

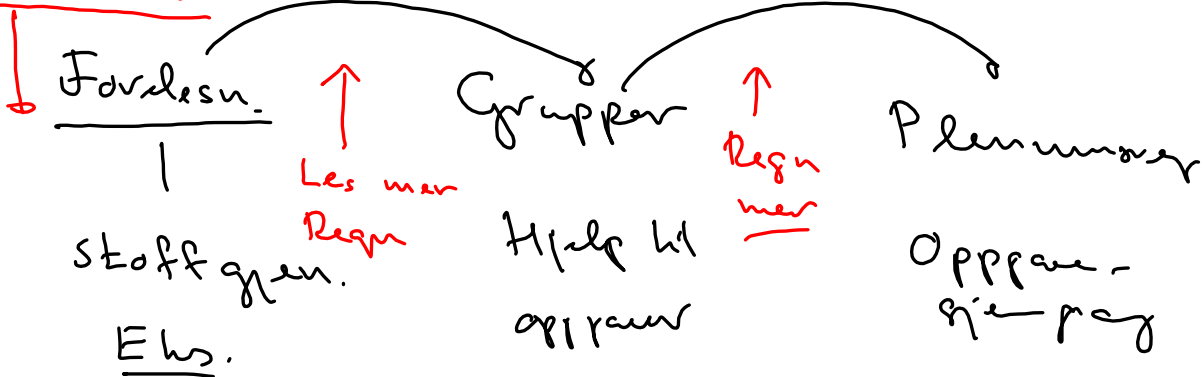
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Föreläsningar SL

Grupper  $\Leftarrow$  grupplös

Plenumsvepning SL

Forberedelser



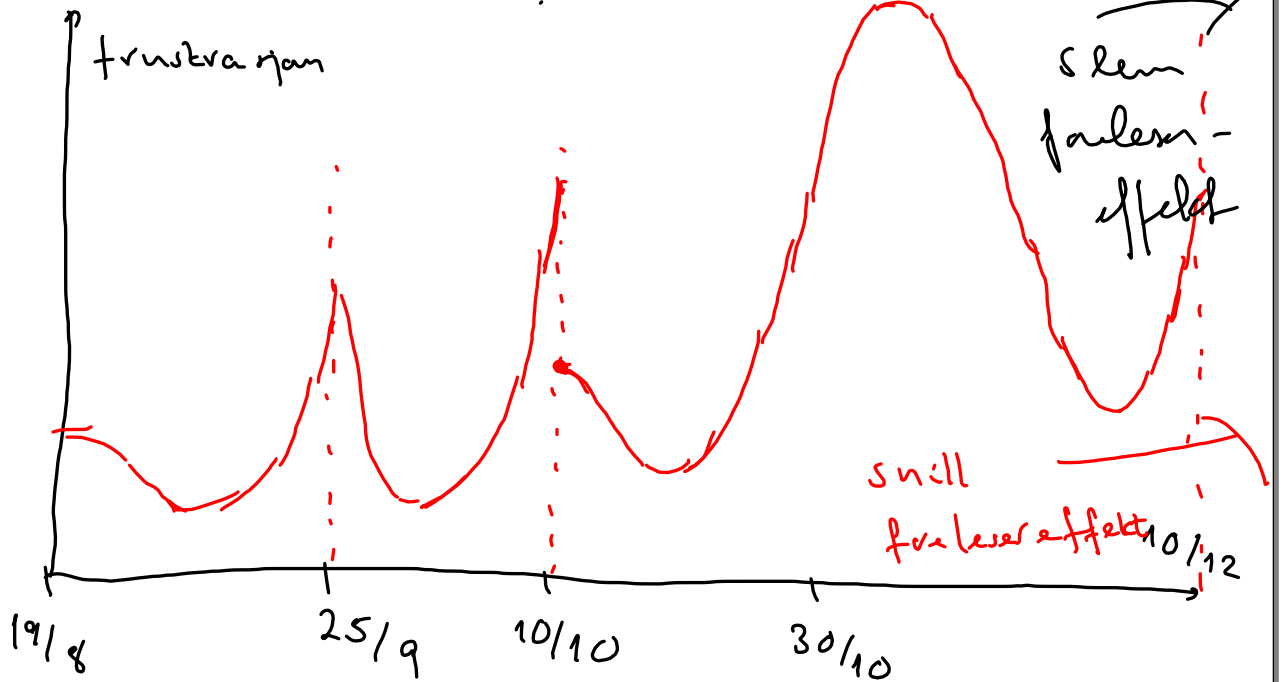
Viktig tidspunkt:

25/9 : Turløring Oblig 1

10/10 : Møte med eksamen 1/3

30/10 : Oblig 2

10/12 : Eksamen 2/3

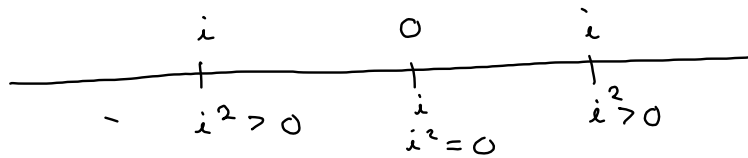


Gode (?) råd:

- Jobb jevent
- Prøv å forstå
- Snublegruppe  
grubleguppe

# Komplekse tall

Hva er  $\sqrt{-1}$ ? Hvis  $i = \sqrt{-1}$ , så  $i^2 = -1$ . Slike tall finnes ikke på tallinjen



Behov for å regne med  $\sqrt{-1}$   $b^2 < 4ac$

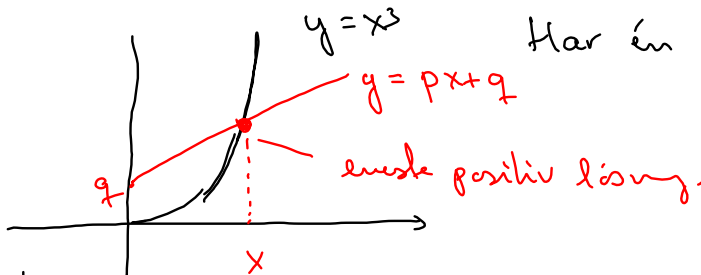
$$ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ingen løsninger

Tredjegradslikninger:

$$x^3 = px + q, \quad p, q > 0$$

Har én positiv rot



Løsning:

$$x = \sqrt[3]{\sqrt{-\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} + \frac{q}{2}} - \sqrt[3]{\sqrt{-\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} - \frac{q}{2}}$$

Eksempel:  $x^3 = 15x + 4$   $p = 15, q = 4$ .

$$x = \sqrt[3]{-11\sqrt{-1} + 2} - \sqrt[3]{-11\sqrt{-1} - 2} = 4$$

$x = 4$ : VS  $4^3 = 64$  — formell verifisering gir 4.  
 HS  $15 \cdot 4 + 4 = 64$

Reelt problem  
 $x^3 = 15x + 4$

imaginært

Reell løsning  
 $x = 4$

## Komplekse tall formelt

Anta at  $i = \sqrt{-1}$  finnes og studer  
konvensene:

$$z = a + i b \quad \text{komplekst tall} \quad a, b \in \mathbb{R}$$

$\uparrow$                        $\swarrow$   
 real delen            imaginærdelen  
 til  $z$                       til  $z$

Addisjon  $z = a + ib, w = c + id$

$$z + w = a + ib + c + id = \underbrace{(a+c)}_{\text{real del}} + i \underbrace{(b+d)}_{\text{imaginær}}$$

Subtraksjon:

$$z - w = a + ib - (c + id) = \underbrace{(a-c)}_{\text{real del}} + i \underbrace{(b-d)}_{\text{imaginær del}}$$

Multiplikasjon

$$\begin{aligned} z w &= (a + ib) \cdot (c + id) = ac + iad + ibc \\ &\quad + \underbrace{i^2}_{-1} bd \\ &= \underbrace{ac - bd}_{\text{real del}} + i \underbrace{(ad + bc)}_{\text{imaginær del}} \end{aligned}$$

Division:  $\frac{z}{w} = \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)}$

$$= \frac{ac - iad + ibc - i^2 bd}{c^2 - i^2 d^2} = \frac{(ac+bd) + i(bc-ad)}{c^2 + d^2}$$

$$= \frac{ac+bd}{c^2+d^2} + i \frac{bc-ad}{c^2+d^2}$$

realteil                      imaginärteil

Beispiel:  $z = 3 + 2i$ ,  $w = 1 - 4i = 1 + (-4)i$

$$z \cdot w = (3 + 2i)(1 - 4i) = \underline{3 \cdot 1} - 3 \cdot 4i + 2i$$

$$= 11 - 10i = 11 + (-10)i$$

$$\frac{z}{w} = \frac{(3 + 2i)(1 + 4i)}{(1 - 4i)(1 + 4i)} = \frac{\underline{3} + 12i + 2i + \underline{8i^2}}{1^2 - \underbrace{4^2 \cdot i^2}_{+4^2}}$$

$$= \frac{-5 + 14i}{17} = \underline{\underline{-\frac{5}{17} + \frac{14}{17}i}}$$