\left(\frac{b}{a}\right)$$

To find the voltage we include *M* in the expression and get:

$$V_N = j\omega MI_1 = j\omega I_1\cdot 4\times10^{-7}\ln\left(\frac{b}{a}\right)$$

Example:

$f = 10MHz$

$I_1 = 100\mu A$

$a = 10\mu m$

$b = 3000\mu m$

$V_N = 14mV$

The effect of shielding a conductor

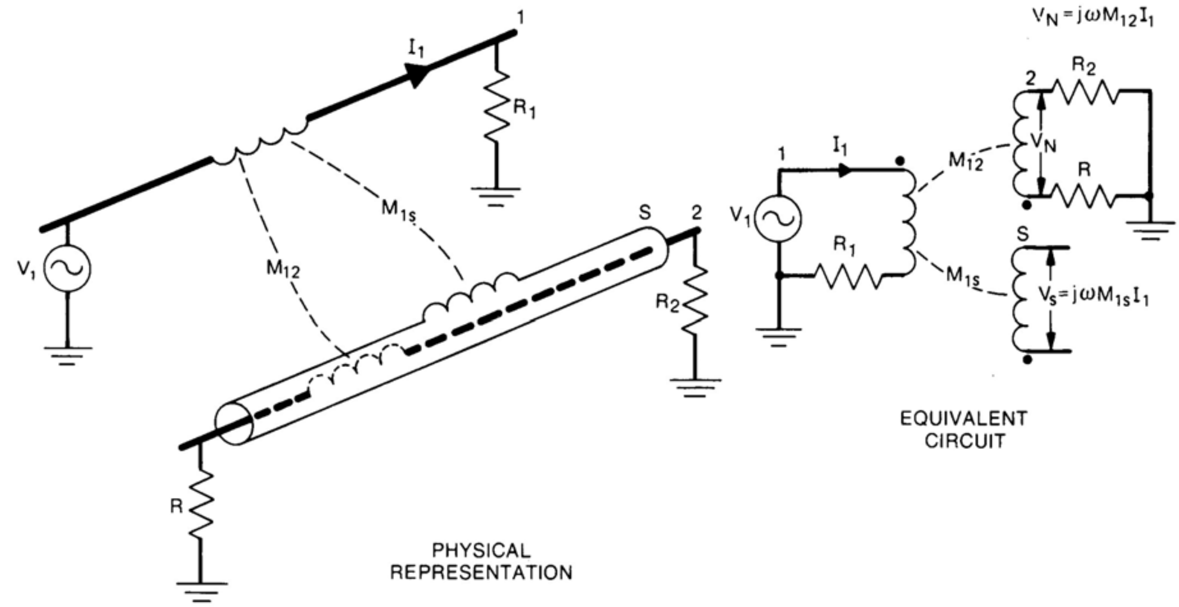

Figure 2-11. *Magnetic coupling when a shield is placed around the receptor conductor.*

Source (1) generates a voltage in the object (2)

a) First we assume:
The shield is not grounded and non-magnetic

=> The shield has no influence on 2
=> The induced shield voltage:

$$V_S = j\omega M_{1S} I_1$$

M_{1S}: Mutal inductance between the shield and 1.

b)

Grounding at one end of the shield will have no influence!! (Presuppose non-magnetic shielding)

c) Assume:
The shield is grouned at both ends:

- ⇨ The voltage induced in the shield will result in a current through the shield.
- ⇨ This current will induce noise in 2.

What about magnetic coupling between shield and centre conductor? This has to be investigated before we carry on.

Magnetic coupling between shield and center conductor

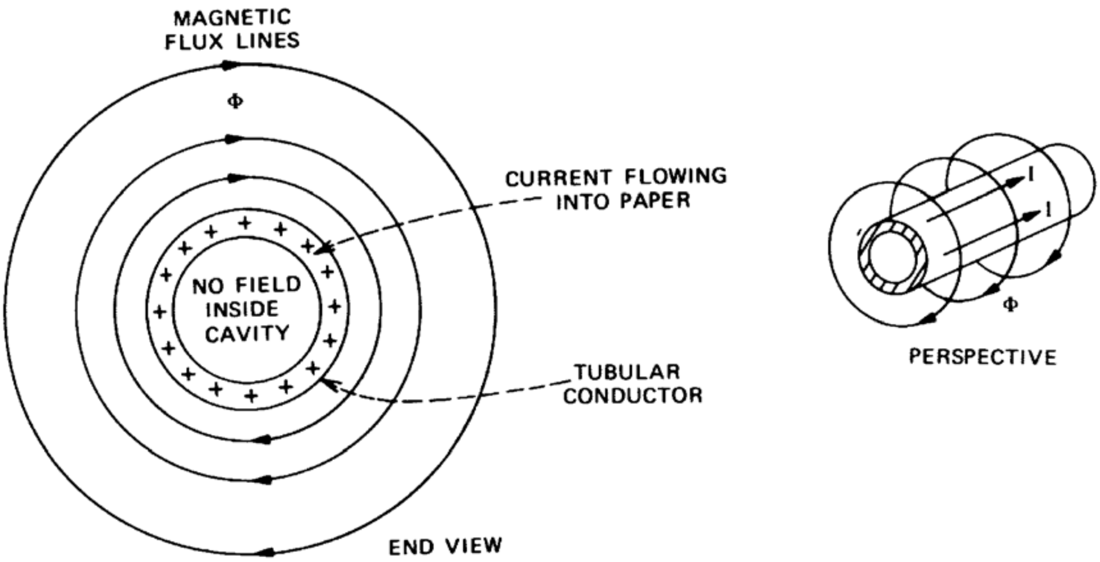

Figure 2-12. *Magnetic field produced by current in a tubular conductor.*

a) Assume a conductor shaped as a tube. (Same thickness everywhere. Uniform current distribution.)

⇨ No magnetic field lines inside the pipe

⇨ Magnetic field outside of the pipe

b) Conductor inside the shield (Say as an coax).

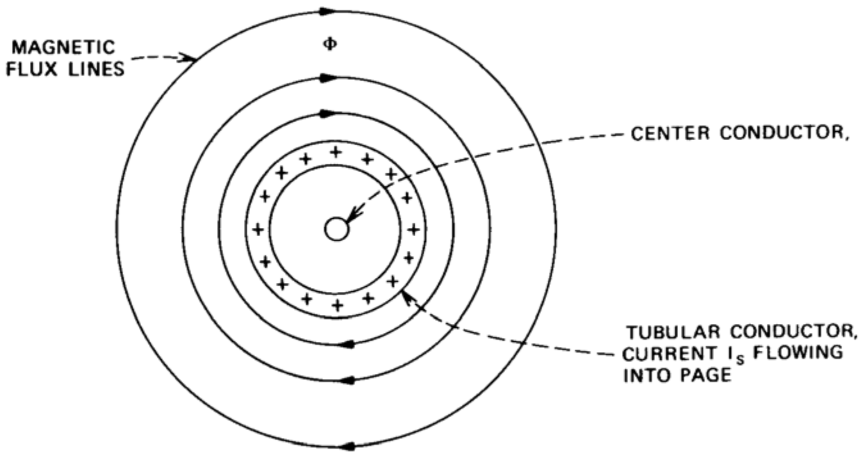

Figure 2-13. *Coaxial cable with shield current flowing.*

$$L_S = \frac{\phi_S}{I_S}$$

L_S: Shield inductance (”the pipe”)
I_S: Current in shield (”røret”)
ϕ: Magnetic field generated from screen: Enclose both shield and inner cennector.

$$M = \frac{\phi_C}{I_S}$$

M: Mutal inductance

$$\phi_S = \phi_C$$

=> We get the important relation:

$$M = L_S$$

*Mutual inductance between shield and inner conductor is equal to the shield inductance! *

Presuppose:

- No magnetice fieldlines in shield
- Uniform distribution of current i shield

Does NOT presuppose::

- That the center wire is in the geometric center of the shield (does not have to be a coax).

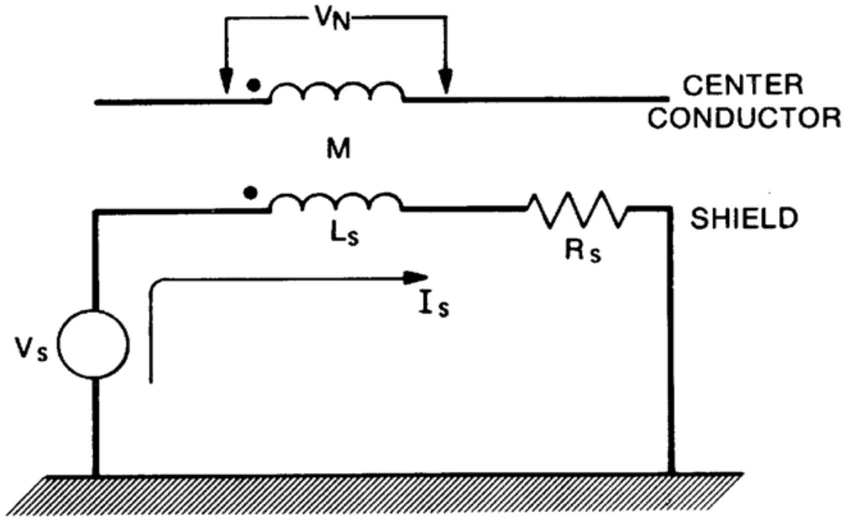

Figure 2-14. *Equivalent circuit of shielded conductor.*

V_S: Voltage in shield induced from an external source (not drawn)
I_S: Current in shield as a result of V_S over L_S and R_S.

$$V_N = j\omega M I_S$$

V_N: Voltage in inner conductor due to I_S

We find the current I_S:

$$V_S = I_S\left(R + j\omega L\right)$$

and change the order to end up with:

$$I_S = \frac{V_S}{L_S}\left(\frac{1}{j\omega + R_S / L_S}\right)$$

Combined with the previous expression we achieve:

$$V_N = \left(\frac{j\omega M V_S}{L_S}\right)\left(\frac{1}{j\omega + R_S / L_S}\right)$$

Since $L_S = M$ we may reduce the expression to:

$$V_N = \left(\frac{j\omega}{j\omega + R_S / L_S}\right) V_S$$

The plott for this expression looks like:

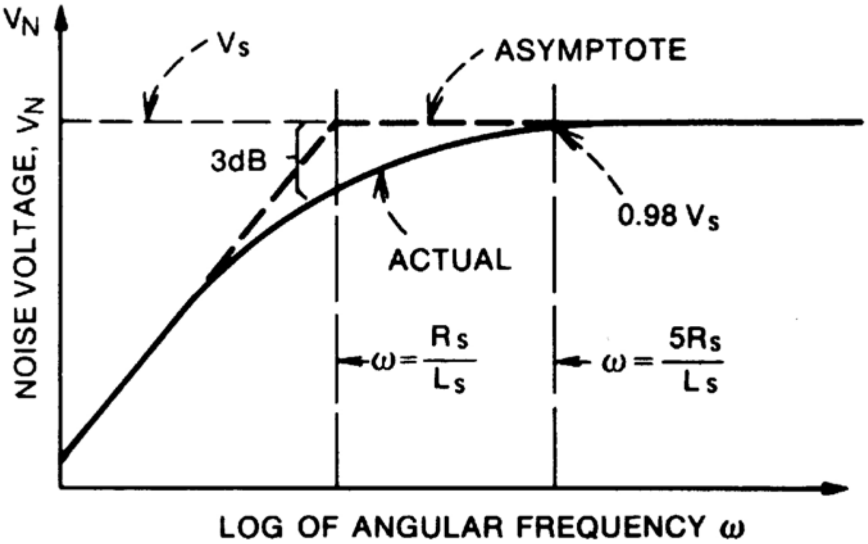

Figure 2-15. *Noise voltage in center conductor of coaxial cable due to shield current.*

Low frequencies: $V_N = (j\omega L_S / R_S) V_S$
High frequencies: $V_N = V_S$
Cut-off frequency:

$$\omega_c = \frac{R_S}{L_S} \qquad\qquad f_c = \frac{R_S}{2\pi L_S}$$

Due we wish R_S/L_S to be small or large?

Small Rs/Ls means
lower cut-off frequency, faster ascent during cut-off.

Achieved through

- Small resistance in shield
- Large resistance in screen

Table 2-1 Measured Values of Shield Cutoff Frequency (f_c)

Cable	Impedance (Ω)	Cutoff Frequency (kHz)	Five Times Cutoff Frequency (kHz)	Remarks
Coaxial cable				
RG-6A	75	0.6	3.0	Double shielded
RG-213	50	0.7	3.5	
RG-214	50	0.7	3.5	Double shielded
RG-62A	93	1.5	7.5	
RG-59C	75	1.6	8.0	
RG-58C	50	2.0	10.0	
Shielded twisted pair				
754E	125	0.8	4.0	Double shielded
24 Ga.	—	2.2	11.0	
22 Ga.[a]	—	7.0	35.0	Aluminum-foil shield
Shielded single				
24 Ga.	—	4.0	20.0	

[a]One pair out of an 11-pair cable (Belden 8775).

frequency than any other. This is due to the increased resistance of its thin aluminum-foil shield.

From the table we see that the cut-off frequency is in the area: 2-20kHz.

$$Z = \sqrt{\frac{L}{C}}$$

When shield is connected at both ends

(We jump back to where we were before we looked at the coax.)

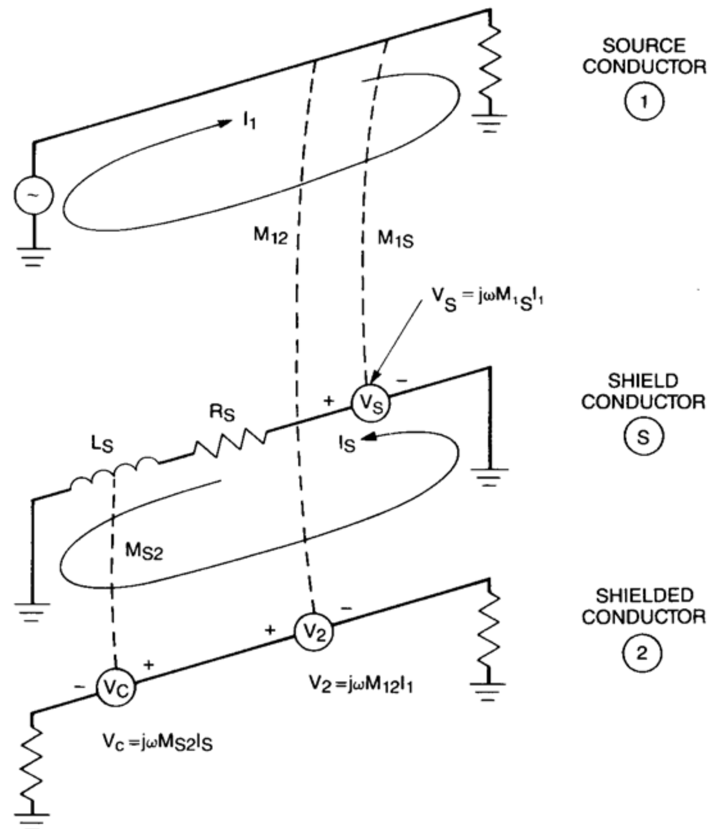

Figure 2-16. *Magnetic coupling to a shielded cable with the shield grounded at both ends.*

V_N is a result of direct radiation from source and from source via shield.

$$V_N = V_2 - V_C$$

Now e can put up the expression for a doubble grounded shield:

- Here the shield contributes with the term in parantehis.
- Without the screen we will only have the term in fron of the paranthesis.

Low frequency: $V_N = j\omega M_{12} I_1$
High frequency: $V_N = M_{12} I_1 (R_S/L_S)$

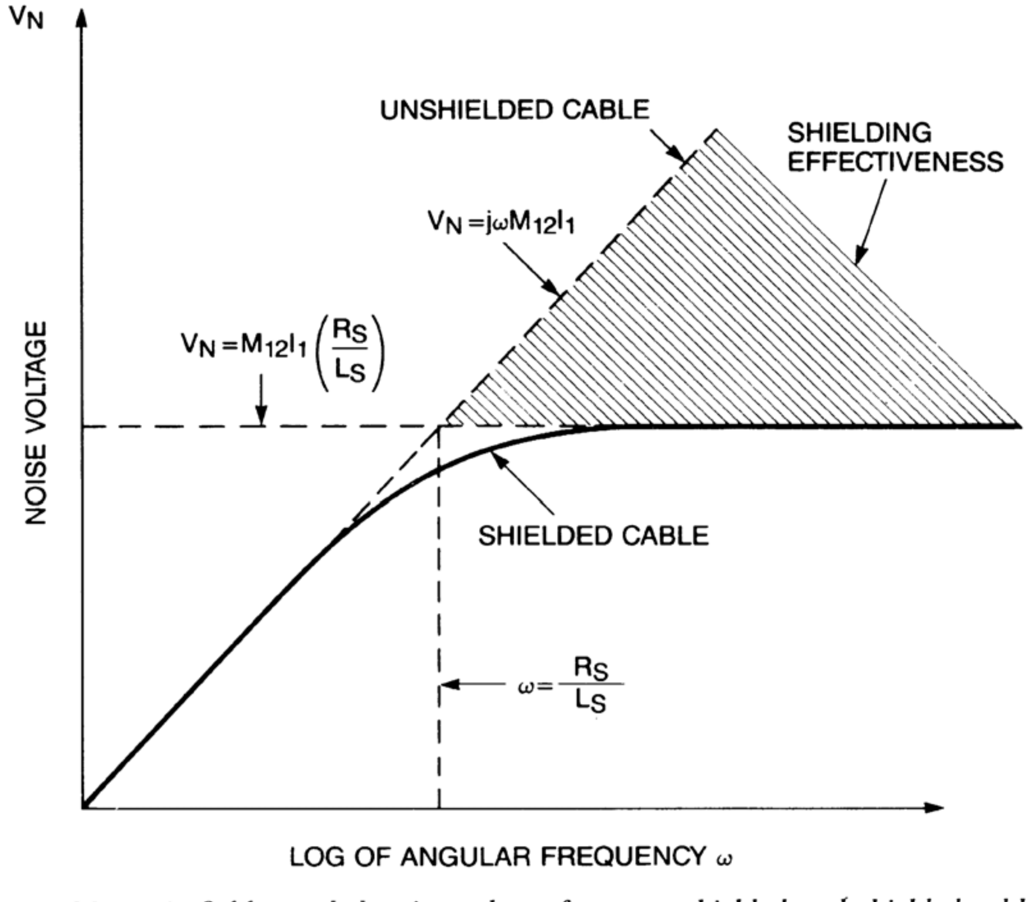

Figure 2-17. *Magnetic field coupled noise voltage for an unshielded and shielded cable* (*shield grounded at both ends*) *versus frequency*.

The drawing shows a transformer model of the system:

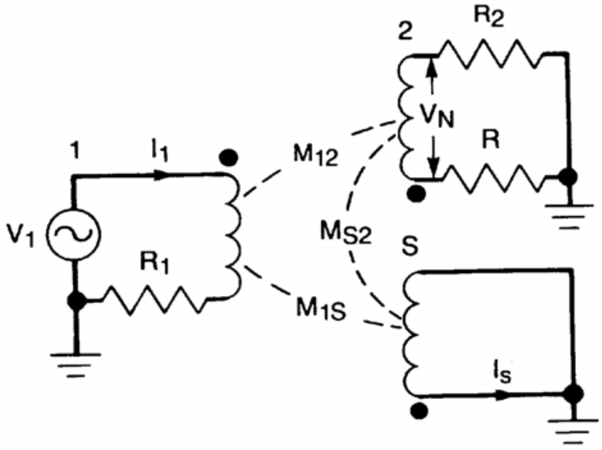

Figure 2-18. *Transformer analogy of magnetic field coupling to a shielded cable when shield is grounded at both ends (M_{S2} is much larger than M_{12} or M_{1S}).*

The shield works as a short-circuit winding that will short connect the voltage in 2.

Shielding to avoid emission:
First some basics:

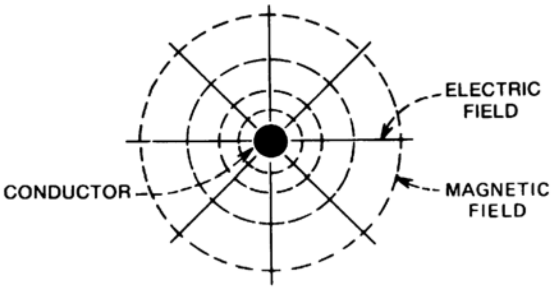

Figure 2-19. *Fields around a current-carrying conductor.*

A conductor will set up a radial electric field and a circular magnetic field.

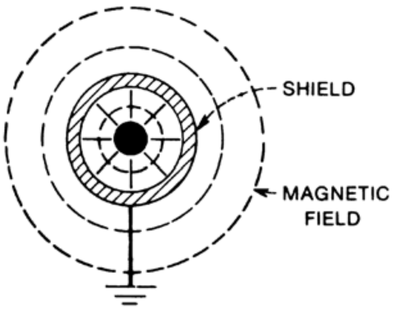

Figure 2-20. *Fields around shielded conductor; shield grounded at one point.*

A screen groundet at one end will terminate the electrical field.

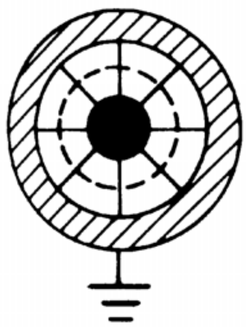

Figure 2-21. *Fields around shielded conductor; shield grounded and carrying a current equal to the conductor current but in the opposite direction.*

Equal and opposite directed current in screen and centre conductor eliminates the external magnetic field.

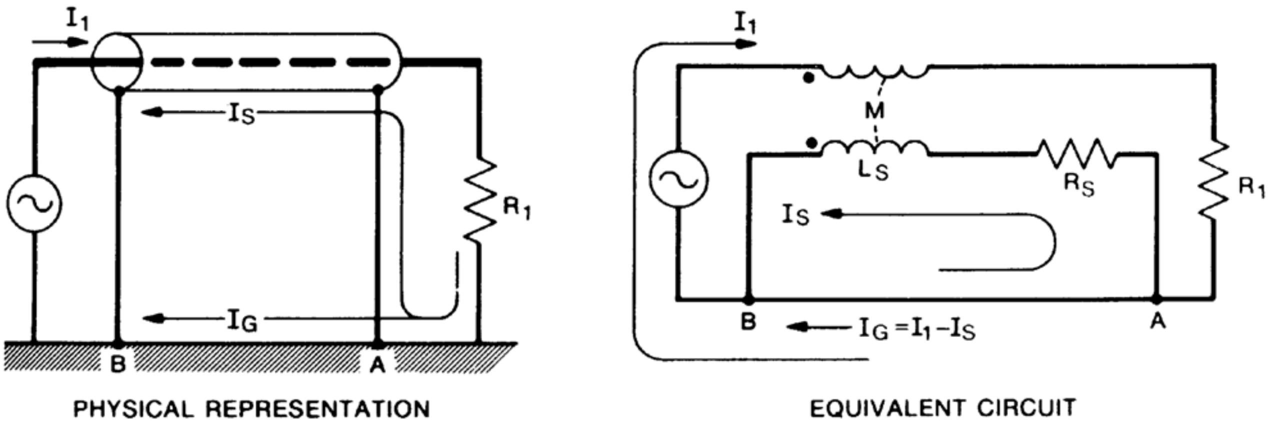

Figure 2-22. *Division of current between shield and ground plane.*

That means that the return current should go in the shield in stead of the ground.

We put up the expression:

$$0 = I_S\left(j\omega L_S + R_S\right) - I_1\left(j\omega M\right)$$

Last term is the voltage in the screen as a function of the current in the center conductor. First term is the same voltage as a function of the current it generates in the shield. No resistance in the ground-plane.

We have at $M=L_S$.

This result in the following expression for I_S:

$$I_S = I_1\left(\frac{j\omega}{j\omega + R_S/L_S}\right) = I_1\left(\frac{j\omega}{j\omega + \omega_C}\right)$$

Low frequencies: $I_S = I_1 j\omega L_S/R_S$

High frequencies: $I_S = I_1$

At higher frequencies we achieve what we want without any effort. The return current passes through the shield instead of through the ground (although the ground may have almost 0Ω in resistance). The return current is equal to the current in the centre conductor and the magnetic field external to the shield disappears.

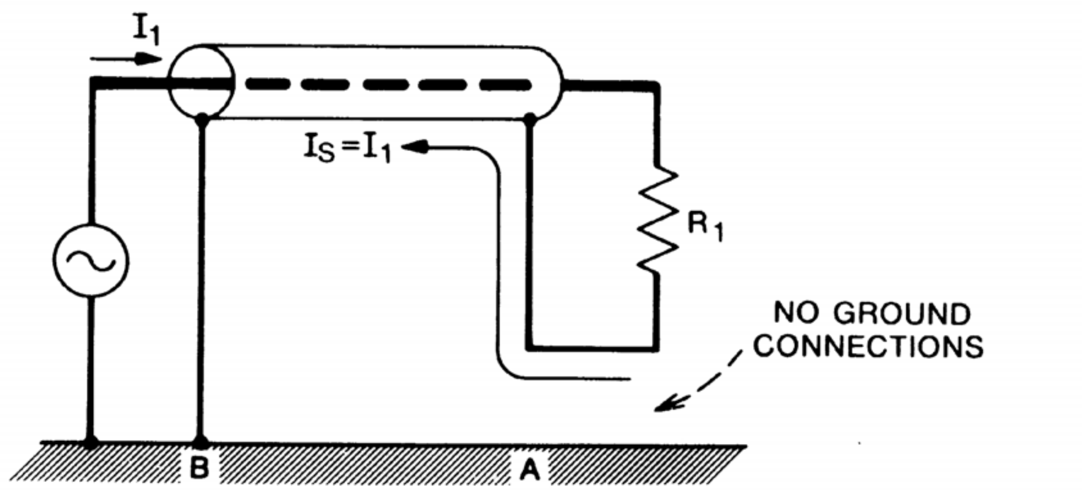

Figure 2-23. *Without ground at far end, all return current flows through shield.*

If the electronic circuitry on the right side is not grounded neither should the shield be grounded. If so all return current pass throught the shield and the external magnetic field will be eliminated also at lower frequencies.

Shielding towards external fields.

<u>To protect towards external magnetic fields the best alternative is to reduce the size of the loops.</u>

NB! In particular the return current through ground has to be considered. In may find other paths than what the designer expect and result in larger loop areas. .

About shields:
If a shield results in a smaller loop we achieve a better protection. However in this case the increased protection is due to the smaller loop and not due to a magnetic shielding of the centre wire.

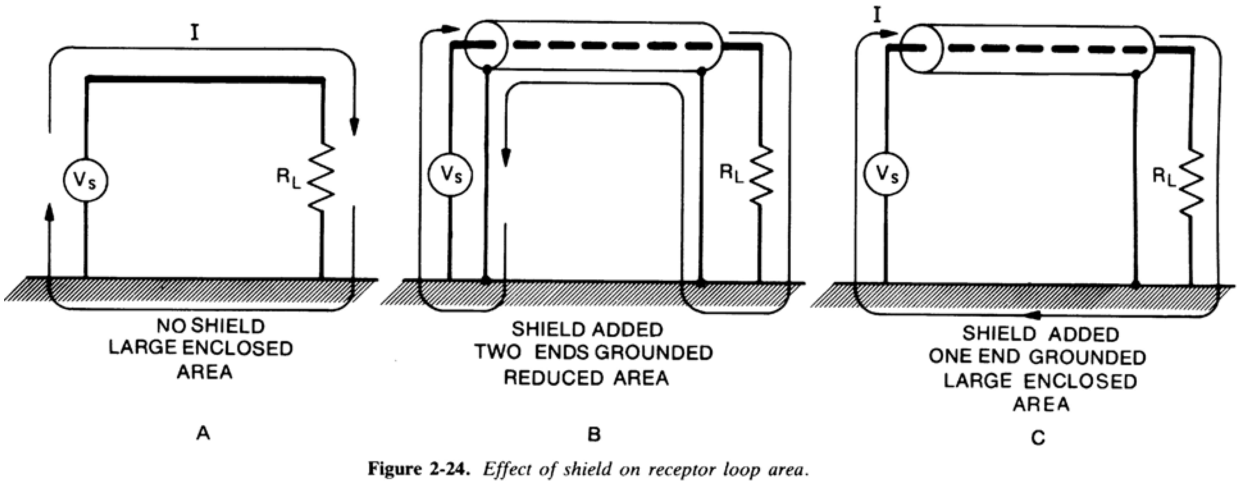

Figure 2-24. *Effect of shield on receptor loop area.*

The solution in Fig 2-24 B is the best alternative of the three but have the following limitations:

- Low efficiency at lower frequencies
- Noise generated in the screen will result a voltage drop in the shield and hence be a noise source.
- If the ground potential is different at the two connection points we will have a noise current in the circuit.

Hence it is magnetic good but electronically bad.

The noise voltage induced is the shield current times the screen impedance.