Seksjon 3.1

1)

b) $(4+8i)-(7-3i) = 4+8i-7+3i$

$= 4-7+8i+3i$

$= \underline{\underline{-3+11i}}$

d) $(5+2i)(3+i) = 5(3+i)+2i(3+i)$

$= 15+5i+6i+\underline{2i^2}\,(=-2)$

$= 15-2+5i+6i$

$= \underline{\underline{13+11i}}$

f) $\dfrac{4+3i}{2+i} = \dfrac{(4+3i)(2-i)}{(2+i)(2-i)}$

$= \dfrac{8-4i+6i-3i^2}{4+\cancel{2i-2i}-i^2}$ $-3i^2 = 3$

$= \dfrac{11+2i}{5} = \underline{\underline{\dfrac{11}{5}+\dfrac{2}{5}i}}$

$\overline{2+i} = 2-i$

3. Regn ut:

d) $\overline{(3-i)(-2i)} = \overline{-6i+2i^2} = \overline{-2-6i}$

$$= \underline{\underline{-2+6i}}$$

e) $\overline{\left(\frac{4-3i}{i}\right)} = \overline{\frac{(4-3i)(-i)}{i\cdot(-i)}} = \overline{\frac{-4i+3i^2}{-i^2}}$

$$= \overline{\frac{-3-4i}{1}} = \underline{\underline{-3+4i}}$$

5. Løs likningene

a) $\frac{\cancel{2i}z}{\cancel{2i}} = \frac{3+4i}{2i}$ Løse for z.

$$z = \frac{(3+4i)\cdot(-2i)}{2i(-2i)}$$

$$= \frac{-6i-8i^2}{4} = \frac{8-6i}{4} = \underline{\underline{2-\frac{3}{2}i}}$$

$$\underline{\underline{z = 2 - \frac{3i}{2}}}.$$

c) $\frac{z-2}{z+1} = 3i$ | $\cdot(z+1)$ på begge sider. Husk: $z \neq -1$.

$z - 2 = 3i(z+1)$.

$z-2 = 3iz + 3i$.

$\rightarrow z - 3iz = 2 + 3i$.

$z(1-3i) = 2+3i$ | deler på $1-3i$

$$z = \frac{2+3i}{1-3i} = \frac{(2+3i)(1+3i)}{(1-3i)(1+3i)} \quad a=1, b=-3$$

$$= \frac{2+6i+3i+9i^2}{1^2+3^2} \quad (=-1)$$

$z = a+ib$
$\bar{z} = a-ib$.
$z\bar{z} = a^2+b^2$

$$= \frac{2-9+9i}{10} = \frac{-7+9i}{10}$$

$$\underline{\underline{z = -\frac{7}{10} + \frac{9}{10}i}}$$

Standardformen $\underline{z = a+ib}$.

6. Finn komplekse tall $z$ og $w$ slik at

1) $z+w=2i$ , 2) $z-w=3+i$.

•) Innsetting

1) $z+w=2i$ | Isolerer $z$

$z=2i-w$

2) $z-w=3+i \Rightarrow \underline{(2i-w)-w=3+i}$

$2i-w-w=3+i$

$2i-2w=3+i$

$\frac{-2w}{-2}=\frac{3+i-2i}{-2}$

$=\frac{3-i}{-2}$

$w=-\frac{3}{2}+\frac{1}{2}i=\frac{-3+i}{2}$

$z=2i-\left(\frac{-3}{2}+\frac{1}{2}i\right)$

$=2i+\frac{3}{2}-\frac{1}{2}i$

$=\frac{3}{2}+i\left(2-\frac{1}{2}\right)$

$2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$

$z=\frac{3}{2}+\frac{3}{2}i=\frac{3}{2}(1+i)$

---

$z+w=2i$ , $z-w=3+i$.

$(z+w)+(z-w)=2z=2i+3+i=3+3i$

$\Rightarrow 2z=3+3i \Rightarrow z=\frac{3+3i}{2}$

Metode nr 2

8. Bevis regnereglene for konjugasjon:
Teorem 3.1.5 i Kalkulus. $z$ og $w$ komplekse.

(i) $\overline{z}+\overline{w}=\overline{z+w}$ addisjon

(ii) $\overline{z}-\overline{w}=\overline{z-w}$ subtraksjon

(iii) $\overline{z}\,\overline{w}=\overline{zw}$ multiplikasjon

(iv) $\dfrac{\overline{z}}{\overline{w}}=\overline{\left(\dfrac{z}{w}\right)}$ divisjon $(w\neq 0)$

---

Bevise (iv). Kan skrive $z=a+ib$, og $w=c+id$. Først: regner ut $\dfrac{\overline{z}}{\overline{w}}$:

$\overline{z}=a-ib$. $\overline{w}=c-id$.

$$\frac{\overline{z}}{\overline{w}}=\frac{a-ib}{c-id}=\frac{(a-ib)(c+id)}{(c-id)(c+id)}$$

$$=\frac{ac+adi-bci-i^2bd}{c^2+\underbrace{cdi-cdi}_{\text{stryker}}-i^2d^2}$$

$$\longrightarrow\ \frac{\overline{z}}{\overline{w}}=\frac{(ac+bd)+(ad-bc)i}{c^2+d^2}$$

$\overline{\left(\dfrac{z}{w}\right)}$. Regner ut $\dfrac{z}{w}=\dfrac{a+ib}{c+id}$

$$\frac{z}{w}=\frac{(a+ib)(c-id)}{(c+id)(c-id)}=\frac{ac-adi+bci-bdi^2}{c^2+d^2}$$

$$=\frac{(ac+bd)+(-ad+bc)i}{c^2+d^2}$$

$$\frac{z}{w}=\frac{ac+bd}{c^2+d^2}-\frac{(ad-bc)}{c^2+d^2}i.$$

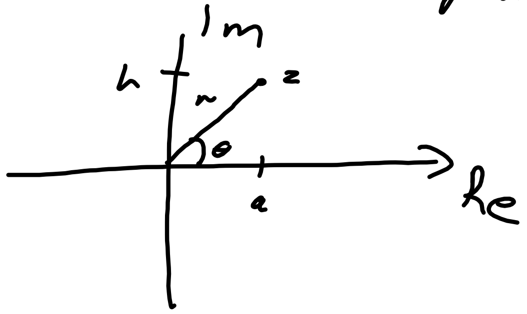
$$\Rightarrow\ \overline{\frac{z}{w}}=\frac{ac+bd}{c^2+d^2}+\frac{(ad-bc)}{c^2+d^2}i.$$

$$\Rightarrow\ \overline{\left(\frac{z}{w}\right)}=\frac{\overline{z}}{\overline{w}}.$$

Seksjon 3.2. Modulus og argument.

$z = a + ib = r(\cos\theta + i\sin\theta)$

modulus argumentet



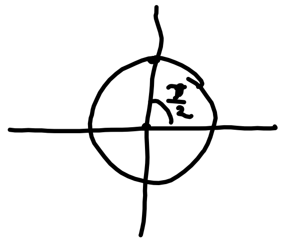

$(a, b)$ koordinater

$(r, \theta)$ polar —"—

5. a) Skriv $r$ på formen $a+bi$ når $r = 4$, $\theta = \frac{\pi}{2}$.

$a + bi = r(\cos\theta + i\sin\theta)$

$= 4(0 + i \cdot 1) = 4i.$




7. Finn $zw$ når

$\rightarrow z = 2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ | $r_1 = 2$, $\theta_1 = \frac{\pi}{12}$

$\rightarrow w = 3\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$ | $r_2 = 3$, $\theta_2 = \frac{5\pi}{12}$.

Teorem 3.2.3: hvis $z_1, z_2$ er komplekse tall, hvor modulusen er $r_1$ og $r_2$, og argument $\theta_1$ og $\theta_2$, da er modulusen til produktet $z_1 z_2$ lik $r_1 r_2$, og argumentet er $\theta_1 + \theta_2$.

---

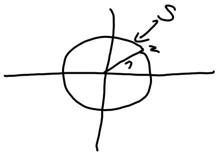
Modulus til $zw$: $r = r_1 \cdot r_2 = 2 \cdot 3 = 6$

Argument til $zw$: $\theta = \theta_1 + \theta_2 = \frac{\pi}{12} + \frac{5\pi}{12} = \frac{\pi}{2}$.

$zw = 6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = \underline{\underline{6i.}}$

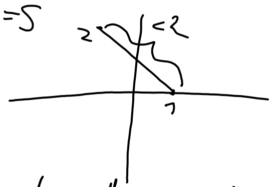
10. Skisser områdene i planet.
1) Algebraisk 2) Geometrisk.

a) $\{z \mid |z| = 1\} = S$



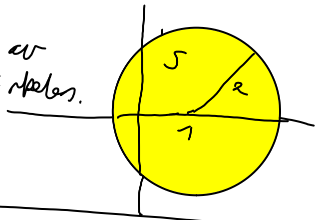

S består av de $z$ med lengde 1 fra origo.
Tolker modulus som lengde.

b) $\{z \mid |z-1| < 2\} = S$




Hvordan tolke $|z-1|$?
Avstanden mellom $z$ og 1 i planet.
$z \in S$ hvis avstanden mellom $z$ og 1 < 2.

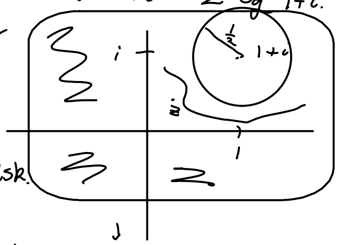
S = mengden av punkter i sirkelen.




c) $S = \{z \mid |z-(i+1)| \geq \frac{1}{2}\}$

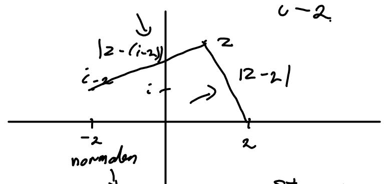
$|z-(i+1)|$ = avstanden mellom $z$ og $1+i$.

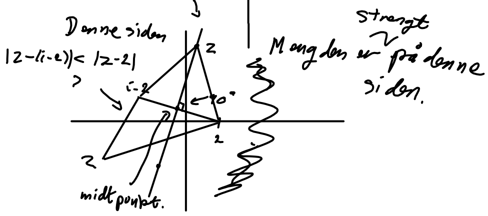
S = hele planet utenom det som ligger i sirkelen.
Heter også en disk.




d) $\{z \mid |z-2| < |z-i+2|\}$ $(|z-(i-2)|)$

avstand til 2 avstand til $i-2$




Algebraisk løsning:
$\{|z-1| < 2\}$. Skriver $z = a+ib$.

$$|z-1| = |a+ib-1| = |(a-1)+ib|$$
$$= \sqrt{(a-1)^2+b^2}$$
$$\sqrt{(a-1)^2+b^2} < 2 \Rightarrow (a-1)^2+b^2 < 2^2 \qquad = r^2$$

$(a-1)^2+b^2 = 2^2$ er en sirkel med sentrum $(1,0)$ og radius 2.

Seksjon 3.3.

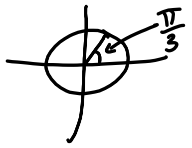
de Moivre's formel:

$$z = re^{i\theta} = r(\cos\theta + i\sin\theta).$$

Oppgv 2:

a) Skriv på formen $a+ib$.

$$e^{2+i\frac{\pi}{3}} = e^{2}\cdot e^{i\frac{\pi}{3}} = e^{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$



$$= e^{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

---

8. Regn $(1+i)^{804}$. Bruke de Moivre.

$$1+i = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right).$$

$$a+ib = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + i\frac{b}{\sqrt{a^2+b^2}}\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}e^{i\frac{\pi}{4}}.$$

$$(1+i)^{804} = \left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{804} = \sqrt{2}^{804}e^{\frac{i\pi\cdot 804}{4}}$$

$$= 2^{\frac{1}{2}\cdot 804}e^{i\pi\cdot 201}$$

$$= 2^{402}e^{201\pi i}$$

$$201\pi i = \pi i + 200\pi i$$

$$= \pi i + 100\cdot(2\pi i).$$

sin og cos periodiske $\Rightarrow \cos(201\pi)$

$$= \cos(\pi + 100\cdot(2\pi))$$

$$= \cos\pi = -1$$

$$\sin(201\pi) = \sin\pi = 0.$$

$$2^{402}e^{i\pi\cdot 201} = 2^{402}(\cos(201\pi) + i\sin(201\pi))$$

$$= 2^{402}(-1 + i\cdot 0) = -2^{402}.$$