F5 (Mot1) Motchenbacher

(Main focus on component noise)

Definition of noise:

- **Unwanted:**

Most definitions of noise focus on that it is unwanted like in Motchenbacher: ”Any unwanted disturbance that obscures or interfaces with a desired signal”. In general this is a describing characteristic with the small exception that randomness of noise is wanted and utilised in random number generators.

- **Unpredictable:**

Another important feature that one should include when defining noise is that it is unpredictable. In advance it is not possible to say what strength the unwanted signal will have at a specific time in the future. The exception is the low-frequency components that are present also immediately beforehand. But even if one can not predict exactly the undesirable components, one can describe the statistical probability and distribution with respect to amplitude and frequency.

Before Motchenbacher discusses component noise the book briefly discusses coupling noise entitled as “external noise”.

:

<u>External sources:</u>

- ♦ Electrostatic (capacitive coupling)
- ♦ Electromagnetic (inductive coupling)
- ♦ AC-power/DC-power
- ♦ Signal wires
- ♦ radio transmitters
- ♦ electrical storms
- ♦ galactic radiation
- ♦ mechanical vibrations

May be “eliminated” through adequate

- ♦ shielding
- ♦ filtering
- ♦ altering layout
  - distance
  - make parallel
  - make serial
- ♦ change external components like adding a separate power supply for the front-end amplifiers.

Internal sources:

"Noise" in this book means component noise. It is also entitled as "internal noise", "true noise" and "fundamental noise"

- ♦ basic random-noise generators
- ♦ spontaneous fluctations from the physics of the devices and materials that make up the electrical system.

Example:

Thermal noise in resistors (and parasitic resistance in transistors, conductors, coils and capacitors etc.)

- ⇨ More difficult (impossible) to eliminate than coupling noise. However it is helpful and necessary to estimate the size of this noise.
- ⇨ The resolution of the sensor signal is typically decided by the sensor noise.

Example: "Snow storm" on TV-screens

Main cause: Thermal noise in the input amplifier.

Not necessarily the preamplifier that generates most noise but typically this is where the noise has the greatest effect. However noise can also come from elsewhere.

Characteristics of noise

While noise from the 50Hz mains can be very predictable, thermal noise are unpredictable when it comes to amplitude and phase. However we can in both cases find statistical characteristics like the RMS (root-mean-square).

Gaussian noise

Thermal noise and some other kind of noise have a Gaussian distribution. The figure shows the Gaussian distribution and what it may look like on an oscilloscope.

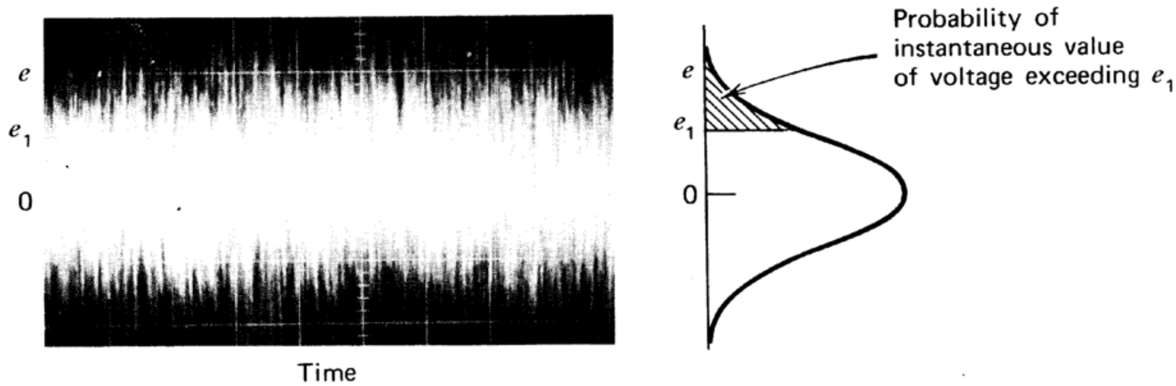


Figure 1-1 Noise waveform and Gaussian distribution of amplitudes.

The Gaussian distribution describes the probability that the noise has a certain value at a given time.
The mathematical expression for the Gauss distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

Here μ is the average value measured. Of all values this is the most probable. The function *f(x)* is a *probability density function* or for short: *pdf*.
For typical considerations we say that the noise is within +/- 3σ of μ.

	Inside	Outside
[-σ,σ]	68%	32%
[-2σ,2σ]	95%	5%
[-3σ,3σ]	99.7%	0.3%
[-4σ,4σ]	99.994%	0.006%

In the case of noise μ is equal to 0. If we find an average value different from zero over some time, this will be something that can be identified and compensated in hardware or software. This average value is not unpredictable and hence not a part of what we identify as noise.

RMS (Root-Mean-Square):

RMS is a general term that applies not only to the Gaussian distribution. The RMS-value for a Gaussian-distribution is equal to σ.

RMS is defined as:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

V(t): The signal (or noise) voltage as a function of time.

T: the time we are integrating over.

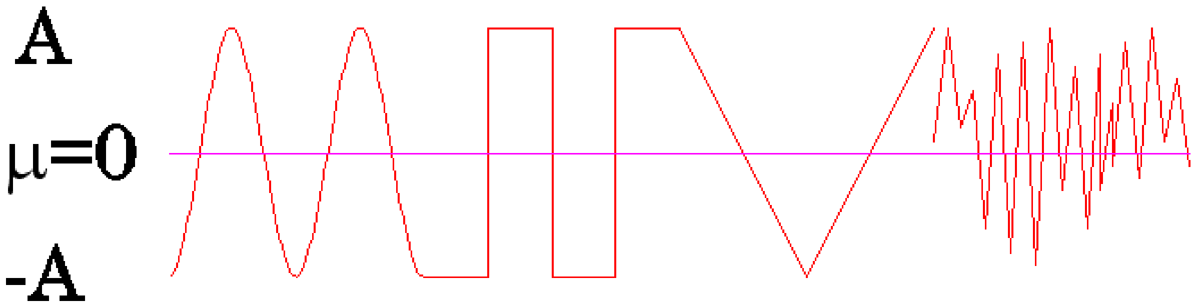
- The integral will grow with T to infinity (as long as V(t) =/= 0). When we divide by T, we find that the average slope of the integral is equal to the square of Vrms.
- If v(t) has a cyclic behaviour we have to integrate over a whole number of periods for the answer to be completely accurate. If we do not integrate a whole number of periods, the last unfinished period contributes with an error. However this error will be relative to the contribution from all the whole periods. This is useful knowledge if one does not know the exact frequencies to integrate a signal over: Hence instead of integrating over a complete number of periods we integrate for a time that is many times longer than the longest expected period. A possible part of a period will give only a small contribution.

The RMS value can be seen as an expression of the effective heating effect of a signal. The RMS value of a random signal is the DC value that will give as rapid heating through a heating element in water as the signal itself.

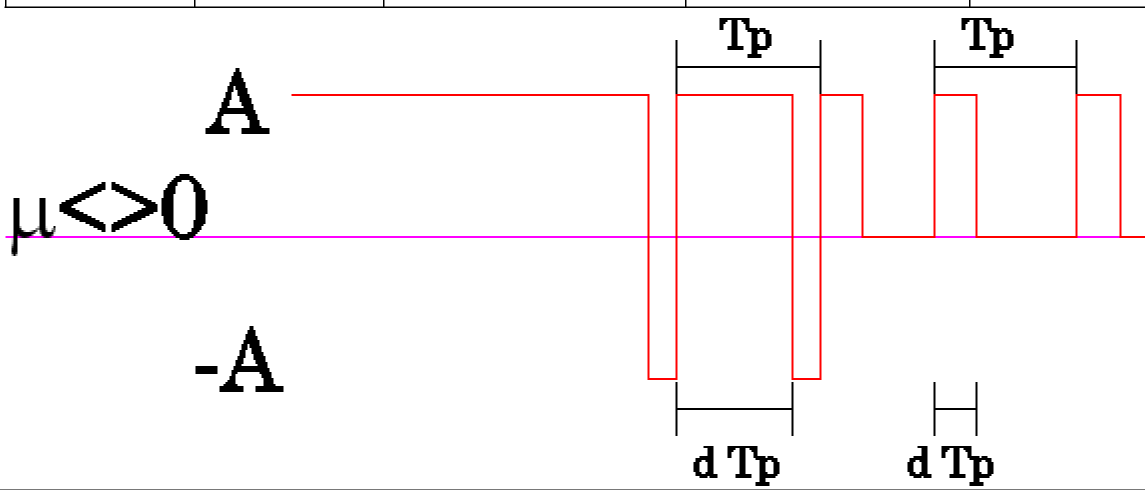
Example: The mains power network
Our mains power network carry sinus voltages with a frequency of 50Hz. The average value is 0V, the peak value is 310V and the RMS value equal to 220V. When we find the maximum effect in a 10A-circuit we multiply the 10A with the RMS voltage (220V) and achieve 2200W. *Vrms* is thus the effective "heating tension" and not eg. peak-voltage

An apropos:
Some voltmeters find the RMS value by dividing the RMS value by √2. This is only correct when the curve is a sine. In true-RMS voltmeters the RMS value is found according to the expression given ahead.

Examples:



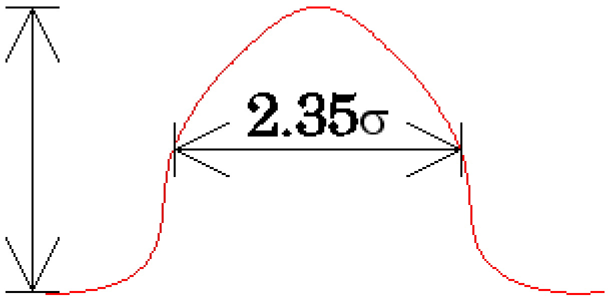
(μ=0)	Sinus	Square (50% duty)	Triangular	Gauss
Vpeak	A	+/-A	+/-A	∞
Vrms	A/√2	A	A/√3	σ



(μ<>0)	DC*	Square High in a part *d* of the time	
Vpeak	A	+/-A	A, (0)
Vrms	A	A	A√d
μ	A	A(2d-1)	Ad

FWHM:

Within some physic fields the notation Full-Width-Half-Maximum is common.



That means the width of the portion of the signal that has a probability that is more than half of the maximum value. This width is a constant scaling of the standard variation and can be expressed as:

$$fwhm = \sigma\sqrt{8\ln 2} \approx 2{,}35\sigma$$

Thermal noise

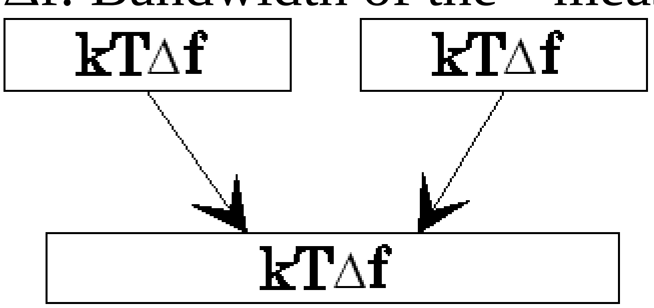
- Due to "Brownian" motion of charges in a conductor.
- First observed by J. B. Johnson in 1927. Theoretically analyzed by H. Nyquist in 1928. Also called "Johnson noise" and "Nyquist noise".
- Over time is the average voltage zero but the random motion of charges results in that we at different time points can measure voltage differences over the terminals.

Available noise power in a conductor can be expressed as: $N_t = kT\Delta f$

k: Boltzmans constant: 1.38E-23 Ws/K

T: Temperatur in Kelvin

Δf: Bandwidth of the "measurement system"



Examples of Nt:

We assume both of:

- Room temperature (17°C or 290K)
- 1Hz bandwidth

$\Rightarrow$ Nt = $4 \cdot 10^{-21} W$ =-204dB relative to 1W

In RF communication we give the noise power relative to 1mW and we have:

$$Noise\_power\_in\_dB_m = 10\log_{10}\left(\frac{4 \times 10^{-21}}{10^{-3}}\right) = -174 dB_m$$

-174dBm is generally called the noise floor and is the minimum noise level one can achieve in a system that operates at room temperature.

(1Hz = 1Hz)

Basic equations

Power:

$$P = UI = \frac{U^2}{R} = RI^2$$

Energy:

$$W = P \cdot t$$

Relative signal strength (decibels):

$$RSS_{dB} = 10\log\frac{P}{P_{ref}} = 10\log\frac{U^2/R}{U_{ref}^2/R_{ref}} =$$

$$10\log\left(\frac{U}{U_{ref}}\right)^2 + 10\log\left(\frac{R_{ref}}{R}\right) =$$

$$20\log\left(\frac{U}{U_{ref}}\right) + 10\log\left(\frac{R_{ref}}{R}\right)$$

Noise Voltage:

It is often easier to calculate and measure noise voltage power than noise power.

The available noise power is the amount of power a resistive source can supply a noiseless resistive load when both resistances are equal.

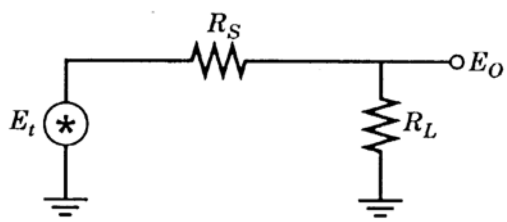


Figure 1-2 Circuit for determination of noise voltage.

Method:
Often, the load resistance is not equal to the source resistance. The method used is that we first assume that these are equal, calculates backward to a theoretical voltage at the source resistance and then use this voltage with the actual load resistance.

$$N_t = \frac{E_0^2}{R_L} = \frac{E_t^2}{4R_L} = \frac{E_t^2}{4R_S} = kT\Delta f$$

$$N_t = \frac{E_0^2}{R_L} = \frac{E_t^2}{4R_L} = \frac{E_t^2}{4R_S} = kT\Delta f$$

We make an expression where the deposed power on the theoretical load is expressed only by values in relation to the source:

$$E_t = \sqrt{4kTR\Delta f}$$

$$4kT = 1.61 \times 10^{-20} \_(at \_ 290K)$$

Example:

1kΩ, 1Hz and 290K (17°C),

$\Rightarrow$ 4nV

(5kΩ? => Multiply with√5)

Noise bandwidth

Signal Bandwidth = / = Noise Bandwidth
Signal Bandwidth: The frequency range with a attenuation of less than -3dB of the centre or maximum signal.
Noise Bandwidth: The noise bandwidth is set so that the product of the noise bandwidth and the maximum noise signal is equal to an area. The area is equal to the integral of the noise integrated over all frequencies. This can be expressed with the formula:

$$\Delta f = \frac{1}{G_0}\int_0^{\infty} G(f)df$$

Here is:
G(f) noise power as a function of frequency.
$G_{O:}$ maximum noise effect.
Δf: noise bandwidth..
However since often the voltage is measured instead of the power it can be useful to express the noise bandwidth as a function of the noise voltage:

$$\Delta f = \frac{1}{A_{vo}^2}\int_0^{\infty} \left|A_v(f)\right|^2 df$$

Examples of "bandwidths"

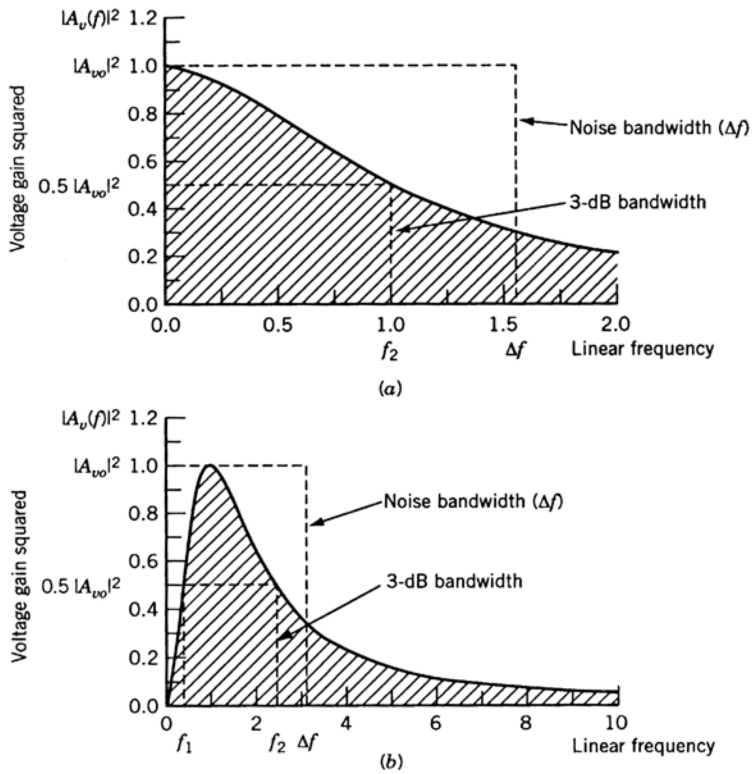


Figure 1-3 Definition of noise bandwidth.

a) Low pass filter
b) Band pass filter
NB! The scale on the frequency axis is linear.

Calculation for low pass filter:

Filter function for first order low pass filter:

$$A_v(f) = \frac{1}{1 + jf/f_2}$$

f_2:-3dB frequency
Normalized: Gain equal to 1 at DC.

Size of gain:

$$|A_v(f)| = \frac{1}{\sqrt{1 + (f/f_2)^2}}$$

Interprets the signal as a noise signal:

$$\Delta f = \int_0^\infty \frac{df}{1 + (f/f_2)^2}$$

Substitutes $f = f_2 \tan\theta$ and $df = f_2 \sec^2\theta d\theta$

and get $\Delta f = f_2 \int_0^{\pi/2} d\theta = \frac{\pi f_2}{2} = 1.571 f_2$

I.e. interpreted as noise the frequency width is 57.1% greater than if it were an ordinary signal.

Calculation for two first order low pass filters

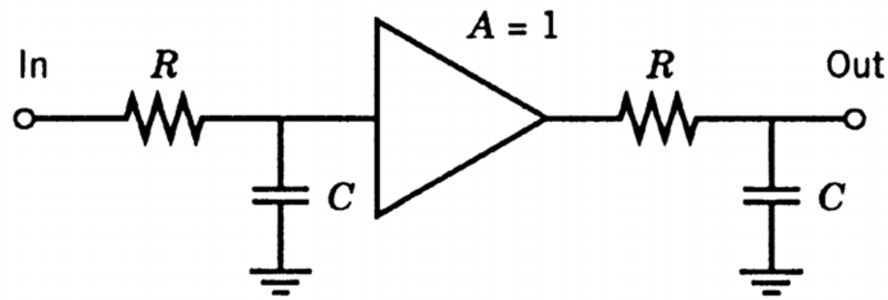


Figure 1-4 Cascaded low-pass filter.

The total gain is now: $A_v(f) = \left| \frac{1}{1 + jf / f_2} \right|^2$

Interpreted as noise the noise bandwidth is:

$$\Delta f = \int_0^\infty \left| \frac{1}{1 + (f / f_2)^2} \right|^2 df$$

with the same substitutions we get

$$\Delta f = \int_0^{\pi/2} \frac{f_2 d\theta}{1 + \tan^2 \theta} = \frac{\pi f_2}{4} = 0.785 f_2$$

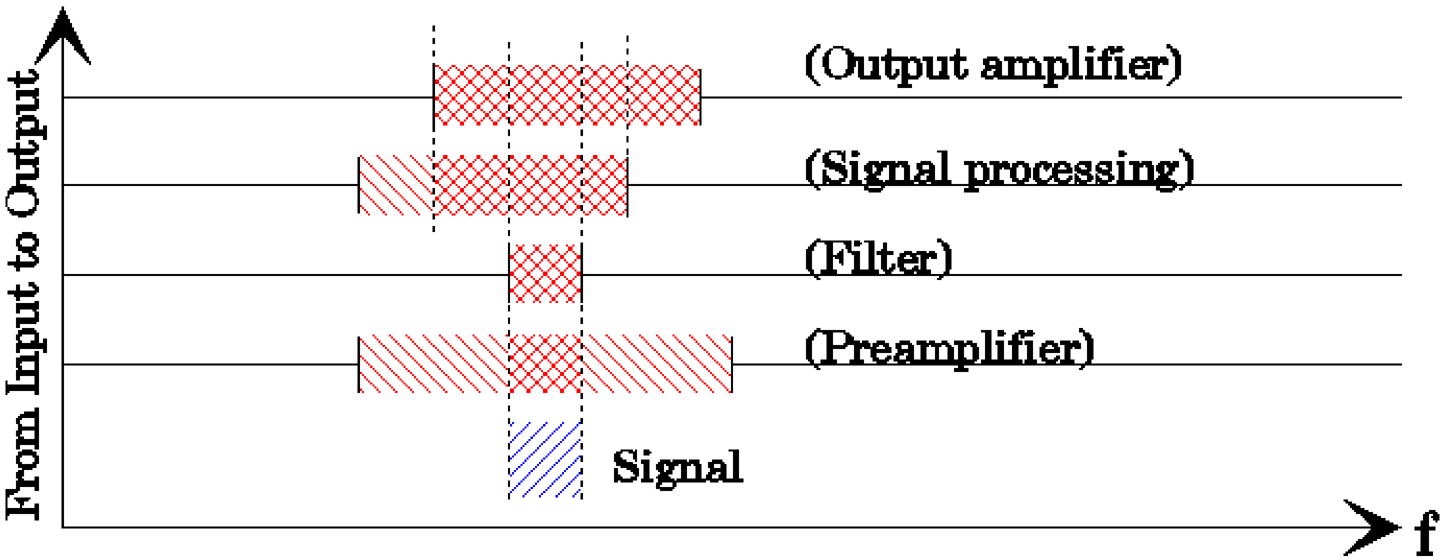
However here f_2 is the -3dB limit for each step and not for the entire system. We must find the -3dB limit to the system: $\frac{1}{\sqrt{2}} = \frac{1}{1 + \left(f_a / f_2 \right)^2}$. This gives us $f_a = 0.6436 f_2$.

At the end we get: $\Delta f = \frac{\pi f_2}{4} = \frac{\pi f_a}{4 \times 0.6436} = 1.222 f_a$

I.e. Δf (noise) is 1.22 fa (signal).

Conclusion:
With sharper edges (higher order) the "noise bandwidth" will approach "the signal bandwidth."

An illustrative example of how the noise generated at the different steps propagate further on towards the exit.



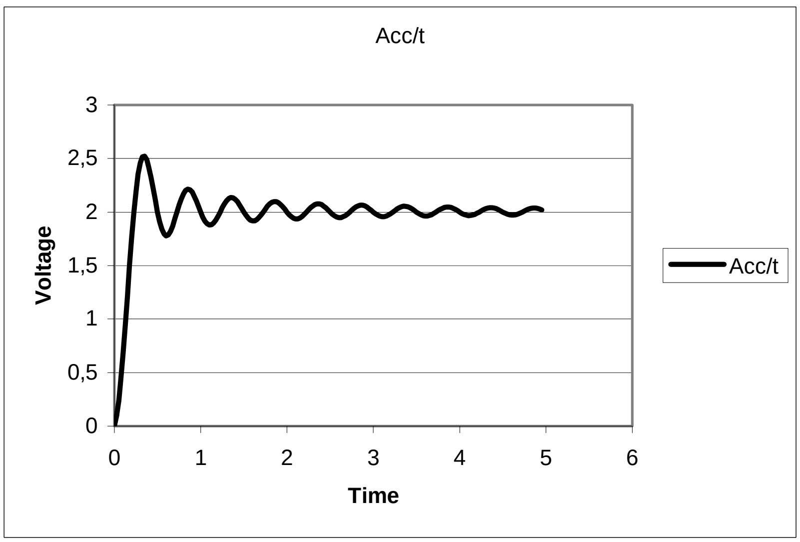
$$V_{Srms}^2 = \frac{1}{T}\int_0^T V_S(t)^2\,dt$$

$$V_S(t) = V_A(t) + V_B(t)$$

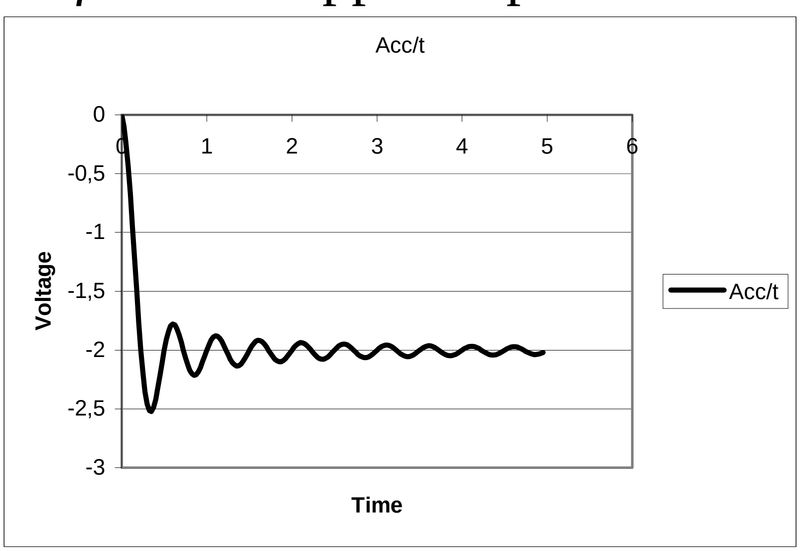
$$V_{Srms}^2 = V_{Arms}^2 + V_{Brms}^2 + \frac{2}{T}\int_0^T V_A(t)V_B(t)\,dt$$

Examples of correlation term (last term):

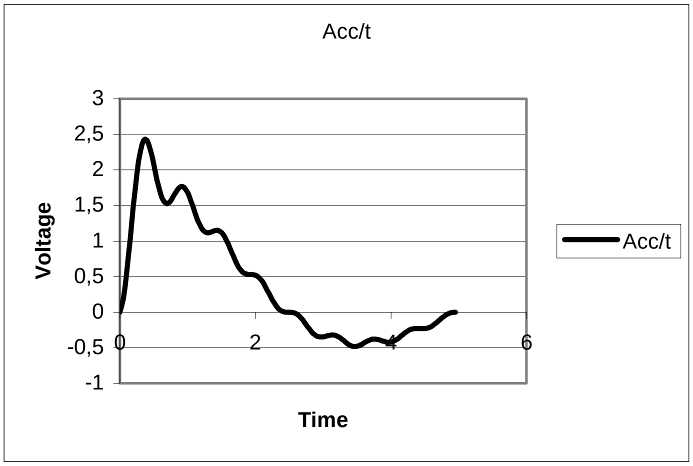
$f_A = f_B$ and in phase:



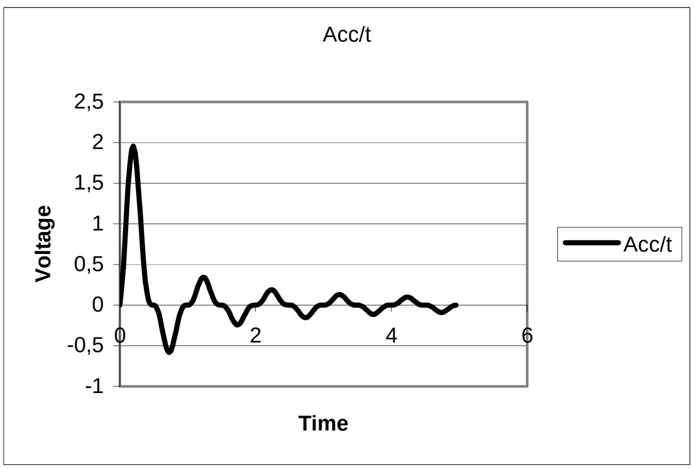
$f_A = f_B$ but in opposite phase:



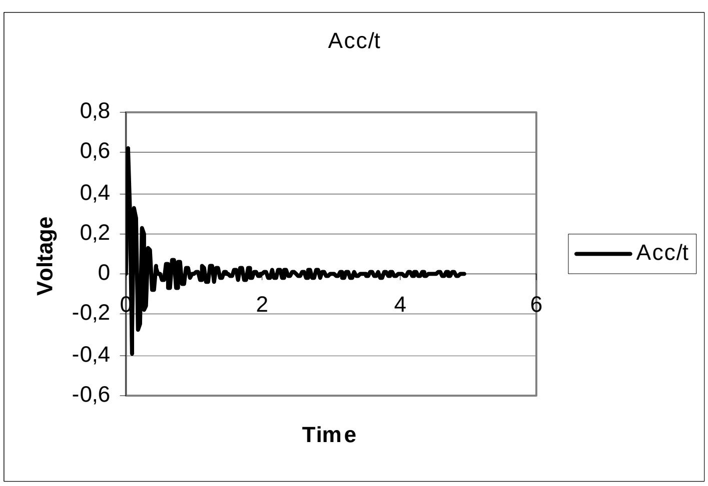
$f_A=0.8 \cdot f_B$:



$f_A=2 \cdot f_B$:



$f_A=10 \cdot f_B$:



Examples of results with and without correlation

Vs (t) = Va (t) + Vb (t)

$$V_s(t)^2 = (V_a(t) + V_b(t))^2 = V_a(t)^2 + V_b(t)^2 + 2V_a(t)V_b(t)$$

Example: Assume that

• Va (t) and Vb (t) has the same amplitude

• and that Va (t) and Vb (t) has the same form (both are a sine, both are a triangle, etc.)

$V_s(t)_{rms}{}^2 = \frac{1}{T}\int_0^T V_s(t)^2\,dt =$		$\frac{1}{T}\int_0^T V_a(t)^2\,dt$	$+\frac{1}{T}\int_0^T V_b(t)^2\,dt$	$+\frac{2}{T}\int_0^T V_a(t)V_b(t)dt$		
Un correlated: $\omega_a = / = \omega_b$		$V_a(t)_{rms}{}^2$	$+V_a(t)_{rms}{}^2$	+0	$= 2V_a(t)_{rms}{}^2$	√2
Correlated:	$\theta_a = \theta_b$	$V_a(t)_{rms}{}^2$	$+V_a(t)_{rms}{}^2$	$+2V_a(t)_{rms}{}^2$	$= 4V_a(t)_{rms}{}^2$	2
$\omega_a = \omega_b$	$\theta_a = \theta_b + \pi$	$V_a(t)_{rms}{}^2$	$+V_a(t)_{rms}{}^2$	$-2V_a(t)_{rms}{}^2$	=0	0

We see that the two noise sources that are correlated will be able to provide between 0% and 141% of noise to the same sources if they were uncorrelated.

Addition of uncorrelated noise voltages

Example 1:

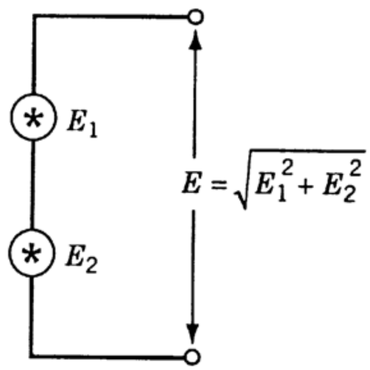


Figure 1-8 Addition of uncorrelated noise voltages.

We must here make an rms addition and not a standard linear addition. We will then have:

$$E^2 = E_1^2 + E_2^2$$

As an approach we can ignore noise contributions that are less than 1/10 of more dominating noise contributions.

Example 2:

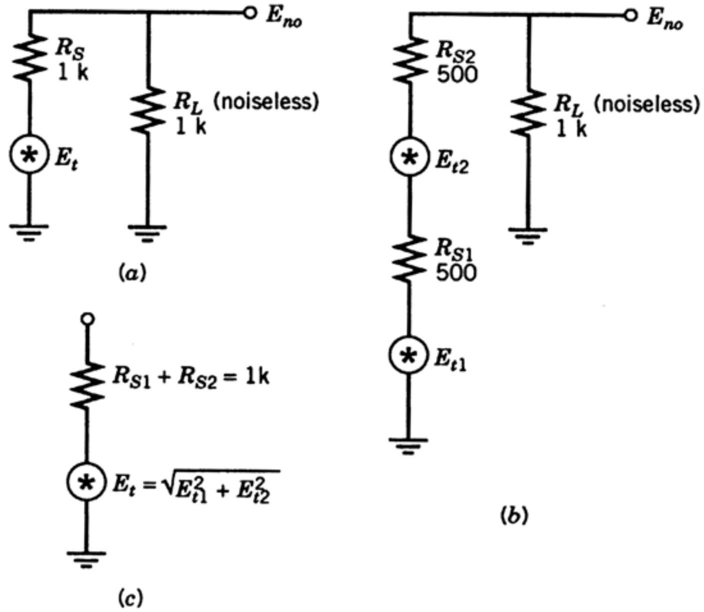


Figure 1-9 Circuits with noise voltages: (*a*) simple circuit, (*b*) equivalent circuit, and (*c*) correct resultant circuit.

In b) is the source resistance in a) split into two equal parts. First, we calculate with standard (and incorrect) linear mathematics.

a): $E_{no} = \dfrac{R_L}{R_S + R_L} E_t = 0.5\left(4nV/Hz^{1/2}\right) = 2nV/Hz^{1/2}$

b):

$$E_{no} = \frac{R_L}{R_{S1} + R_{S2} + R_L} E_{t1} + \frac{R_L}{R_{S1} + R_{S2} + R_L} E_{t2} =$$

$$0.5\left(2.82nV/Hz^{1/2}\right) + 0.5\left(2.82nV/Hz^{1/2}\right) = 2.82nV/Hz^{1/2}$$

We would expect that the answers should be equal but get a difference. The reason is that we use linear calculus although we should have practiced squared calculus of the rms values.

We now perform the same calculation again but with the square of the noise voltage:

$$E_{no}{}^{2} = \left(\frac{R_L}{R_{S1} + R_{S2} + R_L} \right)^2 E_{t1}^2 + \left(\frac{R_L}{R_{S1} + R_{S2} + R_L} \right)^2 E_{t2}^2$$

$$= (0.5)^2 \left(2.82 nV / Hz^{1/2}\right)^2 + (0.5)^2 (2.82 nV / Hz^{1/2})^2$$

$$= (0.5)\left(2.82 nV / Hz^{1/2}\right)^2 = 4 \times 10^{-18} V^2 / Hz$$

So we take the square root:

$$E_{no} = 2 nV / Hz^{1/2}$$

which is the same as we did when we calculated for a).

When there are resistors in series or in parallel like in b) one should calculate the total resistance first and calculate the noise for the resulting resistance.

Partly correlated

When some of the noise in the two noise voltages comes from the same source (cause), while some come from different sources the sources are partly correlated. We may in this case use the expression:

$$E^2 = E_1^2 + E_2^2 + 2CE_1E_2$$

Here is C a correlation coefficient that can have any value between -1 and +1. When C is equal to 0 the voltages are uncorrelated and we have the relationship as discussed earlier. When C is equal to -1 the voltages are correlated but in opposite phase.

Typically the correlation is zero and this is considered as default. If one incorrectly assumes no correlation the maximum error will be when the two noise voltages are equal and completely correlated. For equally large signals (the same rms value) we will have:

- Two correlated signals => 2 rms value.
- Two fully uncorrelated signals => 1.4 of the rms value.

(i.e. the error will be 2/1.4 = 1.4 which gives that the noise level is 40% more than we assumed).

Example 3:

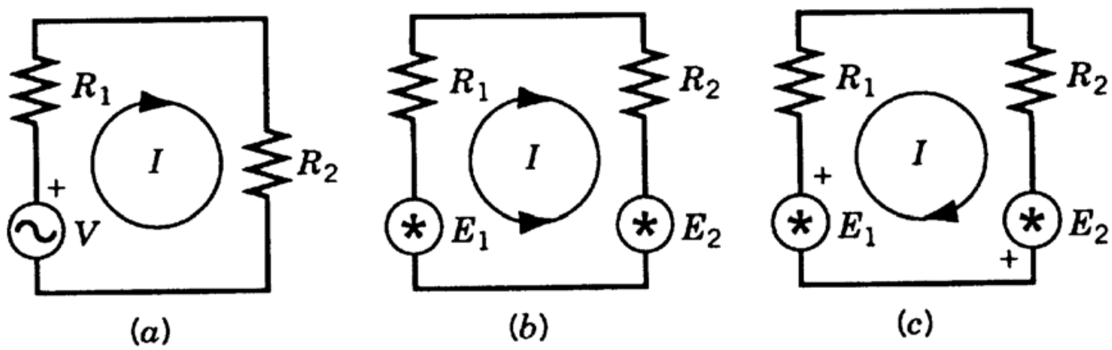


Figure 1-10 Circuits for analysis examples.

a)

We put up the following expression for a):

$$V = IR_1 + IR_2$$

We square all terms and get:

$$V^2 = (IR_1)^2 + (IR_2)^2$$

But this is not correct! Why?

Because it is the same current *I*! Hence all terms are 100% correlated.

We must therefore have a correlation part:

$$V^2 = \left(IR_1\right)^2 + \left(IR_2\right)^2 + 2CIR_1IR_2$$

In this case C = 1 and we can write the expression:

$$V^2 = I^2(R_1 + R_2)^2$$

The rule of thumbs for serial resistors and impedances is that they should first be summed before they are squared. !

b)

Two noise sources (or sine generators with different frequency) is in series with two noise-free resistors. This is also the same current that passes through both resistors. We therefore add the resistances before they are squared.

$$I^2 = \frac{E_1^2 + E_2^2}{(R_1 + R_2)^2}$$

The voltage sources are uncorrelated and hence squared before they are summed.

For the voltage source, there is no correlation term.

b) Calculated with the superposition principle:
The current *I* consists of *I1* and *I2*. We use the super-position principle, which says:
In a linear network will the response from two or more sources be the sum of the response from each source alone with (the other) voltage sources short circuit, and the (other) current sources left open.

That gives us: $I_1 = \frac{E_1}{R_1 + R_2}$ and $I_2 = \frac{E_2}{R_1 + R_2}$

The currents are uncorrelated and we add:

$$I^2 = I_1^2 + I_2^2$$

When we insert for I_1 and I_2 so we get:

$$I^2 = \frac{E_1^2}{(R_1 + R_2)^2} + \frac{E_2^2}{(R_1 + R_2)^2} = \frac{E_1^2 + E_2^2}{(R_1 + R_2)^2}$$

which is what we found earlier.

c) Syntax for partial correlation
E1 and E2 have some correlation. On the figure this is marked with a plus sign. The position of these signs show that they support each other and that the correlation is positive i.e. 0 < 1.≤C

$$I^2 = \frac{E_1^2 + E_2^2 + 2CE_1E_2}{(R_1 + R_2)^2}$$

Example 4: Two uncorrelated sources

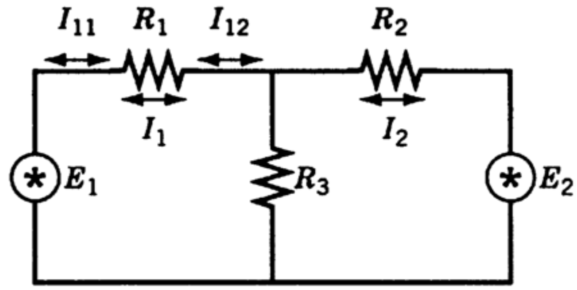


Figure 1-11 Two-loop circuit.

Target: Find the total current I_1 through R_1.
Method: Super-position
Syntax: I_1 consists of two sections: I_{11} from E_1 and I_{12} from E_2.Obs! In the book is I_2 only the contribution from E_2.

We have:

$$E_1^2 = I_{11}^2 \left[R_1 + \frac{R_2 R_3}{(R_2 + R_3)} \right]^2$$

which we can write as:

$$I_{11}^2 = \frac{E_1^2 (R_2 + R_3)^2}{(R_1 R_2 + R_1 R_3 + R_2 R_3)^2}$$

We have also

$$E_2^2 = I_2^2\left[R_2 + \frac{R_1R_3}{\left(R_1 + R_3\right)}\right]^2$$

the part of I_2 that passes through R_1 is:

$$I_{12} = I_2R_3/(R_1 + R_3)$$

The last two expressions can put together to:

$$I_{12}^2 = \frac{E_2^2R_3{}^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

Since

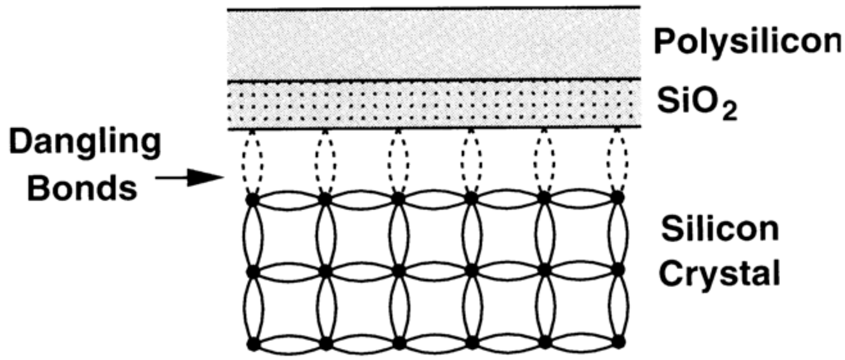
$$I_1^2 = I_{11}^2 + I_{12}^2$$

we can put together the two current contributions on the right side and we get:

$$I_1^2 = \frac{E_1^2(R_2 + R_3)^2 + E_2^2R_3^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

Flicker noise

Flicker noise =1/f-noise = low frequency noise = pink noise



Flicker noise is a sort of noise that is related to irregularities at interfaces between different atom structures. One such place is the interface between the silicon crystal and silicon dioxide. The free electron pairs will collect charge that is trapped for a variable time.

Flicker noise is observed primarily in semiconductors but can also be seen in radio tubes and some sorts of resistors.

Flicker noise have a 1/f-characteristic i.e. it is weakest at the highest frequencies and grow to infinity when the frequency decreases.

To find the noise in a frequency band from f_l to f_h we can integrate as follows:

$$N_f = K_1 \int_{f_l}^{f_h} \frac{df}{f} = K_1 \ln \frac{f_h}{f_l}$$

N_f is the noise power in Watts.
K is a constant in Watt.

Example:
Let $f_h = 10f_l$.
Then we have: $N_f = 2.3K_1$
This shows that the noise level within each decade is constant. Thus the flicker noise between 0.01 and 0.1 Hz is equal to the noise from 100kHz to 1MHz.

Most other types of noise is given pr √Hz and the multiplication with Δf is done at the end. It is often advisable to do the same with the flicker noise. We make the approximation: $\frac{\Delta f}{f_l} \approx \ln\left(\frac{f_h}{f_l}\right)$

This is correct as long as $\Delta f \ll f_l$

Example: Flicker noise in MOSFET

$$I_f^2(f) = \frac{K_F I_{DS}^{AF}}{Cox \cdot L_{eff}^2} \cdot \frac{1}{f}$$

$$I_f^2(f_l, f_h) = \frac{K_F I_{DS}^{AF}}{Cox \cdot L_{eff}^2} \int_{f_l}^{f_h} \frac{1}{f} = \frac{K_F I_{DS}^{AF}}{Cox \cdot L_{eff}^2} \ln \frac{f_h}{f_l}$$

	AF	KF	Cox
N	1.5	2.3e-26	2.2fF/μm²
P	1.3	6.3e-29	2.2fF/μm²

KF: 2.3e-26 --- 6.3e-29
AF: 1.3 --- 1.8
m²μ / μCox: 2.1fF / m² --- 4.6fF

1.11 Shot noise

Shot noise occurs in the pn-interfaces in the transistors and diodes.
This type of noise describes fluctuations in the current running.
It is expressed as:

$$I_{sh} = \sqrt{2qI_{DC}\Delta f}$$

where q = 1.602 e-19 Coulomb.
We see that the noise level increases with the square root of the current. We also see that this type of noise is "white" i.e. it is constant and independent of the frequency (but not the bandwidth).

Bipolar transistors:

From the equation for Shot-noise one could assume that the shot-noise level was almost zero when the power is zero. This is not correct. We will now study this further.

In bipolar transistors, we find the most shot-noise in the emitter-base interface.
The VI behaviour follows the familiar diode expression:

$$I_E = I_S(e^{qV_{BE}/kT} - 1)$$

where I_E is the emitter current in Ampere, I_S is the reverse current in Ampere and V_{BE} is the voltage between the base and emitter.
We share the current I_E in two parts ...

$$I_E = I_1 + I_2$$

so that $I_1 = -I_S$ and

$$I_2 = I_S \exp(V_{BE}/kT)$$

I_1 is due to thermally generated minority carriers while I_2 is due to diffusion of majority carriers over the pn-interface.
NB! Both these currents have full shot-noise even if the current itself eliminate each other at $V_{BE} = 0$ *Volt!*

(During reversed bias voltage I_1 will dominate while under strong forward voltage I_2 will dominate.).

At $V_{BE} = 0$ we have that $I_E = 0$ while the noise is

$$I_{sh}^2 = 4qI_S\Delta f$$

Shot noise model:

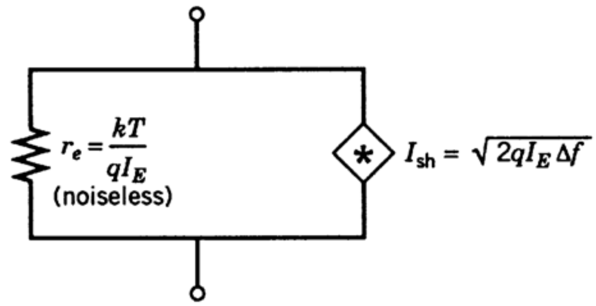


Figure 1-12 Shot noise equivalent circuit for forward-biased *pn* junction.

The circuit model for shot noise consists of a current source in parallel with a (noiseless) resistance.

We find the size of the resistance by derivative the expression for diode current by V_{BE}. By this derivation, we will have conductivity. The resistance is found by finding the inverse of the conductivity.

$$r_e = kT / qI_E$$

Capacitive shunting of the thermal noise: kT / C noise

The expression for thermal noise

$$E_e = \sqrt{4kTR\Delta f}$$

indicates that an open circuit with infinite resistance will generate an infinite noise voltage. This will not be the case since there will always be (parasitic) capacitances between the terminals.

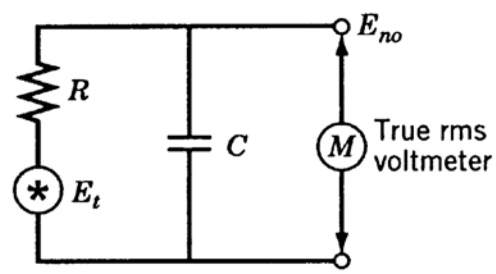


Figure 1-14 Thermal noise of a resistor shunted by a capacitance.

The resistance and capacitor will together act as a low pass filter. When the resistance grows so do E_t. But at the same time decreases the filter cut-off frequency and hence reduces the bandwidth.

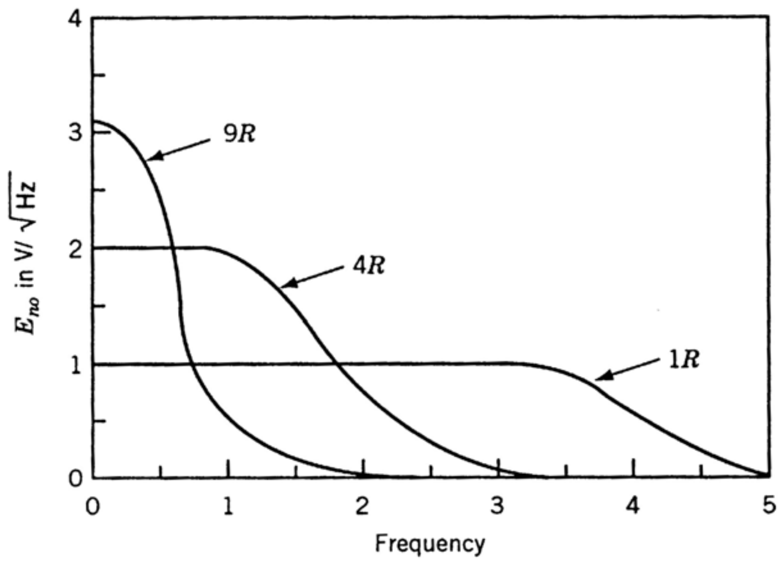


Figure 1-15 Noise spectral density for a resistance shunted by a capacitance.

The figure shows three curves with the same *C* but with three different *R*-values. The integral under the curves are equal.

First we calculate the integrated noise voltage:

$$E_{no}^2 = \int_0^\infty E_t^2 \left| \frac{1/j\omega C}{R + 1/j\omega C} \right|^2 df = \int_0^\infty \frac{E_t^2 df}{1 + (\omega RC)^2}$$

Then we need to change some variables:

$f = f_2 \tan\theta$, $f_2 = 1/2\pi RC$, $df = f_2 \sec^2\theta d\theta$

and change the upper limit to $\pi/2$. Then we get

$$E_{no}^2 = \int_0^{\pi/2} \frac{E_t^2 f_2 \sec^2\theta d\theta}{1 + \tan^2\theta} = \int_0^{\pi/2} E_t^2 f_2 d\theta = \int_0^{\pi/2} 4kTRf_2 d\theta = 2\pi kTRf_2$$

and when we insert for f_2 we get:

$$E_{no}^2 = kT/C$$

C sets an upper limit for the noise voltage

Example of a kT/C-calculation.

Assume an input signal of 1µVrms amplified by 30dB. This gives an output signal of 31.6µ Vrms. The output will be sampled and measured and we would like to have a capacitance large enough to limit the noise voltage to -15dB below the signal level. With a 200pF capacitor and a temperature of 290°K capacitor is limiting the noise to 4.5µVrms which is -17dB relative to the signal level.

$$E_n = \sqrt{\frac{kT}{C}} = \sqrt{\frac{1.38 \cdot 10^{-23} W / Ks \cdot 290^{\circ} K}{200\, pF}} = 4.5 \mu Vrms$$