9.1.5

a) Hvorfor er tilnærmingen $\sin x \approx x$ rimelig?

Taylor polynomet av grad 2 om $a=0$ til $\sin x$

er $T_2 \sin x = x$.

Derfor vil $x \approx \sin x$ gi liten feil for ~~stor~~ x nærme $a=0$.

b) Feilen er begrenset av

c er i intervallet mellom 0 og x

$$\left|R_2 \sin x\right| = \left|\frac{-\cos(c)}{3!}x^3\right| \le \frac{x^3}{6}$$

For $x \in [0, 0{,}1]$ har vi da at

$$\left|R_2 \sin x\right| \le \frac{(0{,}1)^3}{6} = \frac{1}{6000}.$$

9.2.3

x	0	1	3	4
$f(x)$	1	0	2	1

a) Skriv det interpolerende polynomet på formen

$$P_3(x) = c_0(x-1)(x-3)(x-4) + c_1 x(x-3)(x-4) + c_2 x(x-1)(x-4) + c_3 x(x-1)(x-3)$$

$$f(0) = P_3(0) = 1 = c_0(-1)(-3)(-4) = -12c_0 \qquad \Rightarrow c_0 = -\frac{1}{12}$$

$$f(1) = P_3(1) = 0 = c_1(-2)(-3) = 6c_1 \qquad c_1 = 0$$

$$f(3) = P_3(3) = 2 = c_2(3)(2)(-1) = -6c_2 \qquad c_2 = -\frac{1}{3}$$

$$f(4) = P_3(4) = 1 = c_3\, 4\cdot 3\cdot 1 = 12c_3 \qquad c_3 = \frac{1}{12}$$

Det gir

$$\cancel{P_3(x) = (x-1)(x-3)(x-4) + 0 + 2x(x-1)(x-4) + x(x-1)(x-3)}$$
$$\cancel{= (x-1)(x^2-7x+12) + 2x(x^2-5x+4) + x(x^2-4x+3)}$$
$$\cancel{= x^3-7x^2+12x-x^2+7x-12+2x^3-10x^2+8x+x^3-4x^2+3x}$$
$$\cancel{= 4x^3-22x^2+30x-12}$$

$$P_3(x) = -\frac{1}{12}(x-1)(x-3)(x-4) + 0 + -\frac{1}{3}x(x-1)(x-4) + \frac{1}{12}x(x-1)(x-3)$$

$$= -\frac{1}{12}(x-1)(x^2-7x+12) - \frac{1}{3}x(x^2-5x+4) + \frac{1}{12}x(x^2-4x+3)$$

$$= -\frac{1}{12}(x^3-7x^2+12x-x^2+7x-12) - \frac{1}{3}(x^3-5x^2+4x) + \frac{1}{12}(x^3-4x^2+3x)$$

$$= \frac{(\cancel{-x^3}+7x^2-12x+x^2-7x+12) - 4x^3+20x^2-16x+\cancel{x^3}-4x^2+3x}{12}$$

$$= \frac{-4x^3+24x^2-32x+12}{12} = -\frac{1}{3}x^3+2x^2-\frac{8}{3}x+1$$

9.2.3 (c) Newton formen

x	0	1	3	4
$f(x)$	1	0	2	1

$$q_3(x) = c_0 + c_1(x-0) + c_2(x-0)(x-1) + c_3(x-0)(x-1)(x-3)$$

$f(0) = q_3(0) = 1 = c_0 \quad \Rightarrow c_0 = 1$

$f(1) = q_3(1) = 0 = c_0 + c_1 \quad \Rightarrow c_1 = -1$

$f(3) = q_3(3) = 2 = \overset{1}{c_0} + 3\overset{(-1)}{c_1} + 6c_2 \quad \Rightarrow 2 = 1 - 3 + 6c_2$

$c_2 = \frac{2}{3}$

$f(4) = q_3(4) = 1 = c_0 + 4c_1 + 12c_2 + 12c_3$

$$1 = \underbrace{1 - 4}_{= -3} + \underbrace{\frac{12 \cdot 2}{3}}_{= 8} + 12c_3$$

$$-4 = 12c_3$$

$$-\frac{1}{3} = c_3$$

Det gir

$$q_3(x) = 1 - x + \cancel{\frac{2}{3}x^2 - \frac{2}{3}x}\ \frac{2}{3}(x)(x-1) - \frac{1}{3}x(x-1)(x-3)$$

$$= 1 - x + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{1}{3}x(x^2 - 4x + 3)$$

$$= \cancel{1}\ x + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{1}{3}x^3 + \frac{4}{3}x^2 \cancel{} - x$$

$$= \cancel{-\frac{1}{3}x^3 + 2x^2 - \frac{5}{3}x}$$

$$= -\frac{1}{3}x^3 + 2x^2 - \frac{8}{3}x + 1$$