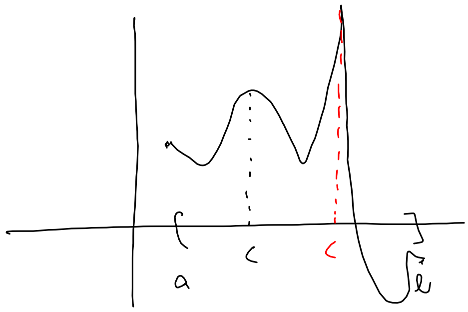
Repetisjon 2

Kurvedrøfting

Maks/min

voksende/avtagende



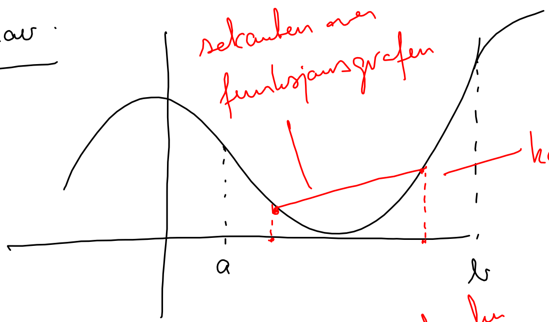
Kritiske punkter:

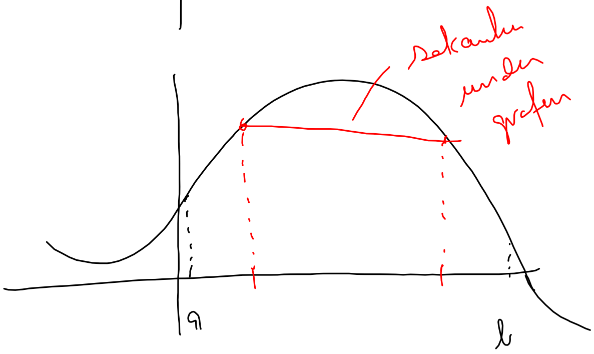
(i) $f'(c) = 0$

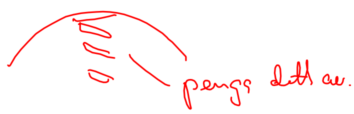
(ii) $f'(c)$ ikke definert

(iii) endepunkter a, b

Konveks/konkav:

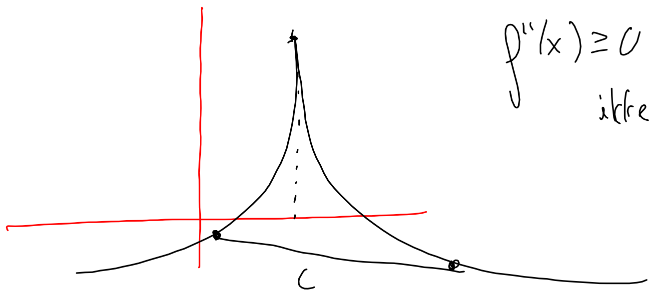






Setningen: Dersom $f''(x) \geq 0$ for alle $x \in I$, så er f konveks på I.

Dersom $f''(x) \leq 0$ for alle $x \in I$, så er f konkav på I.



$f''(x) \geq 0$ for alle $x \neq c$

ikke konveks

Uoppstilte maks/min:
Koblede hastigheter:
} Husk!

Nye funksjoner

Cotangens:

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

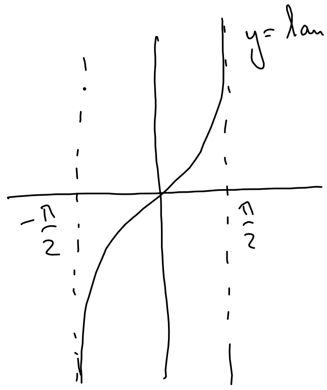
$$(\cot x)' = -\frac{1}{\sin^2 x} \qquad \int \frac{1}{\sin^2 x}\,dx = -\cot x + C$$

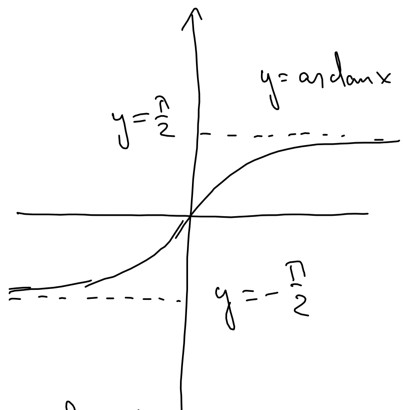
Arcus-funksjoner:

$$\arctan x = \tan^{-1} x$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$





$$\arctan 0 = 0$$

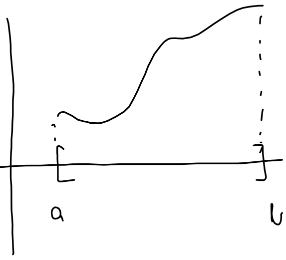
$$\arctan 1 = \frac{\pi}{4}$$

$$\lim_{x \to \pm\infty} \arctan x = \pm\frac{\pi}{2}$$

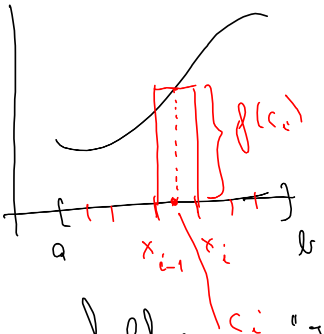
Integrasjon

Bestemte integraler:

$\int_a^b f(x)\,dx$ = definert ved trappesummer / alternativt ved Riemannsummer



Riemannsum:



$$R(\Pi, c) = \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1}) \longrightarrow \int_a^b f(x)\,dx$$

Analysens fundamentalteorem:

"Integrasjon og derivasjon er motsatte regningsarter: Integrasjon = antiderivasjon"

$$\int_a^b f(x)\,dx = H(b) - H(a)$$

Finner en antiderivert H

Problemtype: $G(x) = \int_0^{x^2} e^{-t^2}\,dt$

Hva er $G'(x)$?

Anta at H er en antiderivert til e^{-t^2}; dvs $H(x) - H(0) = \int_0^x e^{-t^2}\,dt$.

Altså: $H'(x) = e^{-x^2}$

$$G(x) = \int_0^{x^2} e^{-t^2}\,dt = H(x^2) - H(0)$$

$$G'(x) = H'(x^2)\cdot 2x = e^{-(x^2)^2}\cdot 2x = \underline{\underline{2x\,e^{-x^4}}}$$

Integrasjonsteknikk

Substitusjon: $\int f(g(x))\,dx$ $\quad \boxed{u=g(x)} \Rightarrow x = h(u)$

$\frac{dx}{du} = h'(u) \Rightarrow dx = h'(u)\,du$

$= \int f(u)\,h'(u)\,du$

$\int_a^b f(g(x))\,dx = \int_{g(a)}^{g(b)} f(u)\,h'(u)\,du$

Delvis integrasjon

Delbrøkoppspalting

$\int u v'\,dx = uv - \int u' v\,dx$

$\int_a^b u v'\,dx = \left[uv\right]_a^b - \int_a^b u' v\,dx$

Eksempel: $I = \int 1 \cdot \arcsin\sqrt{x}\,dx =$ Planer

$u = \sqrt{x}$ utviklingsstrukk

Delvis integrasjon: $u = \arcsin\sqrt{x}$, $v' = 1$

tematisk

Desperasjon: $u = \arcsin\sqrt{x}$
kanskje skjer det et under!

$u = \arcsin\sqrt{x} \Rightarrow \sqrt{x} = \sin u \Rightarrow x = \sin^2 u$

$dx = 2\sin u \cos u$

$I = \int u \cdot \underbrace{2\sin u \cos u}_{\sin 2u}\,du =$

Delvis integrasjon
$U = u \quad V' = \sin 2u$

$U' = 1 \quad V = -\frac{1}{2}\cos 2u$

$= -\frac{u}{2}\cos 2u + \frac{1}{2}\int \cos 2u$

$= -\frac{u}{2}\cos 2u + \frac{1}{4}\sin 2u + C$

$=$

$\cos 2u = 1 - 2\sin^2 u = 1 - 2x$

$\sin 2u = 2\sin u \cos u = 2\sin u\sqrt{1-\sin^2 u}$
$= 2\sqrt{x}\sqrt{1-x}$

Delbrøkoppspalting

$$\frac{2x^3-4}{(x-3)(x+4)^2(x^2+2x+3)^2} = \frac{A}{x-3} + \frac{B}{x+4} + \frac{C}{(x+4)^2} + \frac{Dx+E}{x^2+2x+3} + \frac{Fx+G}{(x^2+2x+3)^2}$$

$$\int \frac{x+4}{x^2+2x+3}\,dx = \frac{1}{2}\int \frac{2x+8}{x^2+2x+3}\,dx$$

$N = x^2+2x+3$

$N' = 2x+2$

Sample denne inn i telleren.

$$= \frac{1}{2}\int \frac{2x+2}{x^2+2x+3}\,dx + \frac{1}{2}\int \frac{6}{x^2+2x+3}\,dx$$

$u = x^2+2x+3$

$du = (2x+2)\,dx$

$$= \frac{1}{2}\int \frac{du}{u} + \underbrace{\int \frac{3}{x^2+2x+3}\,dx}$$

$$= \frac{1}{2}\ln(x^2+2x+3) + I_2$$

Mellomregning: $I_2 = \int \frac{3}{x^2+2x+3}\,dx = \int \frac{3}{\underbrace{x^2+2x+1}_{(x+1)^2}+2}\,dx$

$$= \int \frac{3}{(x+1)^2+2}\,dx = \frac{1}{2}\int \frac{3}{\frac{(x+1)^2}{2}+1}\,dx = \frac{3}{2}\int \frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1}\,dx$$

synd dette ikke er 1

$u = \frac{x+1}{\sqrt{2}}$

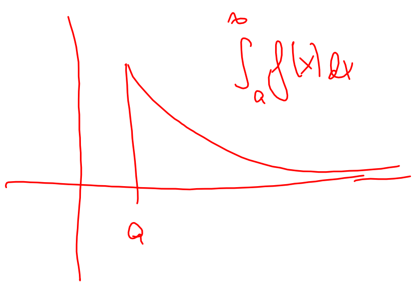
$du = \frac{1}{\sqrt{2}}\,dx$

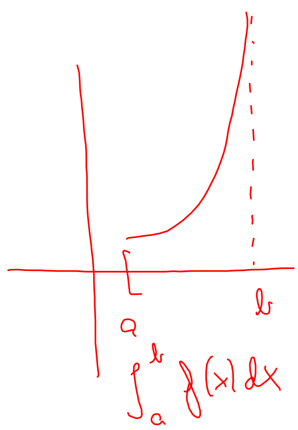
$dx = \sqrt{2}\,du$

$$= \frac{3}{2}\int \frac{\sqrt{2}}{u^2+1}\,du = \frac{3\sqrt{2}}{2}\arctan u + C$$

$$= \frac{3\sqrt{2}}{2}\arctan \frac{x+1}{\sqrt{2}} + C$$

Uegentlige integraler:





Vektorer, matriser, determinanter

n-tupler / vektorer $\quad \vec{a} = \underbrace{(a_1, a_2, \dots, a_n)}_{\text{radvektor}} \quad \vec{a} = \underbrace{\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}}_{\text{søylevektor}}$

matriser: $\quad A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \\ a_{m1} & \dots & & a_{mn} \end{pmatrix} \quad m \times n$

determinanter: $\quad \det(A) = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix} \leftarrow$ et tall

Operasjoner: $\vec{a} + \vec{b}$, $\vec{a} \cdot \vec{b}$, $\underbrace{\vec{a} \times \vec{b}}_{\text{bare dim 3.}}$

$\underbrace{A+B}_{\text{samme dimensjoner}}$, AB — $m \times n \quad n \times k \Rightarrow \underline{m \times k}$

Matriser transformerer vektorer

$$\begin{tikzcd} \vec{x} \arrow[r, bend left, "A"] & A\vec{x} \arrow[l, bend left, "A^{-1}"] \end{tikzcd}$$

Hvis A er en $n \times n$-matrise, så er B inversen til A ($n \times n$) dersom

$$AB = BA = I_n$$

nok å sjekke én av disse!

$$(AB)^{-1} = B^{-1}A^{-1}$$

Arealer, volumer:

- ved vektorprodukter
- determinanter

parallellogram
trekanter: $\frac{1}{2}$

parallellepipeder:
pyramider: $\frac{1}{6}$

Funksjoner av flere variable

$f'(\vec{a};\vec{r})$ - retningsderiverte

$\frac{\partial f}{\partial x_i}(\bar{a}) = f'(\vec{a};\vec{e}_i)$ — partiellderivert, deriver mhp. x_i som om de andre variablene var konstante

$$\nabla f(\vec{a}) = \left(\frac{\partial f}{\partial x_1}(\bar{a}), \frac{\partial f}{\partial x_2}(\bar{a}), .., \frac{\partial f}{\partial x_n}(\bar{a})\right)$$

$f'(\vec{a};\vec{r}) = \nabla f(\vec{a}) \cdot \vec{r}$ (forutsatt at f er deriverbar)

$\nabla f(\bar{a})$ peker i den retningen hvor funksjonen vokser raskest ut fra $\vec{a}$, og stigningstallet i den retningen er $|\nabla f(\bar{a})|$

$$\vec{F}(x_1,..,x_n) = \begin{pmatrix} F_1(x_1,..,x_n) \\ F_2(x_1,..,x_n) \\ \vdots \\ F_m(x_1,..,x_n) \end{pmatrix}$$

Jacobi-matrisen:

$$\vec{F} = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \cdots & & \vdots \\ \vdots & & & \vdots \\ \frac{\partial F_m}{\partial x_1} & \frac{\partial F_m}{\partial x_2} & \cdots & \frac{\partial F_m}{\partial x_n} \end{pmatrix} \begin{array}{l} \leftarrow \nabla F_1 \\ \leftarrow \nabla F_2 \\ \vdots \\ \leftarrow \nabla F_m \end{array}$$

Eksamen

10 flervalg, 5 alt., 3 poeng = 30 poeng 27

7 vanlige punkter, 10 poeng = 70 poeng 51 ←

Midtveis 50 poeng 42

150 poeng

$$\int_0^{\pi} \sin x \, dx = e^2$$