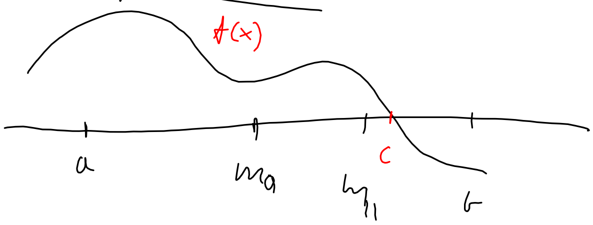
10.2: 1, 2a,b, 3, 5
10.3: 1, 2, 3, 4
10.4: 1, 2a,b, 4, 7

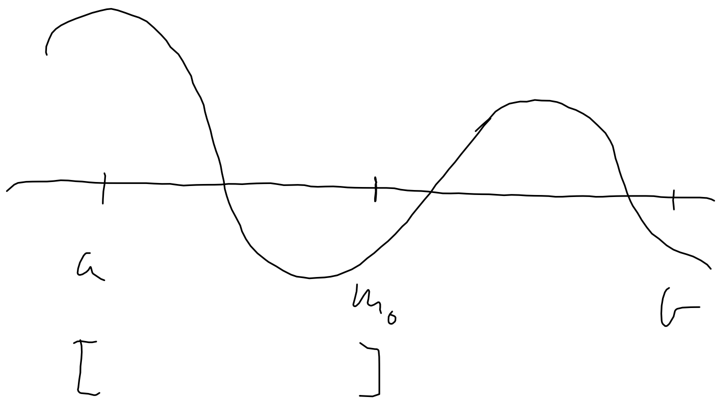
Halveringsmetoden



10.2.1

a) Error: $|c - m_N| \le \frac{b-a}{2^{N+1}}$ Sann

c)



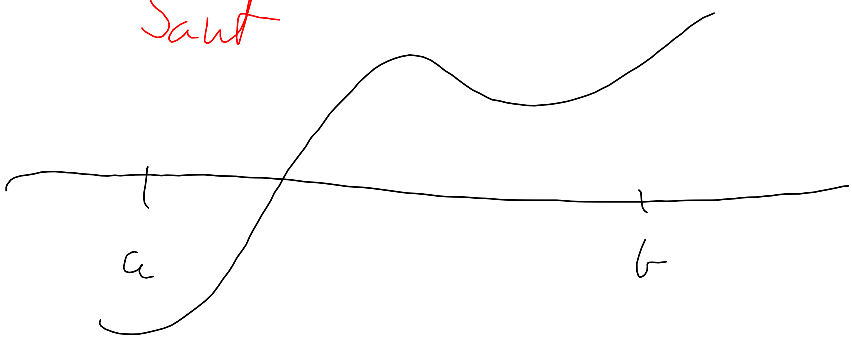
Sann

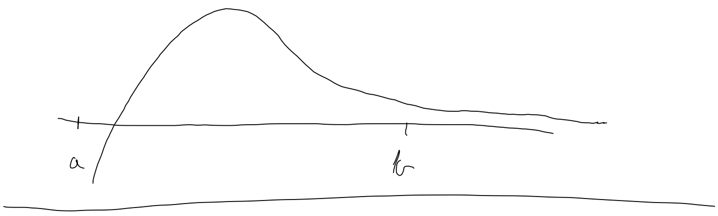
b)

Usant

d) Usant, se oppg. c)

e) Sant





10.2.2

$$f(x) = (x-3)(x^2-3x+2)$$

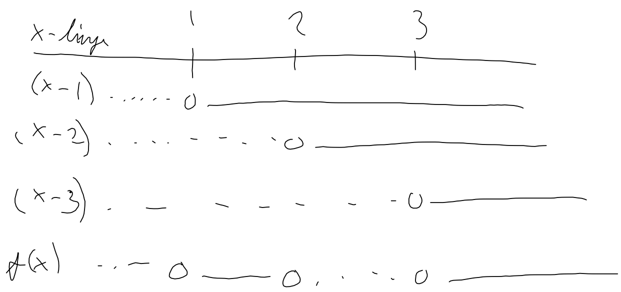
$[a, b] = [0,\ 3{,}5]$

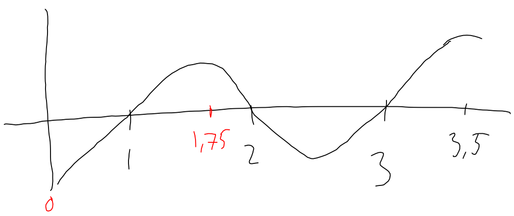
Kjører 1000 iterasjoner med Halveringsmetoden
Hvilket nullpunkt går den mot?

$$x^2 - 3x + 2$$

$$x = \frac{3 \pm \sqrt{9-4\cdot 2}}{2} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3\pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

$$f(x) = (x-3)(x-2)(x-1)$$





$$f(0) = (-1)(-2)(-3) = -6$$

$$f(3{,}5) = (3{,}5-1)(3{,}5-2)(3{,}5-3) = 1{,}875$$

midtpunkt

$$m_0 = \frac{3{,}5-0}{2} = 1{,}75$$

f(x) har bare et nullpunkt mellom $[0,\ 1{,}75]$
Halveringsmetoden vil derfor konvergere mot $c = 1$

10.2.3 $f(x) = x - \cos x$ $\qquad f(c) = 0$ for $c \approx 0{,}739\ldots$

La $[a, b] = [0, 1]$

Hvor mange riktige siffer har vi etter 10 steg med halveringsmetoden

Absolutt feil

$$|c - m_n| \leq \frac{b-a}{2^{n+1}} = \frac{1-0}{2^{n+1}} = \frac{1}{2^{n+1}}$$

Relativ feil

$$\frac{|c - m_n|}{|c|} \leq \frac{1}{|c|\,2^{n+1}} \leq \frac{1}{\frac{1}{2}\,2^{n+1}}$$

$$\frac{1}{c} \approx \frac{1}{0{,}74} = \frac{1}{\frac{74}{100}} = \frac{100}{74} \leq \frac{100}{50} = 2 = \frac{1}{\frac{1}{2}}$$

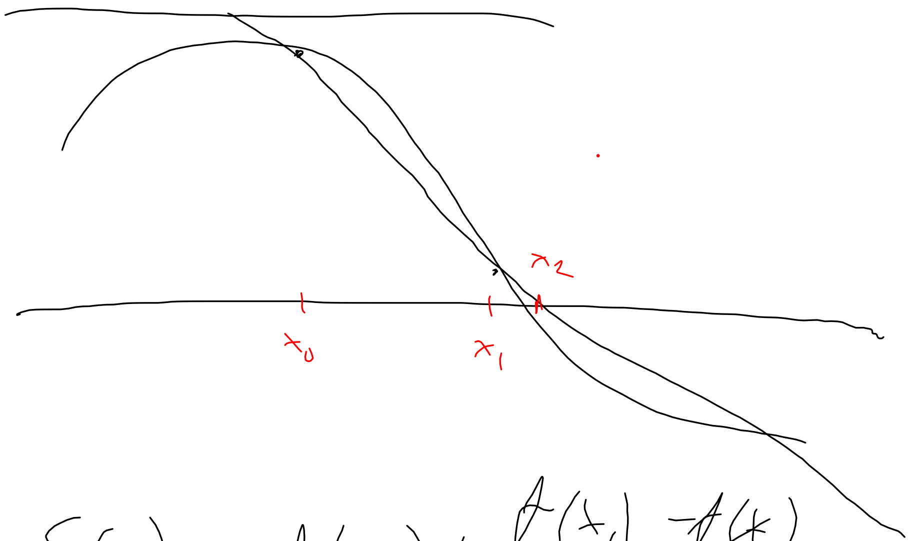
$$= \frac{1}{2^n}$$

$n = 10$

$$\text{Relativ feil} \leq \frac{1}{2^{10}} = \frac{1}{1024} \approx 10^{-3}$$

gir 3-siffers nøyaktighet.

Sekantmetoden



$$S(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$P(x) = ax + b$$

$$S(x) = c_0 + c_1(x - x_0)$$

Finn $S(x) = 0$

$$f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) = 0$$

$$x = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

10.3.2 $\quad f(x) = x^3 - 2 \quad x_0 = -2 \quad x_1 = 2$

Ta et steg med sekantmetoden

$$x_{n+1} = x_{n-1} - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_{n-1})$$

$f(2) = 6$

$n=1$ $\qquad f(-2) = -10$

$$= -2 - \frac{2-(-2)}{f(2) - f(-2)} f(-2)$$

$$= -2 + \frac{4}{6-(-10)} (+10)$$

$$= -2 + \frac{4}{16} 10$$

$$= -2 + \frac{1}{4} \cdot 10 = \frac{-8}{4} + \frac{10}{4} = \frac{2}{4} = \frac{1}{2}$$

$x_2 = \frac{1}{2} \qquad x_1 = 2$

$n=3$

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_1)$$

Relativ feil

$$\frac{|c - m_n|}{|c|} \leq \varepsilon$$

$$|c - m_n| \leq \varepsilon |c|$$

Alternativ form for kode

$$\frac{|c - m|}{m} \leq \varepsilon$$

Halveringsmetode

$$|c - m_n| \leq \frac{|b - a|}{2^{n+1}}$$

i kode

$$\frac{|b - a|}{2^{n+1}} \leq \varepsilon \cdot |m|$$