4.1.16

$$x_{n+2} - 3x_{n+1} - 4x_n = 0$$

$$r^2 - 3r - 4 = 0$$

$$r = \frac{3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2} = \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2} = \begin{cases} 4 \\ -1 \end{cases}$$

$$x_n = C4^n + D(-1)^n$$

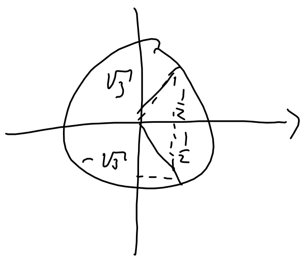
4.1.5b

$$x_{n+2} - x_{n+1} + x_n = 0 \qquad x_0 = 2 \qquad x_1 = 1$$

$$r^2 - r + 1 = 0$$

$$r = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}\, i}{2}$$

$$\rho = \left| \frac{1+\sqrt{3}\, i}{2} \right| = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1+3}{4} = 1$$



$$\frac{1}{2} = \rho \cos\theta \implies \theta = \frac{\pi}{3}$$

$$r = e^{\frac{\pi}{3} i}$$

$$\bar{r} = e^{-\frac{\pi}{3} i}$$

$$x_n = C r^n + D \bar{r}^n$$

$$x_n = C \rho^n \cos(n\theta) + D \rho^n \sin(n\theta)$$

$$x_n = C (1)^n \cos\left(\frac{\pi n}{3}\right) + D (1)^n \sin\left(n \frac{\pi}{3}\right)$$

$$= C \cos\left(\frac{\pi n}{3}\right) + D \sin\left(\frac{\pi n}{3}\right)$$

$$x_0 = 2 = C\cos(0) + D\sin(0) = C \implies C = 2$$

$$x_1 = 1 = 2\cos\left(\frac{\pi}{3}\right) + D \sin\left(\frac{\pi}{3}\right)$$

$$1 = 2 \cdot \frac{1}{2} + D \frac{\sqrt{3}}{2}$$

$$1 = 1 + D \frac{\sqrt{3}}{2}$$

$$0 = \frac{\sqrt{3}}{2} D \implies D = 0$$

Løsning

$$x_n = 2 \cos\left(\frac{\pi}{3} n\right)$$

4.1.15 Bieformering

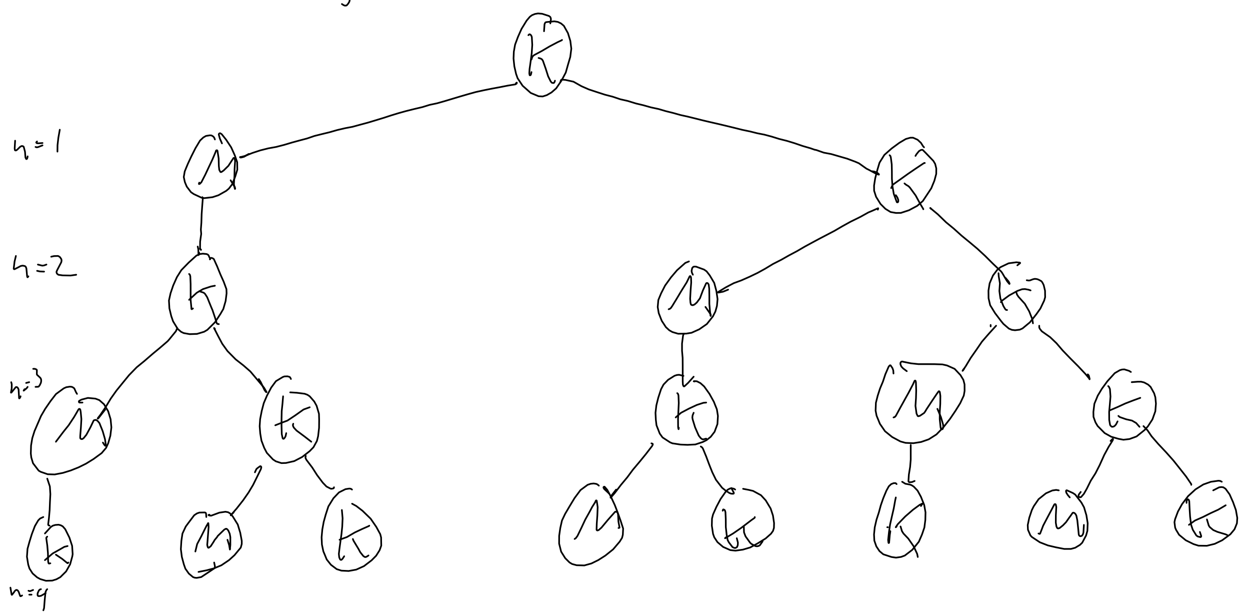
Ubefruktet egg ⇒ Hann (M)
Befruktet egg ⇒ Hunn (K)

x_n ~ Antall forgjengere n generasjoner tilbake

Forklar hvorfor

$$x_n = x_{n-1} + x_{n-2}$$

Ser først på eks



Hver bie i generasjon n-1 har nøyaktig en bie som mor i generasjon n. Antall hunnbier i generasjon n er derfor x_{n-1}. Videre er antall hannbier etter n generasjoner lik antall hunnbier i generasjon n-1.
I generasjon n-1 er det x_{n-2} hunnbier. Altså er antall bier i generasjon n lik

$$x_n = x_{n-1} + x_{n-2}$$

Hvor mange foreldre har en hunnbie n generasjoner tilbake?

$$x_n - x_{n-1} - x_{n-2} = 0$$

$$r^2 - r - 1 = 0$$

$$r = \frac{1 \pm \sqrt{1-4\cdot(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$x_n = C\left(\frac{1+\sqrt{5}}{2}\right)^n + D\left(\frac{1-\sqrt{5}}{2}\right)^n$$

For en hunnbie får vi initialbetingelsene
$x_1 = 2 \qquad x_2 = 3$

$$x_n = C\left(\frac{1+\sqrt{5}}{2}\right)^n + D\left(\frac{1-\sqrt{5}}{2}\right)^n \qquad x_1 = 2 \quad x_2 = 3$$

$$x_1 = 2 = C\left(\frac{1+\sqrt{5}}{2}\right) + D\left(\frac{1-\sqrt{5}}{2}\right)$$

$$x_2 = 3 = C\left(\frac{1+\sqrt{5}}{2}\right)^2 + D\left(\frac{1-\sqrt{5}}{2}\right)^2$$

Mellomregning

$$(1\pm\sqrt{5})^2 = 1 \pm 2\sqrt{5} + 5 = 2(3\pm\sqrt{5}) \quad (*)$$

$$\frac{(1\pm\sqrt{5})^2}{4} = \frac{\cancel{2}(3\pm\sqrt{5})}{\cancel{4}} = \frac{3\pm\sqrt{5}}{2}$$

Det gir

$$C(1+\sqrt{5}) + D(1-\sqrt{5}) = 4 \qquad \mathrm{I}$$

$$C(3+\sqrt{5}) + D(3-\sqrt{5}) = 6 \qquad \mathrm{II}$$

$$\mathrm{II} - \mathrm{I} \Rightarrow 2C + 2D = 2$$

$$D = 1 - C$$

Setter inn i I

$$\sqrt{5}(2C-1) = 3$$

$$2c - 1 = \frac{3}{\sqrt{5}}$$

$$2c = \frac{3}{\sqrt{5}} + 1 \Rightarrow C = \frac{3}{2\sqrt{5}} + \frac{1}{2} = \frac{3\sqrt{5}}{2\sqrt{5}^2} + \frac{1}{2} = \frac{3\sqrt{5}}{10} + \frac{1}{2}$$

$$\Rightarrow C = \frac{3}{10}\sqrt{5} + \frac{1}{2} = \frac{3\sqrt{5} + \sqrt{5}^2}{10} = \frac{\sqrt{5}}{10}(3+\sqrt{5})$$

Bruker (*)

$$= \frac{\sqrt{5}}{10}\cdot\frac{1}{2}(1+\sqrt{5})^2 = \frac{\sqrt{5}}{5}\left(\frac{1+\sqrt{5}}{2}\right)^2$$

Regner ut D på tilsvarende måte

$$D = -\frac{\sqrt{5}}{5}\left(\frac{1-\sqrt{5}}{2}\right)^2$$

$$x_n = \frac{\sqrt{5}}{5}\left(\frac{1+\sqrt{5}}{2}\right)^2\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{\sqrt{5}}{5}\left(\frac{1-\sqrt{5}}{2}\right)^2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$= \frac{\sqrt{5}}{5}\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \frac{\sqrt{5}}{5}\left(\frac{1-\sqrt{5}}{2}\right)^{n+2}$$