Neste uke: Tirsdag: Harald Schjelderups hus, Aud 4

Onsdag: Vel ikke

L'Hôpitals regel

Eksempel forrige gang: $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$ "1^∞"

En faktor går mot 0, den andre mot ∞.

Generelt triks: $\lim_{x\to a} f(x)^{g(x)} = \lim_{x\to a}\left(e^{\ln f(x)}\right)^{g(x)} = \lim_{x\to a} e^{g(x)\ln f(x)}$

Eksempel: $\lim_{x\to 0}|x|^{\sin x} = \lim_{x\to 0}\left(e^{\ln|x|}\right)^{\sin x} = \lim_{x\to 0} e^{\underbrace{\sin x}_{\downarrow\, 0}\underbrace{\ln|x|}_{\downarrow\, -\infty}}$ V

Mellomregningen: $\lim_{x\to 0}\sin x\ln|x| = \lim_{x\to 0}\frac{\ln|x|}{\frac{1}{\sin x}}$ "$\frac{\infty}{\infty}$"

$$\overset{L'H}{=} \lim_{x\to 0}\frac{\frac{1}{x}\cdot x\sin^2 x}{-\frac{1}{\sin^2 x}\cdot\cos x\cdot x\sin^2 x} = \lim_{x\to 0}\frac{\sin^2 x}{-x\cos x}$$

"$\frac{0}{0}$"

$$\underbrace{\lim \frac{\sin^2 x}{-x}}\ \lim\frac{1}{\cos x}$$

$1 \leq$

$$= \lim_{x\to 0}\frac{2\overset{0}{\sin x}\ \overset{1}{\cos x}}{\underbrace{-\cos x}_{-1}+x\underbrace{\sin x}_{\downarrow\, 0}} = \frac{0}{-1} = \underline{\underline{0}}$$

Tilbake igjen til problemet:

$$\lim_{x\to 0}|x|^{\sin x} = \lim_{x\to 0} e^{\overset{\to 0}{\sin x\ln|x|}} = e^0 = \underline{\underline{1}}$$

Eksempel: $\lim_{x\to 0} \frac{\cos x\,(e^{-x}-1+x)}{x\sin x}$ "$\frac{0}{0}$"

$$= \underbrace{\lim_{x\to 0}\cos x}_{1}\;\lim_{x\to 0}\frac{e^{-x}-1+x}{x\sin x} = \lim_{x\to 0}\frac{e^{-x}-1+x}{x\sin x} \quad \text{"}\frac{0}{0}\text{"}$$

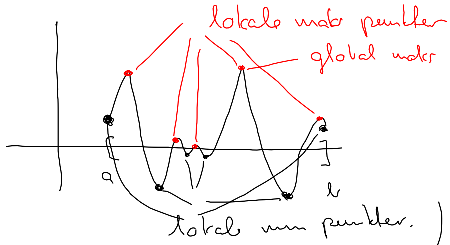
$$= \lim_{x\to 0}\frac{e^{-x}-1+x}{x^2}\cdot\frac{x}{\sin x} = \lim_{x\to 0}\frac{e^{-x}-1+x}{x^2}\cdot\underbrace{\lim_{x\to 0}\frac{x}{\sin x}}_{\to 1}$$

$$\overset{L'H}{=} \lim_{x\to 0}\frac{-e^{-x}+1}{2x}\cdot 1 \overset{L'H}{=} \lim_{x\to 0}\frac{e^{-x}}{2} = \frac{1}{2}$$

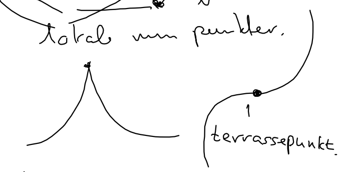
Kurvedrøfting

1. Finne lokale maks og min. punkter
2. Finne hvor funksjonen er voksende og hvor den er avtagende
3. Bestemme hvilken vei funksjonen krummer.

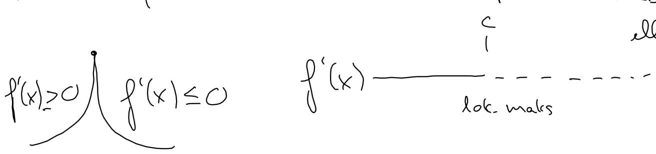
Lokale maks og min. punkter:

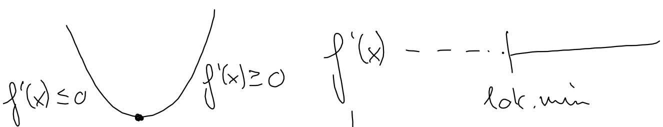


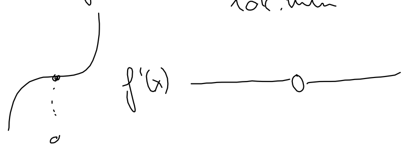
Hvor finner vi lokale maks og min punkter:
1. I punkter der f' er null
2. ——— " ——— ikke er definert.
3. Endepunkter til intervallet

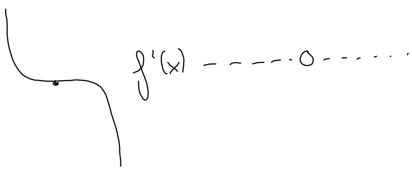


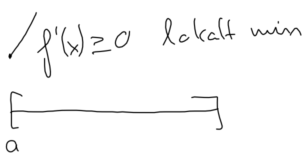
Hvordan sjekker man om et slikt punkt c er maks eller min eller noe annet?

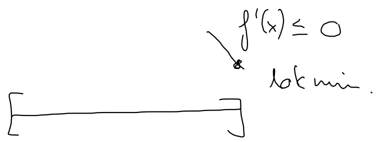




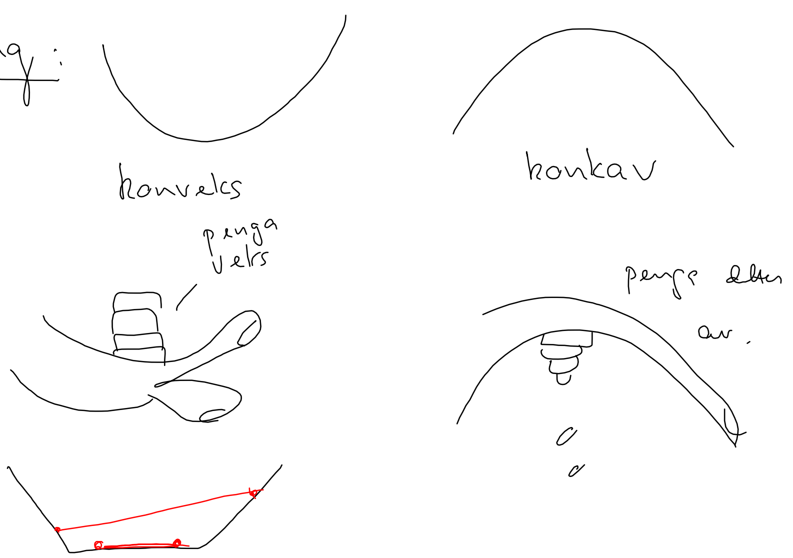




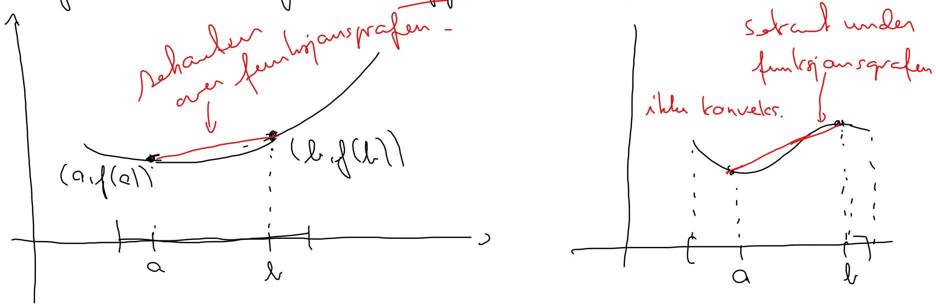




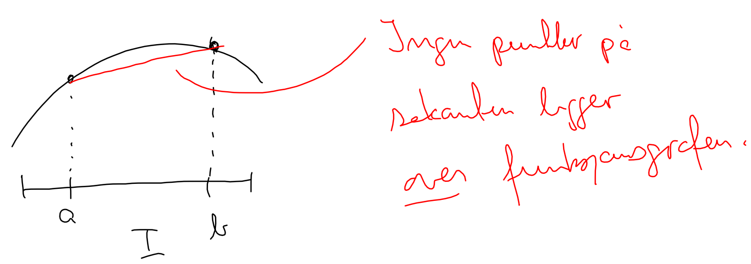
Krumning:



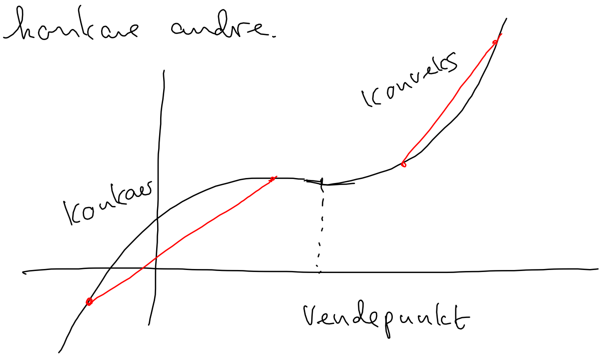
Definisjon: En funksjon f er <u>konveks</u> på intervallet I dersom hver gang vi plukker ut to punkter a og b i I, så vil <u>ingen</u> <u>punkter</u> på <u>linjestykket</u> fra $(a, f(a))$ til $(b, f(b))$ ligge <u>under</u> funksjonsgrafen.



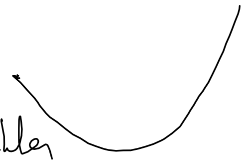
Definisjon: Blah-blah-blah ... Konkav.



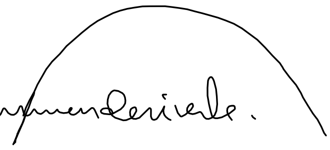
Funksjoner er typisk konvekse noen steder og konkave andre.



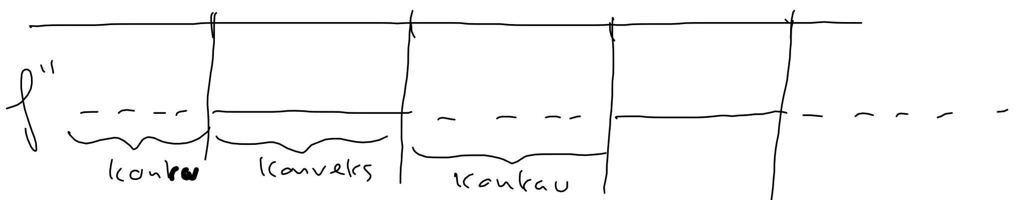
Setning: (i) Anta at f er en kontinuerlig funksjon slik at $f''(a) \geq 0$ for alle indre punkter i et intervall I. Da er f konveks i I.



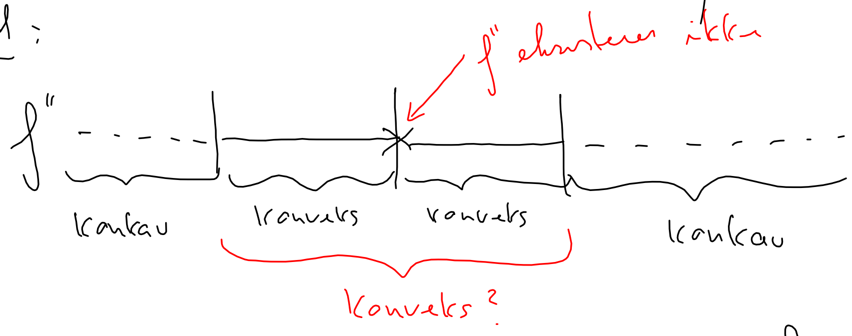
(ii) Dersom $f''(a) \leq 0$ for alle indre punkter i I, så f konkav i I

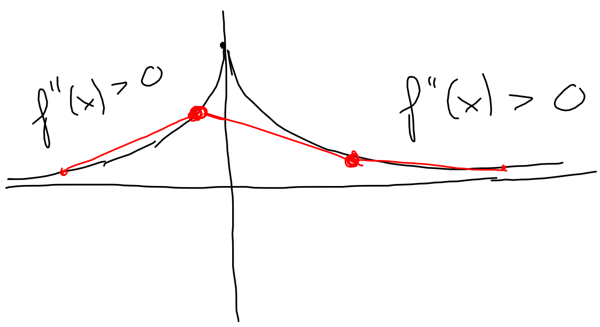


Praktisk metode: Lage fortegnsskjema for den annenderiverte.



Hva med:





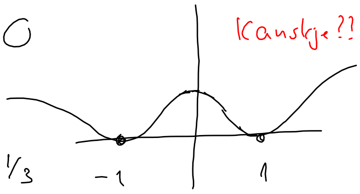
f er konveks på $[-\infty, 0]$ og på $[0, \infty)$, men ikke på hele $\mathbb{R}$

Eksempel: $f(x) = (x^2-1)^{2/3}$, drøft funksjonen

f er positiv unntatt i $x=1$ og $x=-1$ der f er lik 0.

$$f(x) = (x^2-1)^{2/3} = \left[(x^2-1)^{1/3}\right]^2 \ge 0$$

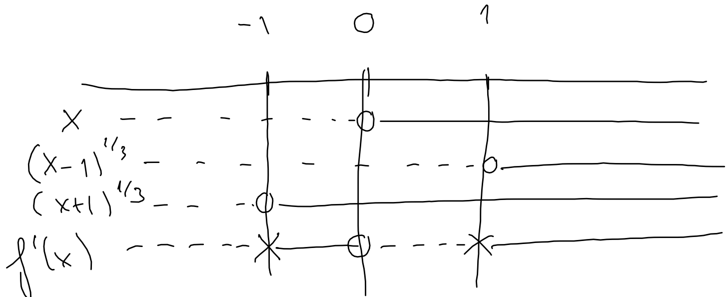
Kanskje?!



Deriverer:

$$f'(x) = \frac{2}{3}(x^2-1)^{2/3-1}\cdot 2x = \frac{4x}{3}(x^2-1)^{-1/3} = \frac{4x}{3(x^2-1)^{1/3}}$$

$(x-1)^{1/3}(x+1)^{1/3}$



Deriverer igjen: $f''(x) = \frac{4}{3}\left[\frac{1\cdot(x^2-1)^{1/3} - x\cdot\frac{1}{3}(x^2-1)^{1/3-1}\cdot 2x}{(x^2-1)^{2/3}}\right]$

$$= \frac{4}{3}\left[\frac{(x^2-1)^{1/3} - \frac{2}{3}x^2(x^2-1)^{-2/3}}{(x^2-1)^{2/3}}\right]\frac{(x^2-1)^{2/3}}{(x^2-1)^{2/3}}$$

ganger med

$$= \frac{4}{3}\left[\frac{(x^2-1) - \frac{2}{3}x^2}{(x^2-1)^{4/3}}\right] = \frac{4}{3}\,\frac{\frac{1}{3}x^2 - 1}{(x^2-1)^{4/3}}$$

$$= \frac{4}{9}\,\frac{x^2-3}{(x^2-1)^{4/3}} = \frac{4}{9}\,\frac{(x-\sqrt{3})(x+\sqrt{3})}{(x^2-1)^{4/3}}$$