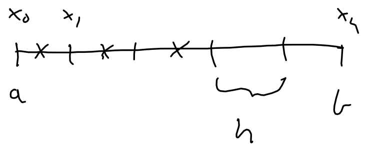
Numerisk integrasjon

Midtpunktsmetoden



$(x_i)_{i=0}^{n}$ uniform partisjon av $[a,b]$

$$\int_a^b f(x)\,dx \approx I_{mid}(h) = h \sum_{i=1}^{n} f(x_{i-1/2})$$

$$x_{i-1/2} = \frac{x_i + x_{i-1}}{2} = a + \left(i - \frac{1}{2}\right) h$$

Lokal feil:

$$\frac{M}{24} h^3$$

Global feil:

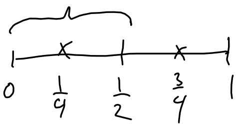
$$\frac{M}{24} (b-a) h^2$$

$$M = \max_{x \in [a,b]} |f''(x)|$$

<u>12.2.2</u> Bruk midtpunktsmetoden til å regne ut

$$\int_0^1 x^2\,dx$$ med 2 delintervaller

$h = \frac{1}{2}$



$f(x) = x^2$

$$h \cdot \sum_{i=1}^{2} f(x_{i-1/2}) = \frac{1}{2}\left(\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2\right) = \frac{1}{2}\left(\frac{1}{16} + \frac{9}{16}\right) = \underline{\underline{\frac{5}{16}}}$$

12.2.1

a) Usant $\qquad h\sum_{i=1}^{n} f(x_{i-1/2})$

vil for det meste gi addisjon av nesten like tall dersom vi velger h liten nok

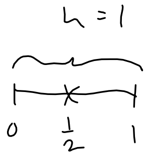
b) Sant $\qquad f(x) = cx + d$

$$\int_a^b f(x)\,dx = \frac{1}{2}cx^2 + dx\Big|_a^b = \frac{1}{2}c(b^2 - a^2) + d(b-a)$$

$$I_{mid}(b-a) = (b-a)\,f\left(\frac{a+b}{2}\right) = \frac{1}{2}c(b^2-a^2) + d(b-a)$$

c) Usant

$$\int_0^1 x^2\,dx = \frac{1}{3}x\Big|_0^1 = \frac{1}{3}$$

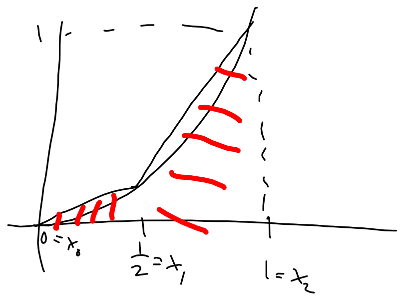


$$h\cdot f\left(\frac{1}{2}\right) = 1\cdot\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

d) Sant

12.3.2 Bruk trapesmetoden til å regne ut

$$\int_0^1 x^2\,dx \quad \text{med 2 delintervaller}$$



$$h \sum_{i=1}^{2} \frac{f(x_{i-1}) + f(x_i)}{2}$$

$$\frac{1}{2} \cdot \left(\frac{f(0) + f(\frac{1}{2})}{2} + \frac{f(\frac{1}{2}) + f(1)}{2} \right)$$

$$= \frac{1}{2} \left(\frac{0 + \frac{1}{4}}{2} + \frac{\frac{1}{4} + 1}{2} \right) = \frac{3}{8}$$

$$h \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} = h \frac{f(x_0) + f(x_n)}{2} + h \sum_{i=1}^{n-1} f(x_i)$$

13.2.3 Hvilke egenskaper kan skape problemer i følgende differensialligninger

a) $x' = \frac{t}{1-x}$ Problem $x = 1$

b) $x' = \frac{x}{1-t}$ Problem: $t = 1$

c) $x' = \ln x$ Problem $x \leq 0$

d) $x'x = 1$ Problem $x = 0$

$\Updownarrow$

$x' = \frac{1}{x}$

e) $x' = \arcsin(x)$

$\sin : \mathbb{R} \to [-1, 1]$

$\arcsin = \sin^{-1} : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

Problem $x \notin [-1, 1]$

f) $x' = \sqrt{1 - x^2}$ Problem $|x| > 1$

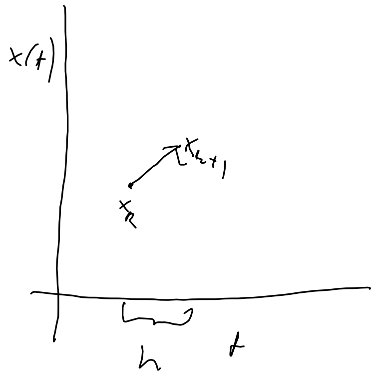
$x' = f(x, t)$

13.3.3 Eulers metode 3 steg og $h = 0,1$ $x' = f(x,t)$

for $k = 0, 1, \dots, n-1$

$x_{k+1} = x_k + h \cdot f(x_k, t_k)$

$t_{k+1} = t_k + h$



a) $x' = t + x$ $x(0) = 1$

$f(x,t) = t + x$

$x_0 = 1$ $t_0 = 0$ $h = 0,1$

$x_1 = x_0 + h \cdot f(x_0, t_0) = 1 + 0,1 \cdot f(1, 0) = 1 + 0,1 \cdot (1+0) = 1,1$ $t_1 = 0 + 0,1 = 0,1$

$x_2 = x_1 + h f(x_1, t_1) = 1,1 + 0,1 \cdot f(1,1, 0,1) = 1,1 + 0,1(1,1 + 0,1) = 1,22$ $t_2 = 0,1 + 0,1 = 0,2$

$x_3 = x_2 + h f(x_2, t_2) = 1,22 + 0,1 f(1,22, 0,2) = 1,22 + 0,1(1,22 + 0,2) = 1,362$ $t_3 = 0,2 + 0,1 = 0,3$

13.3.4 Skriv et program som bruker Eulers metode på

$x'(t) = f(x,t) \qquad x(a) = x_0$

$t \in [a,b]$ med n tidssteg.

La $x(0) = 1$ $\quad [a,b] = [0,1]$ og $x' = x$

$f(x,t) = x$

Eksakt løsning er: $x(t) = e^t$