

# **F8 (Mot 5):**

## **Noise in bipolar transistors**

We have previously found  $E_{ni}$ ,  $E_n$ , and  $I_n$  for an amplifier. We will now do this for a bipolar transistor. We will see that the noise is both depending on the operating point (current and voltage) and the transistor semiconductor process and layout parameters.

## The hybrid- $\pi$ model

Before we look at the noise we will look at a typical model for bipolar transistors. This model applies for both npn and pnp.

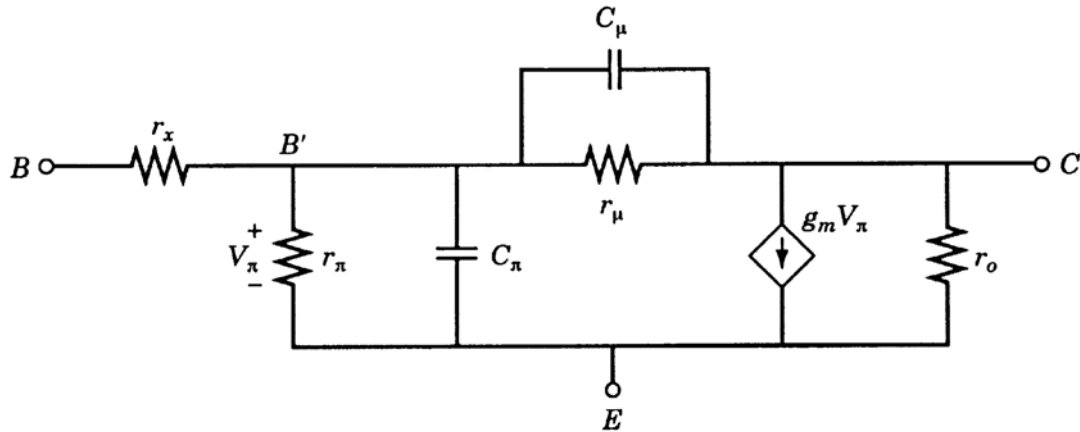


Figure 5-1 Hybrid- $\pi$  bipolar transistor small-signal model.

Note that in the figure is B the external connection point for the base B while B' is the internal, efficient point that best represents the base.

$r_x$ : is the stray resistance in the base.

$r_\pi$ : Real part of impedance B'-E (NB: No thermal noise)

$c_\pi$ : Imaginary part of the impedance B'-E drawn as a capacitance.

$g_m V_\pi$ : Current generator that determines  $I_c$ .

$r_o$ : Dynamic output resistance of C

$r_\mu$  and  $C_\mu$ : Models the depletion zone between B and E and C. Ignored if one wants a simpler low frequency model.

## Some well-known expressions:

$$\beta_0 = \frac{I_C}{I_{BE}} = \frac{g_m V_\pi}{V_\pi / r_\pi} = g_m r_\pi$$

Presuppose  $r_\mu = \infty$  and all  $C \approx 0$  (ie low-frequency consideration.)

$$g_m = \frac{qI_C}{kT}$$

Transconductance. NB! Small signal ac-parameters related to a dc-current => Limited validity range.

$$r_e = \frac{1}{g_m} \cong \frac{0.025}{I_C} \Omega$$

Emitter resistance.

$$r_\pi = \frac{\beta_0}{g_m} = \beta_0 r_e$$

$r_\pi$  in relation to some of the previous mentioned parameters.

$$C_{\pi} = \frac{g_m}{2\pi f_T} - C_{\mu}$$

An important value is the gain-bandwidth product. It is expressed as  $f_T$  and is equivalent to the frequency where the gain is -3dB (1/2). We can modify the expression so that we get:

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$$

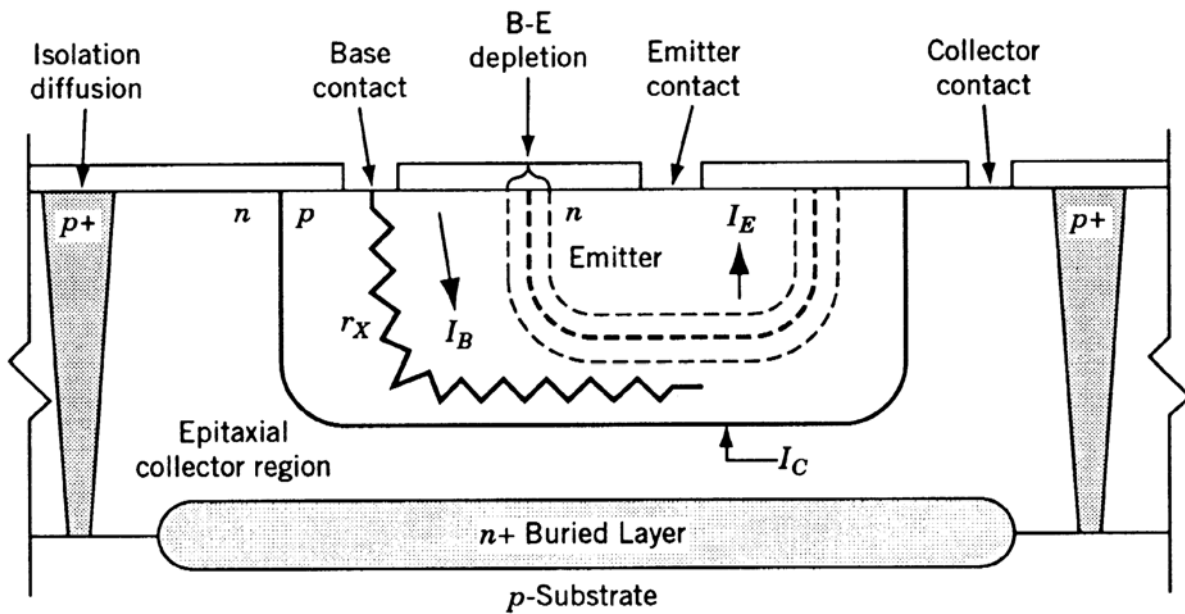
$f_{hfe}$  is the "beta-cut-off frequency". This is the frequency where  $\beta$  is  $1/\sqrt{2}$  of its low frequency value (DC-value)  $\beta_0$ .

$$f_{hfe} = f_{\beta} \cong \frac{f_T}{\beta_0}$$

Some example values:

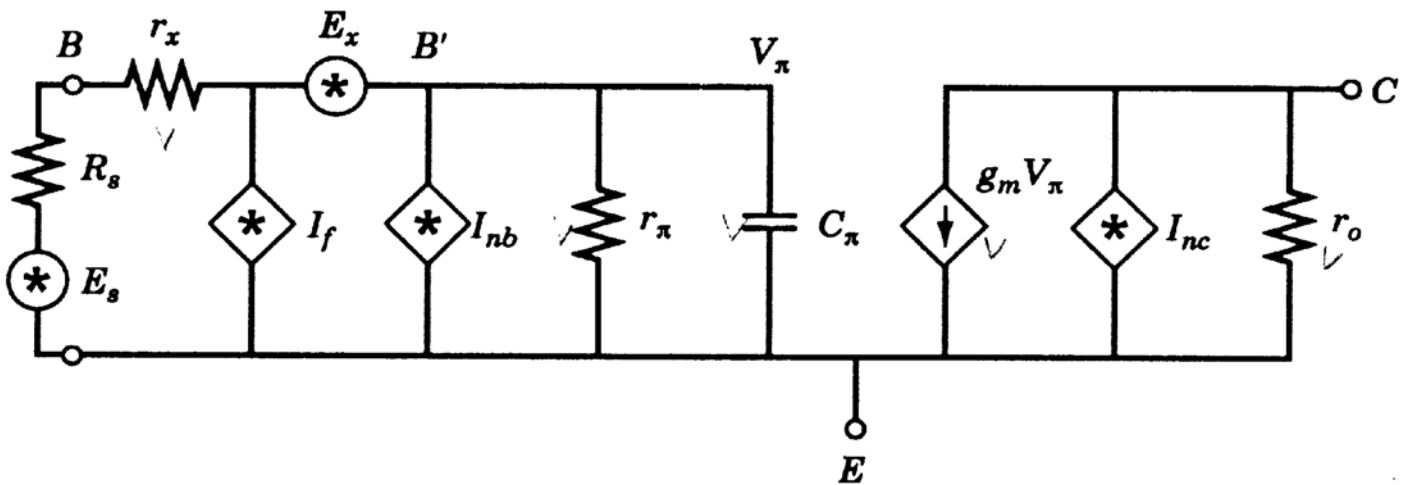
$r_{\pi}=97\text{k}\Omega$	$r_o=1.6\text{M}\Omega$
$r_x=278\Omega$	$r_{\mu}=15\text{M}\Omega$
$g_m=0.0036\text{S}$	$C_{\mu}=4\text{pF}$
$\beta_0=350$	$C_{\pi}=25\text{pF}$
$f_T=19.8\text{MHz}$	$f_{\beta}=56.5\text{kHz}$

# Noise model for the BJT



**Figure 5-2** Cross-section diffused *npn* transistor.

$r_x$  provides thermal noise.  $I_B$  and  $I_C$  provide shot-noise. The current passing through the base-emitter zone closest to the surface will have some flicker noise.



**Figure 5-3** Hybrid- $\pi$  bipolar transistor noise model.

Now we have four noise sources for the transistor. In addition we have the noise in the source.  $r\pi$  and  $r_o$  represent real parts of impedances without having thermal noise. To simplify  $C\mu$  and  $r\mu$  have been omitted i.e. we look at frequencies below  $f_T/\sqrt{\beta}$ . Above this frequency some of the noises become correlated and we will have more noise than the model indicates.

Thermal noise in  $r_x$ :

$$E_x^2 = 4kTr_x$$

Shot-noise due to  $I_B$  and  $I_C$  are, respectively:

$$I_{nb}^2 = 2qI_B$$

and

$$I_{nc}^2 = 2qI_C$$

The flicker noise can be expressed by:

$$I_f^2 = \frac{KI_B^\gamma}{f^\alpha}$$

Often alpha can be set to one and  $K$  can be replaced by  $2qf_L$  where  $f_L$  is a corner frequency typically in the range 3kHz to 7MHz. We will then get:

$$I_f^2 = \frac{2qf_L I_B^\gamma}{f}$$

The noise voltage as a result of this current can be found by multiplying it with the resistance it will go through:  $r_x$ . However experimental data shows that the effective  $r_x$  in this context is less. We therefore create a new  $r'_x$  which is ca.  $r_x/2$ .

$$E_f^2 = \frac{2qf_L I_B^\gamma r_x'^2}{f}$$

## Equivalent input noise.

Method:

- 1) Noise at output
- 2) Gain
- 3) The noise on the input = output noise/gain

### 1) Noise at the output:

First we find the noise at the output. If C (the output) is short circuited to E we will have:

$$I_{no}^2 = I_{nc}^2 + (g_m E_\pi)^2$$

Here is  $E_\pi$  the noise voltage between B' and E.

After substituting the value for  $E_\pi$  so we get:

$$I_{no}^2 = I_{nc}^2 + g_m^2 \left[ \frac{(E_x^2 + E_s^2) Z_\pi^2}{(r_x + R_s + Z_\pi)^2} + \frac{(I_{nb}^2 + I_f^2) Z_\pi^2 (r_x + R_s)^2}{(r_x + R_s + Z_\pi)^2} \right]$$

### 2) Gain:

$$I_o = g_m V_\pi = \frac{g_m V_s Z_\pi}{r_x + R_s + Z_\pi}$$

$$K_t = \frac{I_o}{V_s} = \frac{g_m Z_\pi}{r_x + R_s + Z_\pi}$$



### 3) Noise at the Input

$$E_{ni}^2 = \frac{I_{no}^2}{K_t^2} = E_x^2 + E_s^2 + (I_{nb}^2 + I_f^2)(r_x + R_s)^2 + \frac{I_{nc}^2 (r_x + R_s + Z_\pi)^2}{g_m^2 Z_\pi^2}$$

We put in for the noise voltages and noise currents and let  $\Delta f = 1$ :

$$E_{ni}^2 = 4kT(r_x + R_s) + 2qI_B(r_x + R_s)^2 + \frac{2qf_L I_B^\gamma (r'_x + R_s)^2}{f} + \frac{2qI_C (r_x + R_s + Z_\pi)^2}{g_m^2 Z_\pi^2}$$

At low frequencies the last term may be written as:

$$\frac{2qI_C (r_x + R_s + r_\pi)^2}{\beta_0^2}$$

and at higher frequencies up towards  $f_T/\sqrt{\beta_0}$  as

$$\frac{2qI_C \left( r_x + R_s + \frac{1}{\omega C_\pi} \right)^2}{\frac{g_m^2}{\omega^2 C_\pi^2}} \cong 2qI_C (r_x + R_s)^2 \left( \frac{f}{f_T} \right)^2$$

Divided into a low-frequency component and a high frequency we get:

$$E_{ni}^2 = 4kT(r_x + R_s) + 2qI_B(r_x + R_s)^2 + \frac{2qI_C (r_x + R_s + r_\pi)^2}{\beta_0^2} + \frac{2qf_L I_B^\gamma (r'_x + R_s)^2}{f} + 2qI_C (r_x + R_s)^2 \left( \frac{f}{f_T} \right)^2$$

## En and In for bipolar transistors

### **En:**

We find  $E_n$  by setting  $R_s=0$  in the expression for the noise on the input.

$$E_n^2 = 4kTr_x + 2qI_B r_x^2 + \frac{2qI_C r_\pi^2}{\beta_0^2} + \frac{2qf_L I_B^\gamma r_x'^2}{f} + 2qI_C r_x^2 \left( \frac{f}{f_T} \right)$$

Since  $r_\pi = \beta_0 r_e$  and  $r_x^2 \ll \beta_0 r_e^2$  we can simplify to

$$E_n^2 = 4kTr_x + 2qI_C r_e^2 + \frac{2qf_L I_B^\gamma r_x'^2}{f} + 2qI_C r_x^2 \left( \frac{f}{f_T} \right)^2$$

### **In:**

We find  $I_n$  by letting  $R_s$  be high. We divide all terms by  $R_s^2$  and let  $R_s$  go against  $\infty$ .

$$I_n^2 = 2qI_B + \frac{2qI_C}{\beta_0^2} + \frac{2qf_L I_B^\gamma}{f} + 2qI_C \left( \frac{f}{f_T} \right)^2$$

Since  $I_C/\beta_0^2 \ll I_B$  the first term will dominate the second term which thus can be removed. We will then have:

$$I_n^2 = 2qI_B + \frac{2qf_L I_B^\gamma}{f} + 2qI_C \left( \frac{f}{f_T} \right)^2$$

**Example:**

Find  $E_{ni}^2$  for a 2N4250 transistor where  
 $I_c=1\text{mA}$ ,  $R_s=10\text{k}\Omega$ ,  $\Delta f=10\text{Hz}$ ,  $f_c=1\text{kHz}$

Noise in the source resistance can be calculated:

$$E_t^2 = 4kTR_s = 4 \cdot 1.38 \cdot 10^{-23} \text{Ws} / \text{K} \cdot 300\text{K} \cdot 10\text{k}\Omega = 1.65 \cdot 10^{-16} \text{Ws}\Omega$$

For further calculation, we use:

$$E_{ni}^2 = \left( E_t^2 + E_n^2 + I_n^2 R_s^2 \right) \Delta f$$

Based on the information in the book, we have two ways we can find  $E_n$  and  $I_n$ : From reading the figures or by calculation.

**1) Reading of the figure:**

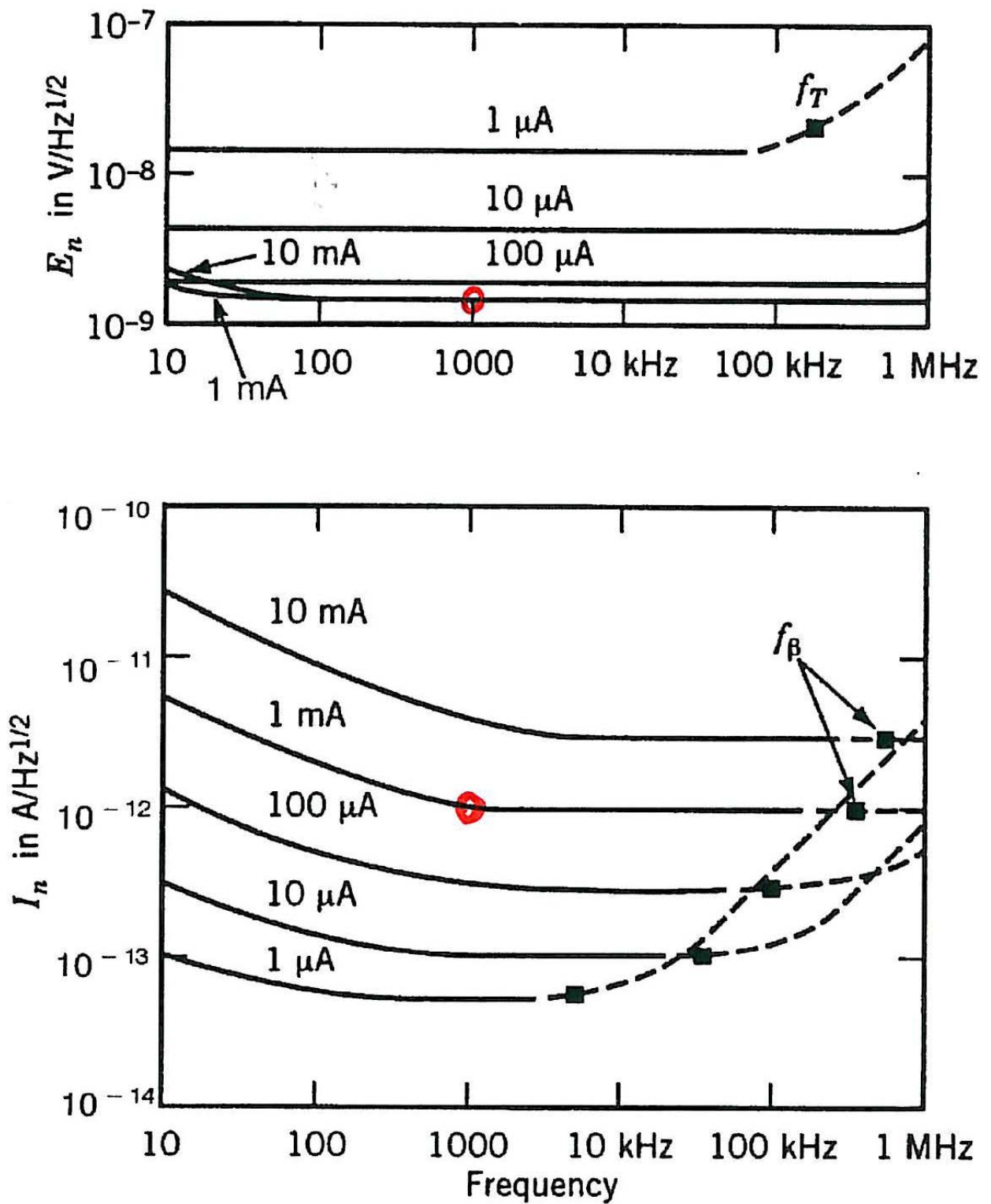
In Figure 5-9 we find  $E_n$  to about  $2\text{nV}/\sqrt{\text{Hz}}$  and  $I_n$  to about  $1\text{pA}/\sqrt{\text{Hz}}$ . Substituted in the equation above we get  $E_{ni}$  to be about  $51.4\text{nV}$ .

$$E_{ni}^2 = \left[ 1.6 \times 10^{-16} + (2 \times 10^{-9})^2 + (10^{-12})^2 (10^4)^2 \right] (10)$$

$$E_{ni}^2 = \left[ 1.6 \times 10^{-16} + 4 \times 10^{-18} + 10^{-16} \right] (10)$$

$$E_{ni}^2 = 2.64 \times 10^{-15} \text{V}^2$$

$$E_{ni} = 51.4 \text{nV}$$



**Figure 5-9**  $E_n$  and  $I_n$  performance of a 2N4250 transistor.

## 2) Calculation

In addition to the specified values, the values in table 5-1 and the known constants,  $r_e$  and  $I_B$  must be calculated. These can be found using the formulas and the other values.

$$E_n^2 = 4kTr_x + 2qI_C r_e^2 + \frac{2qf_L I_B^\gamma r_x'^2}{f} + 2qI_C r_x^2 \left( \frac{f}{f_T} \right)^2$$

$$2,48 \cdot 10^{-18} + 2,14 \cdot 10^{-19} + 1,39 \cdot 10^{-21} + 1,15 \cdot 10^{-27} = 2,70 \cdot 10^{-18}$$

$$I_n^2 = 2qI_B + \frac{2qf_L I_B^\gamma}{f} + 2qI_C \left( \frac{f}{f_T} \right)^2$$

$$8,59 \cdot 10^{-25} + 3,30 \cdot 10^{-25} + 5,11 \cdot 10^{-32} = 1,19 \cdot 10^{-24}$$

$$E_{ni}^2 = (E_t^2 + E_n^2 + I_n^2 R_S^2) \Delta f$$

$$(1,65 \cdot 10^{-16} + 2,70 \cdot 10^{-18} + 1,19 \cdot 10^{-16} \cdot (10 \cdot 10^3)^2) \cdot 10 = 2,867 \cdot 10^{-15} \text{V}^2$$

$$E_{ni} = 52.8 \text{ nV}$$

### Midband noise (= minimum noise)

As we see from the expressions  $E_n$  and  $I_n$  are frequency-dependent. At low frequencies the  $1/f$ -noise will be substantially while at higher frequencies, we will get an additional frequency dependent part of the shot-noise in the collector. We can talk about a midband where the noise is not strongly frequency dependent, and where other contributions than the frequency-dependent is dominant. The midband indicates in a way the minimum noise level we can achieve.

If we remove the frequency-dependent parts of the expressions for  $E_n$  and  $I_n$  so we get:

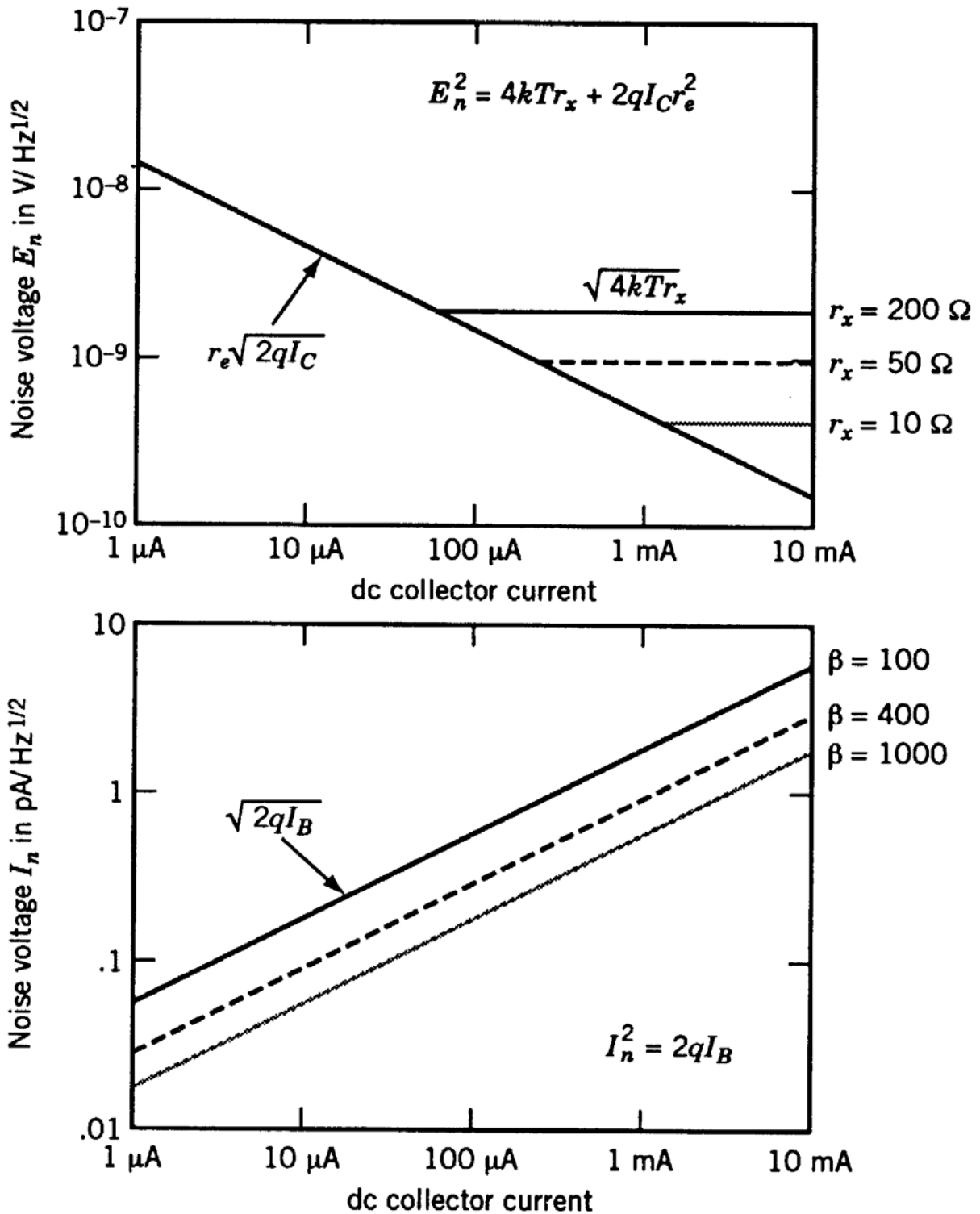
$$E_n^2 = 4kTr_x + 2qI_C r_e^2$$

and

$$I_n^2 = 2qI_B$$

When  $R_s$  is low,  $E_n$  will dominate and it will be desirable with a small base resistance.

When  $R_s$  is large the  $I_n^2 R_s^2$ -term will easily dominate. When this is the case it will be important to have a small  $I_B$ . In order to achieve this, the  $I_C$  should be small and  $\beta$  large.



**Figure 5-4** Limiting noise voltage and noise current.

(Noise current at the Y-axis in the lower figure!)

## Minimizing the noise factor

We found previously the following expressions for optimum noise factor:

$$F_{opt} = 1 + \frac{E_n I_n}{2kT\Delta f}$$

(This can only be achieved when one can select  $R_s$  freely so that  $R_s = R_O = E_n/I_n$ .)

We insert the frequency independent terms we found for  $E_n$  and  $I_n$  and get:

$$F_{opt} = 1 + \sqrt{\frac{2r_x}{\beta_0 r_e} + \frac{1}{\beta_0}}$$

To achieve low noise we must:

Reduce  $r_x$

Increase  $\beta_0$

Reduce  $I_c$  ( $r_e \sim 1/I_c$ )

Normally you will achieve the lowest noise when the collector current is less than 100 $\mu$ A. If the collector current is very small, we are left with:

$$F_{opt} = 1 + \frac{1}{\sqrt{\beta_0}}$$



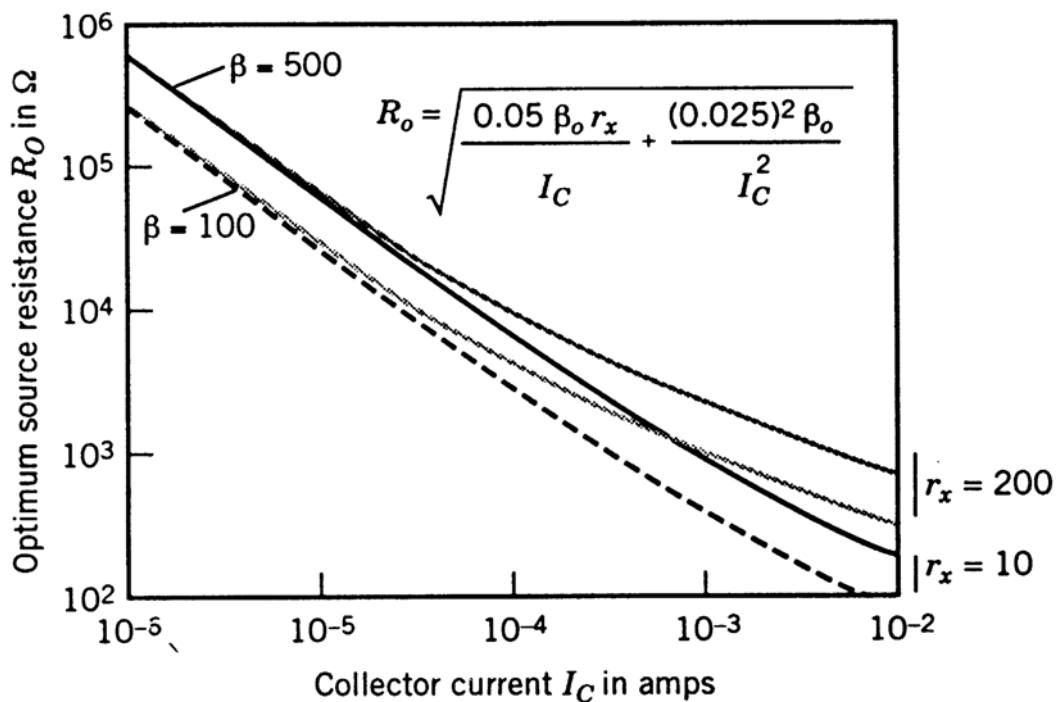
### Optimal $R_s$ :

The optimum condition above presumes that  $R_s = R_o = E_n / I_n$ . We insert the expressions for  $E_n$  and  $I_n$  and get:

$$R_o = \sqrt{\frac{0.05 \beta_o r_x}{I_C} + \frac{(0.025)^2 \beta_o}{I_C^2}}$$

We see that reduced  $I_C$  requires larger  $R_s$ !  
When the base resistance can be neglected, then we have:

$$R_o \cong \frac{0.025 \sqrt{\beta_o}}{I_C}$$



**Figure 5-5** Graph of optimum source resistance versus  $I_C$ .

## The frequency range dominated by 1/f-støy (i.e. low frequencies)

At low frequencies the flicker (1/f)-noise will dominate. We return to our original expression for  $E_n$  and  $I_n$  and retain only the flicker noise. We will then have:

$$E_n^2 = \frac{2qf_L I_B^\gamma r_x'^2}{f^\alpha}$$

and

$$I_n^2 = \frac{2qf_L I_B^\gamma}{f^\alpha}$$

These are the same with the exception of the resistance  $r_x'^2$ .

Optimal  $R_s$  will in this case be:

$$R_s = R_0 = E_n / I_n = r_x'^2$$

We see here that  $R_s$  is independent of all other values than  $r_x'^2$ .

In this frequency range, we have:

$$F_{opt} = 1 + \frac{qf_L I_B^\gamma r_x'}{kTf^\alpha}$$

How to achieve low noise in this frequency band?

Small  $r_x'$

Small  $I_C$  (and thus small  $I_B$ ).

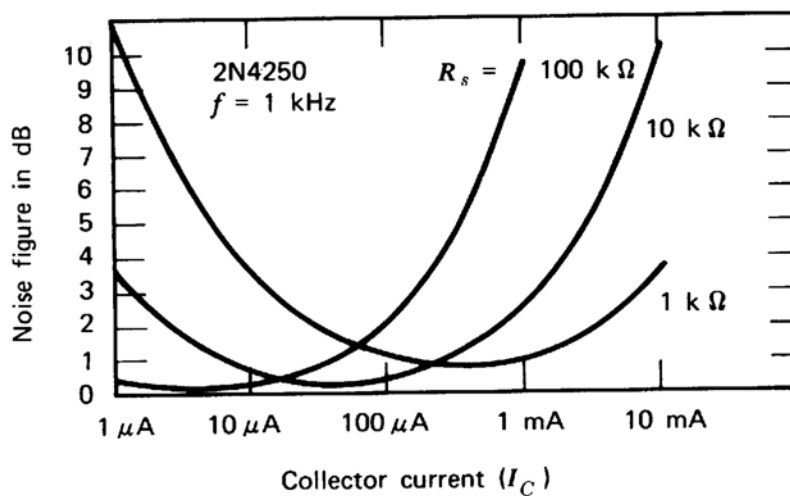
This provides good conditions also in the frequency range we discussed earlier. However it does not ensure good high frequency qualities.

## Operation Conditions and Noise

Equivalent input noise is expressed by:

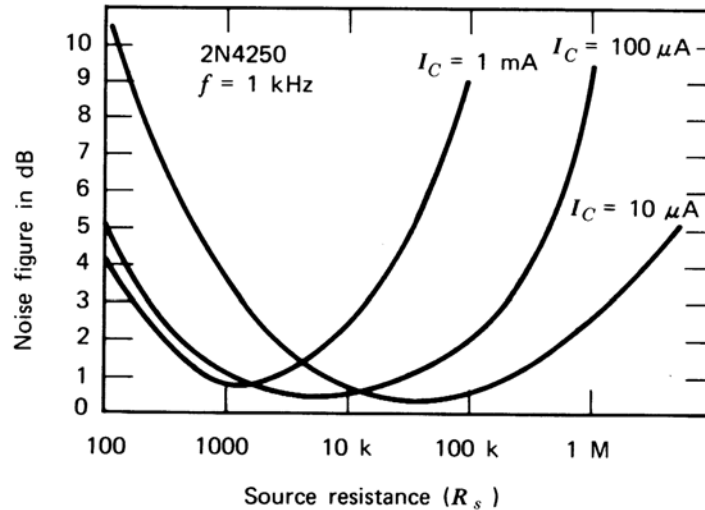
$$E_{ni}^2 = E_t^2 + E_n^2 + I_n^2 R_s^2$$

In figure 5-4 we saw that  $E_n$  declined with growing  $I_c$  while  $I_n$  grows with growing  $I_c$ . Knowing this, one can expect that noise will be great for low and high  $I_c$  and have a minimum in the middle. (Thermal noise in the source ( $E_t$ ) will not be affected by  $I_c$ .) Since the contribution from  $I_n$  scale with  $R_s$  the minimum value will move. Since  $I_n$  is growing with growing  $I_c$ , growing  $R_s$  will give that the minimum point moves towards lower  $I_c$ . This can be seen in the figure below where the horizontal axis is the current while the curves for some selected resistors are outlined.



**Figure 5-6** Effect of collector current and source resistance on noise figure.

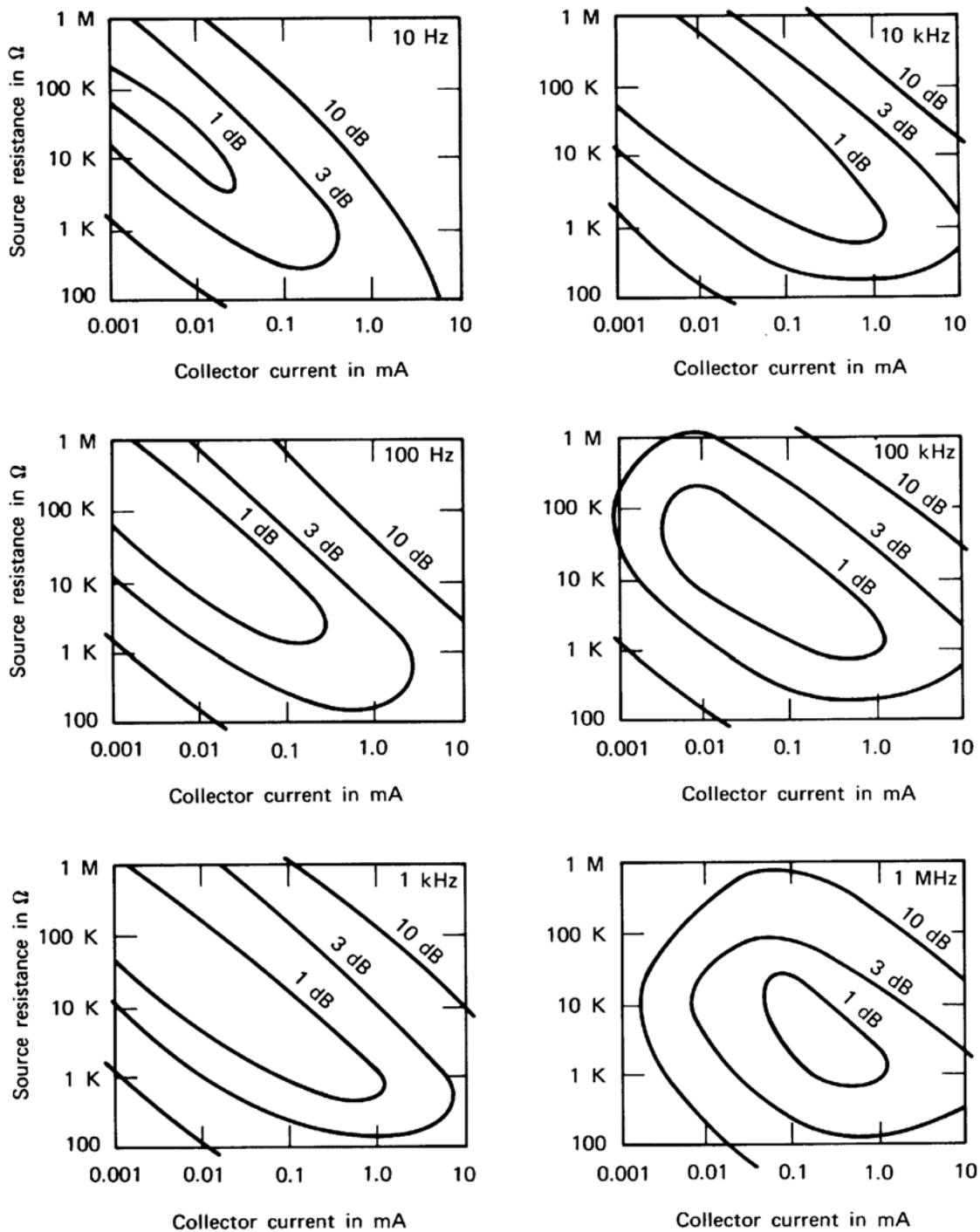
In the curve below we switch and let the horizontal axis be the resistance while the curves for some selected currents are outlined.



**Figure 5-7** Noise figure variation with source resistance versus collector current.

Conclusion:

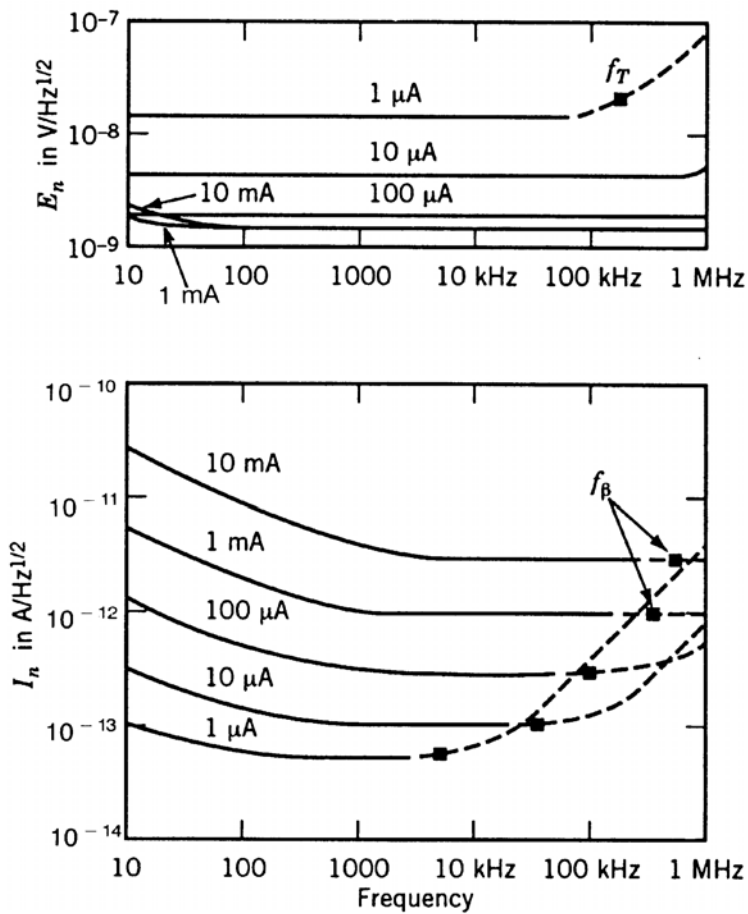
- For a given  $R_s$  there is a minimum noise.
- For a given  $I_C$ , there is a minimum noise.
- Optimal  $R_s$  decreases with increasing  $I_C$ .



**Figure 5-8** Contours of constant narrowband noise figure.

The figure above shows an alternative way to present the curves. The six images are six different frequencies. For a given frequency a combination of  $R_s$  and  $I_c$  should be found that provides the greatest "1dB-area" around the point of the selected  $R_s$  and  $I_c$ .

Often it will be useful to consider the noise as a function of frequency.



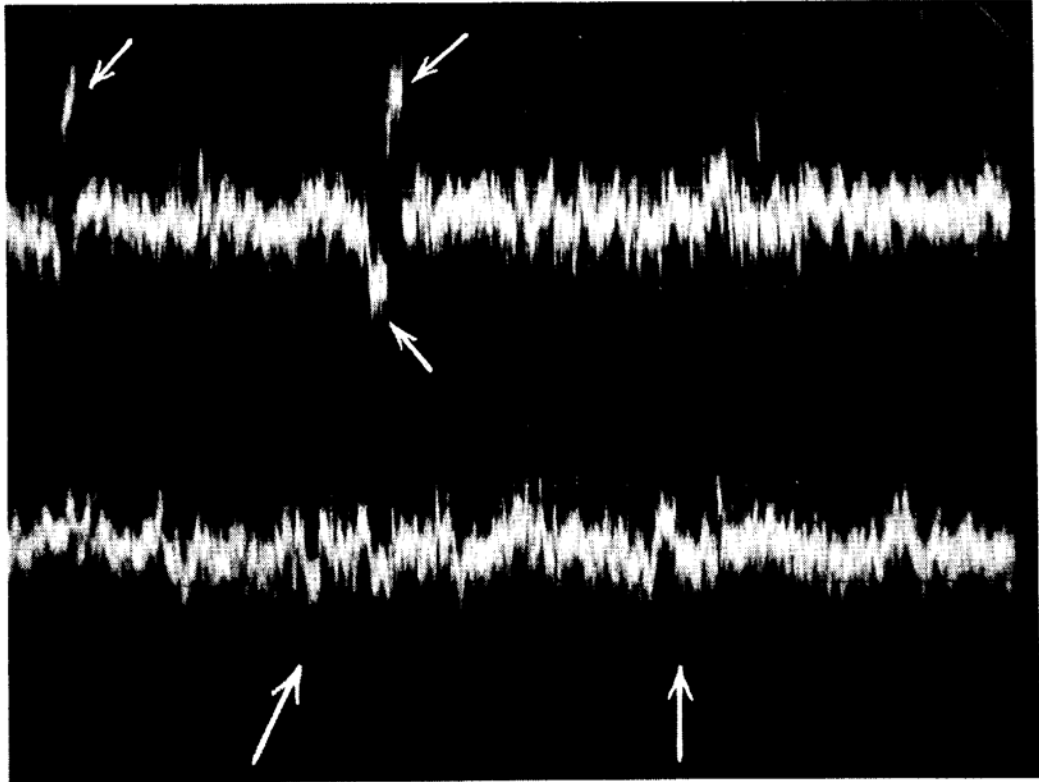
**Figure 5-9**  $E_n$  and  $I_n$  performance of a 2N4250 transistor.

At low frequencies increases the noise (with reduction in frequency) as  $1/\sqrt{f}$  while for high frequencies it is proportional to  $f$ . In the middle area the noise is flat and the curves have their minimum as discussed earlier.

# Popcorn noise

Observed in:

tunnel diodes, diode transitions, film resistors, transistors and integrated circuits.



**Figure 5-11** “Popcorn noise” is shown in the oscilloscope traces. The top trace is considered to represent a moderate level of this noise. The bottom trace is a low level. Some devices exhibit popcorn noise with five times the amplitude shown in the top trace. Horizontal sensitivity is 2 ms/cm.

The spectral density of the effect of this noise is

$$1/f^\alpha$$

where  $\alpha$  is between 1 and 2



In a normal pn-junction is the pulses maximum a few dozen micro ampere, and with a length of few micro seconds.

Suppose the popcorn current noise generator can be expressed as follows:

$$I_{bb}^2 = \frac{K'}{f^2}$$

where  $K'$  is a dimension constant with Ampere as designation.

A more accurate expression is:

$$I_{bb}^2 = \frac{KI_B}{1 + \pi^2 f^2 / 4a^2}$$

where  $K$  is a constant with Ampere per Hertz as designation and the constant  $a$  represents the number of bursts per. seconds.

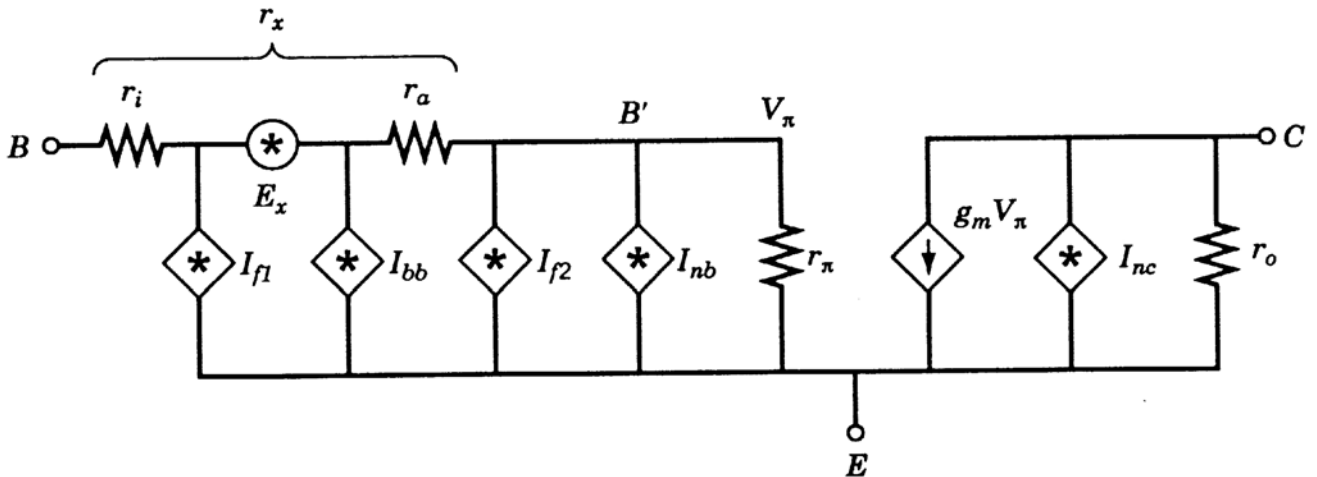
## Decomposition of $r_x$

The base resistance  $r_x$  can be divided into two parts:

i) from the contact (metal) to the nearest base-emitter junction and ii) the effective resistance for the distribution of base current along the base-emitter junction. The first is named as  $r_i$  while the other is named  $r_a$ .

1/f noise that is related with the crystal surface shall only be related to  $r_i$  while 1/f noise that is related to the active base region shall be related to the entire  $r_x$ .

Popcorn noise is related only to  $r_i$ .



**Figure 5-10** Expanded hybrid- $\pi$  bipolar model with excess noise mechanisms.

A new noise model, where  $r_x$  is split up is shown above. Here we have two  $1/f$ -noise sources  $I_{f1}$  and  $I_{f2}$  and a popcorn noise source  $I_{bb}$ .

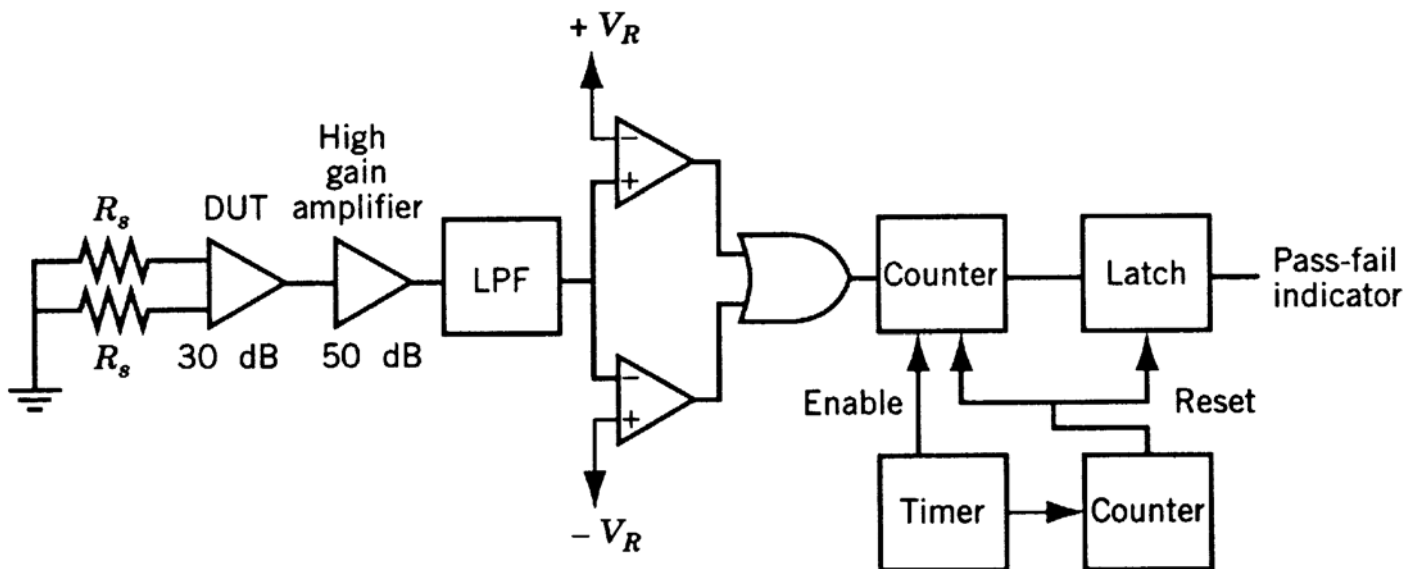
Shot:	$I_{nb}^2 = 2qI_B$
Shot:	$I_{nc}^2 = 2qI_C$
Thermal:	$E_x^2 = 4kTr_x$
Burst:	$I_{bb}^2 = \frac{KI_B}{1 + \pi^2 f^2 / 4a^2}$
$1/f$ :	$I_{f1}^2 = \frac{K_1 I_B^{\gamma_1}}{f}$
$1/f$ :	$I_{f2}^2 = \frac{K_2 I_B^{\gamma_2}}{f}$

(These apply to 1Hz bandwidth.)

## Measurement of popcorn noise.

Popcorn noise is primarily a problem for low frequencies in the audio area.

The form below shows a way to measure this noise. Threshold voltage  $V_R$  must be chosen so that the thermal noise does not trigger. This also means that the lowest values of popcorn-noise are not measured.



**Figure 5-12** Block diagram of system for measuring popcorn noise.

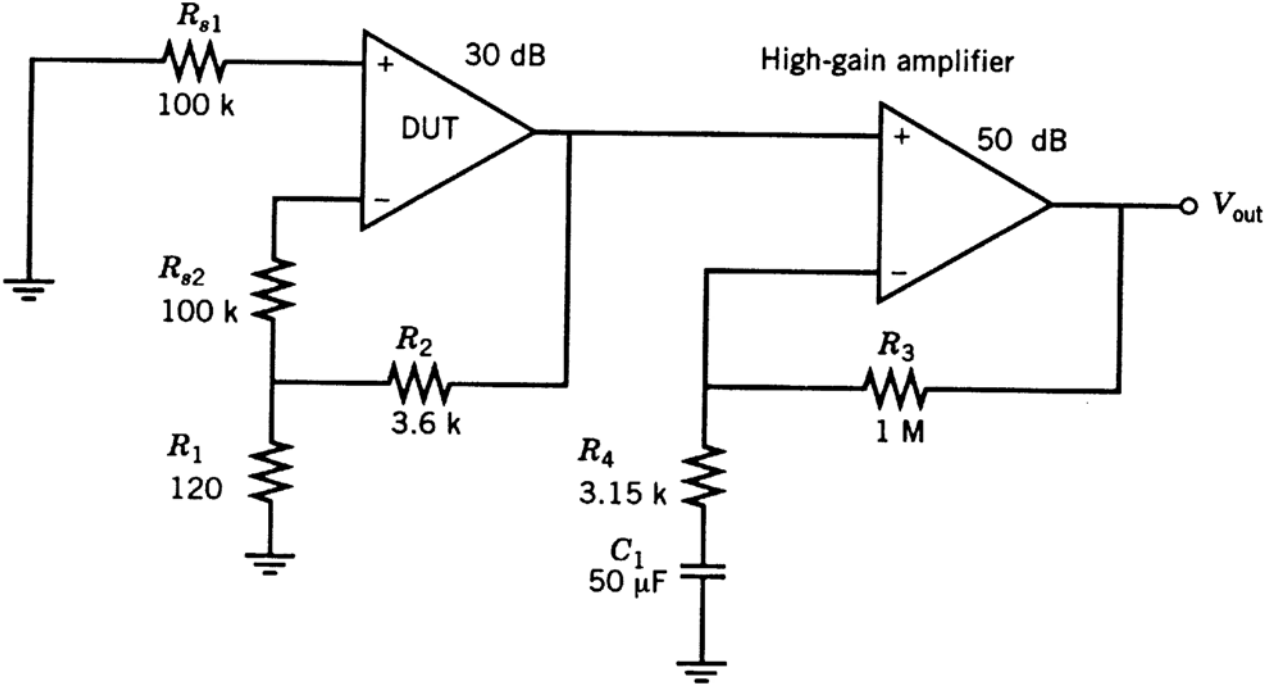


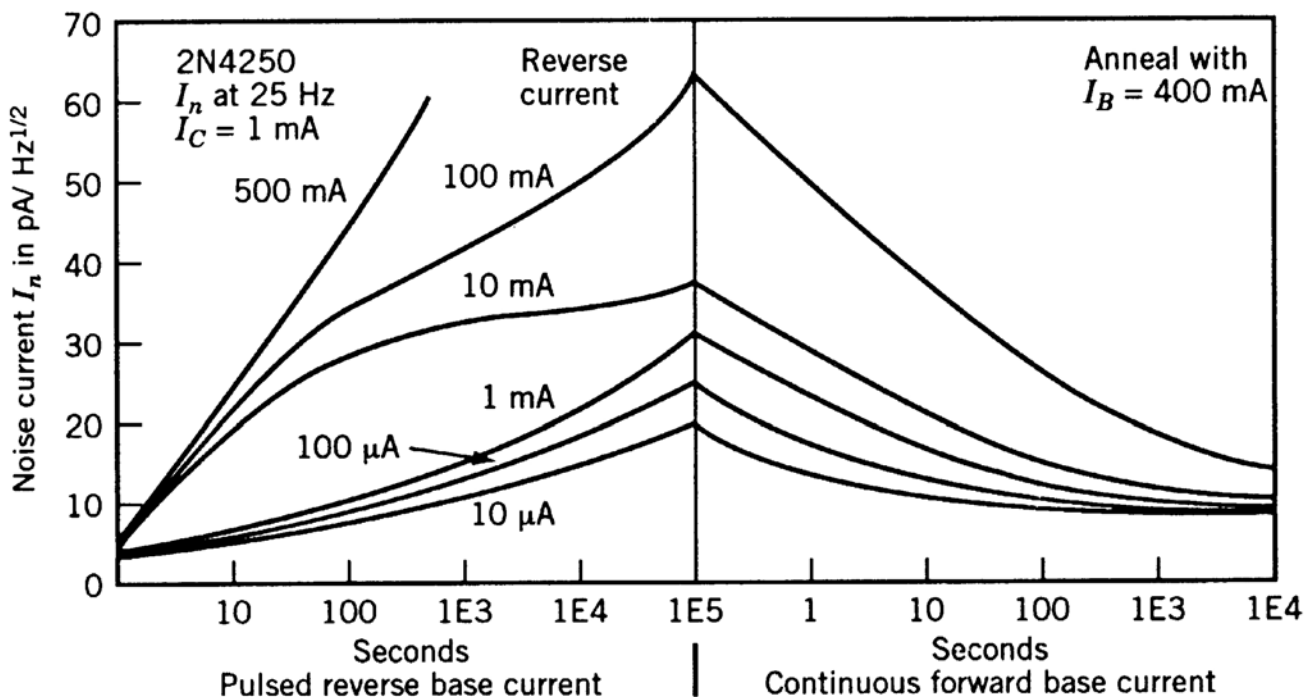
Figure 5-13 Amplifiers used in system for measuring popcorn noise.

## **Flicker noise and reliability**

It turns out that the size of the flicker noise in a component gives a good indication of the component's condition. Comparing two similar components, one could assume that the one with most flicker noise is the least reliable and have the shortest life. Measurement of flicker noise and measurement of the change in flicker noise will thus be able to say something about a system.

## Reverse voltage and noise.

If the reverse voltage over the base-emitter junction passes the breakdown voltage the transistor characteristic will change.  $\beta_0$  will decline somewhat while the  $1/f$ -noise will increase dramatically. The change will depend on the size of the reverse current and how long it is present.

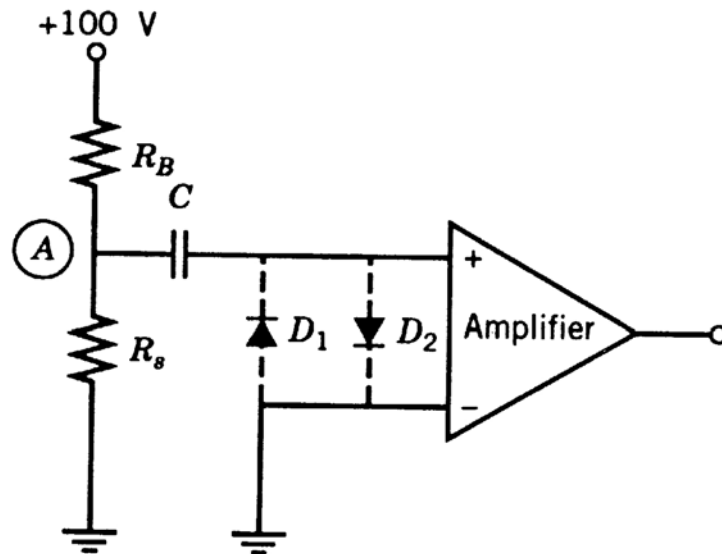


**Figure 5-14** Increase in noise current with avalanching and the decrease resulting from current annealing.

By adding a large forward current the damage may partially be corrected as shown above.

( $10^4 \text{ s} = 2 \text{ h } 46 \text{ m } 40 \text{ s}$ ,  $10^5 \text{ s} = 27 \text{ h } 46 \text{ m } 40 \text{ s}$ )

## Examples of accidental transgression of reverse voltage # 1



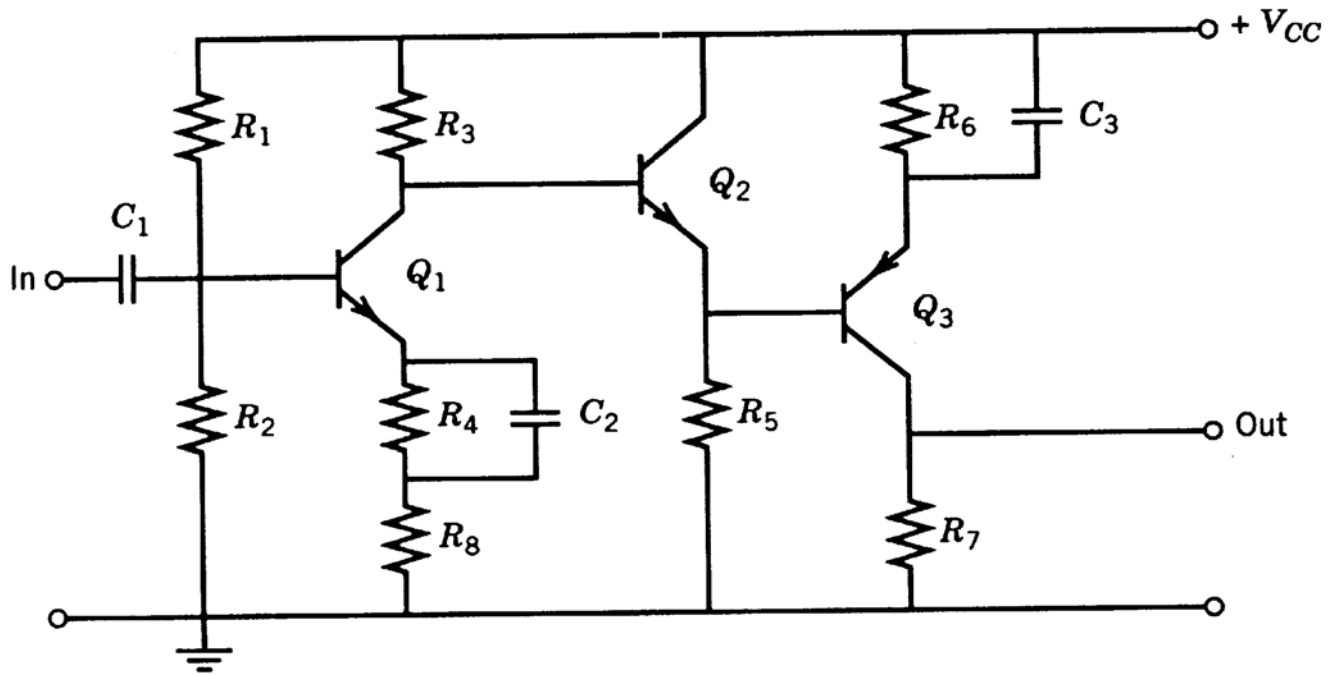
**Figure 5-15** Demonstration of avalanching problem.

If point A is short-circuited accidentally or intentionally against the voltage supply or the amplifier is turned off, the charge over C may lead to that the amplifier input transistors gets too large reverse voltage.

Connecting the diodes as shown will prevent that the voltage becomes too large and that the amplifier becomes damaged.



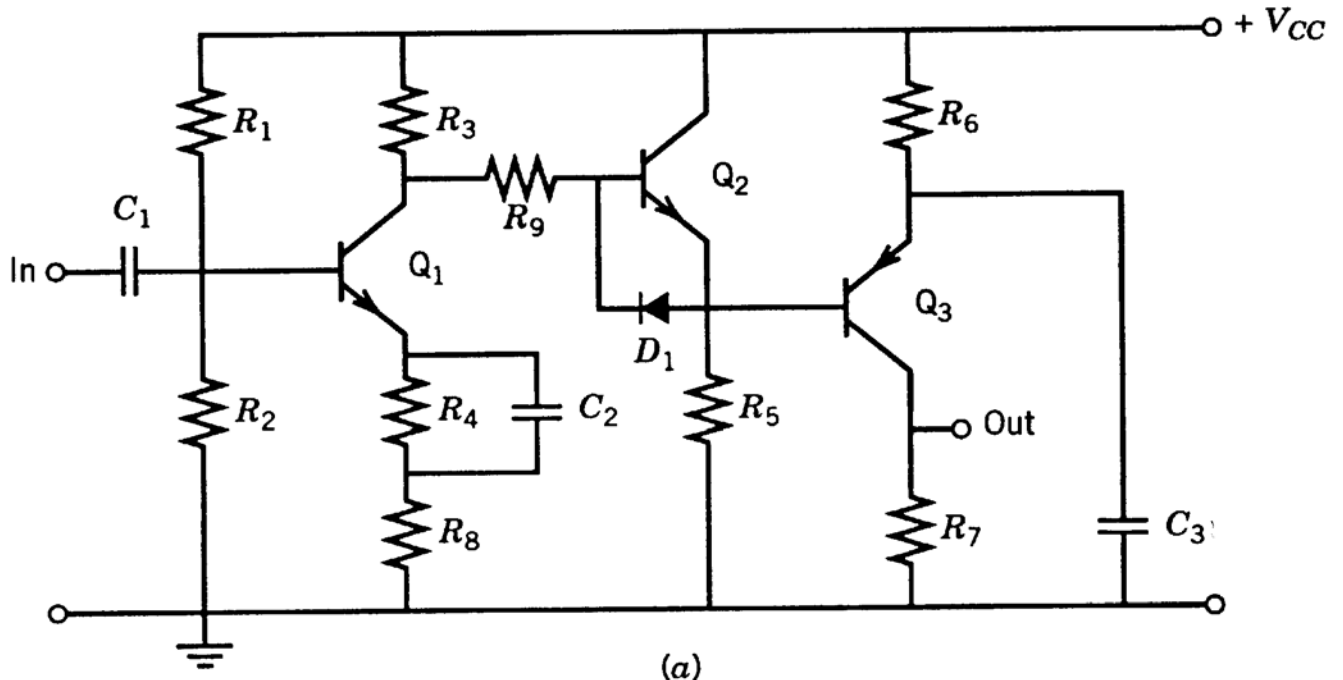
## Examples of accidental transgression of reverse voltage # 2



**Figure 5-16** Direct-coupled complementary amplifier with single supply.

Before  $V_{CC}$  is connected, the voltage across  $C_2$  and  $C_3$  will be zero volts. This will also be the case immediately after  $V_{CC}$  is connected to the power supply. We can then consider  $R_4$  and  $R_6$  to be short circuited.  $Q_1$  and  $Q_3$  will carry large current and  $Q_2$ 's base will have a low voltage while the  $Q_2$ ' emitter will have high voltage. Hence  $Q_2$  may have a reverse voltage larger than the breakdown voltage and will have changed the properties in a negative direction.

It may be counteracted in several ways. Common to all approaches is that they protect  $Q_2$  at power up and then provide normal function.



- $C_3$  is connected to the ground instead of against  $V_{CC}$ . At power up the current will now flow through  $R_6$  and not through  $Q_3$ .  $Q_3$  will not draw as large current and the  $Q_2$ ' emitter will remain low.
- The diode  $D_1$  will ensure that the reverse voltage over the  $Q_2$ 's base-emitter is not too large. At normal operation the diode will have virtually no effect.

If  $Q_2$  can be a *pnp* instead of an *npn* we can have this solution and  $Q_2$  will not be able to be reverse voltage biased.

