

LØSNING AV POLYNOMLIKNINGER MED KOMPLEKSE TALL.

$$\textcircled{1} \quad z^2 + iz + 2 = 0$$

$$\textcircled{2} \quad z^3 = \frac{-8\sqrt{2}}{1+i}$$

$$\textcircled{3} \quad z^2 + 3z + 7 = 0$$

$$\textcircled{4} \quad p(z) = z^5 - z^4 + z^3 - z^2 - 12z + 12$$

LØS LIKNINGEN $p(z) = 0$
FAKTORISER $p(z)$ I REELLE FAKTORER

①

$$z^2 + iz + 2 = 0$$

$$z = \frac{-i \pm \sqrt{i^2 - 4 \cdot 2}}{2}$$

$$= \frac{-i \pm \sqrt{-1 - 8}}{2} = \frac{-i \pm \sqrt{-9}}{2}$$

$$= \frac{-i \pm \sqrt{-1} \cdot \sqrt{9}}{2} = \frac{-i \pm i \cdot 3}{2} = \begin{cases} i \\ -2i \end{cases}$$

LÖSUNGEN $z = i, z = -2i$

②

$$z^3 = \frac{-8\sqrt{2}}{1+i}$$

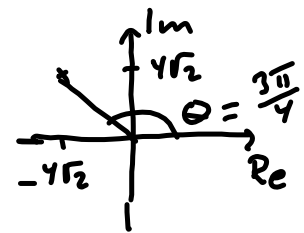
FINNER REAL OG IMAGINÆR DEL TIL HØYRE SIDE
($a + b \cdot i$)

$$\begin{aligned} \text{H.S.: } \frac{-8\sqrt{2}}{1+i} &= \frac{-8\sqrt{2}(1-i)}{(1+i)(1-i)} = \frac{-8\sqrt{2}(1-i)}{1-(-1)} = \frac{-8\sqrt{2}}{2}(1-i) \\ &= -4\sqrt{2} + 4\sqrt{2} \cdot i \end{aligned}$$

FINNER SÅ POLARKOORDINATENE
TIL H.S. ($r e^{i\theta}$)

$$r = \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{32+32} = 8$$

$$\theta = \frac{3\pi}{4}$$



LIGNINGEN KAN SKRIVES: $z^3 = 8 e^{i \frac{3\pi}{4}}$

$$z^3 = 8 e^{i \cdot \frac{3\pi}{4}}$$

$$z = 8^{\frac{1}{3}} e^{i \cdot \theta_0}, \quad 8^{\frac{1}{3}} e^{i \cdot \theta_1}, \quad 8^{\frac{1}{3}} e^{i \cdot \theta_2}$$

$$\begin{aligned} \text{der } \theta_0 &= \frac{1}{3} \cdot \frac{3\pi}{4} & \theta_1 &= \frac{1}{3} \frac{3\pi}{4} + \frac{2\pi}{3} & \theta_2 &= \frac{1}{3} \frac{3\pi}{4} + \frac{4\pi}{3} \\ &= \frac{\pi}{4} & &= \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12} & &= \frac{\pi}{4} + \frac{4\pi}{3} = \frac{19\pi}{12} \end{aligned}$$

MELIOM REGNING:

$$\cos \frac{\pi}{4} = \frac{1}{2} \sqrt{2}$$

$$\sin \frac{\pi}{4} = \frac{1}{2} \sqrt{2}$$

$$\cos \frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \frac{1}{2} \sqrt{2} \cdot \left(-\frac{1}{2}\right) - \frac{1}{2} \sqrt{2} \left(\frac{1}{2} \sqrt{3}\right) = \frac{1}{4} (-\sqrt{2} - \sqrt{6}) \quad \left(\cos \frac{2\pi}{3} = -\frac{1}{2}\right)$$

$$\sin \frac{11\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \frac{1}{2} \sqrt{2} \cdot \left(-\frac{1}{2}\right) + \frac{1}{2} \sqrt{2} \cdot \left(\frac{1}{2} \sqrt{3}\right) = \frac{1}{4} (-\sqrt{2} + \sqrt{6}) \quad \left(\sin \frac{2\pi}{3} = \frac{1}{2} \sqrt{3}\right)$$

$$\sin \frac{19\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) = \frac{1}{2} \sqrt{2} \cdot \left(-\frac{1}{2}\right) - \frac{1}{2} \sqrt{2} \left(-\frac{1}{2} \sqrt{3}\right) = \frac{1}{4} (-\sqrt{2} + \sqrt{6})$$

$$\cos \frac{19\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) = \frac{1}{2} \sqrt{2} \cdot \left(-\frac{1}{2}\right) + \frac{1}{2} \sqrt{2} \left(-\frac{1}{2}\right) = \frac{1}{4} (-\sqrt{2} - \sqrt{6})$$

LÖSNINGAR

$$z = 2 \left(\frac{1}{2} \sqrt{2} + \frac{1}{2} \sqrt{2} i \right) = \sqrt{2} + \sqrt{2} \cdot i$$

$$z = 2 \left(\frac{1}{4} (-\sqrt{2} - \sqrt{6}) + \frac{1}{4} (-\sqrt{2} + \sqrt{6}) \cdot i \right) = \frac{1}{2} (-\sqrt{2} - \sqrt{6}) + \frac{1}{2} (-\sqrt{2} + \sqrt{6}) \cdot i$$

$$z = \frac{1}{2} (\sqrt{2} - \sqrt{6}) + \frac{1}{2} (-\sqrt{2} - \sqrt{6}) \cdot i$$

③

$$z^2 + 3z + 7 = 0$$

$$z = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 7}}{2} = \frac{-3 \pm \sqrt{-19}}{2}$$

$$= \frac{-3 \pm \sqrt{-1} \cdot \sqrt{19}}{2} = \frac{-3 \pm i\sqrt{19}}{2}$$

LÖSNUNGEN:

$$z = -\frac{3}{2} + \frac{\sqrt{19}}{2} \cdot i$$

$$z = -\frac{3}{2} - \frac{\sqrt{19}}{2} \cdot i$$

$$(4) \quad p(z) = z^5 - z^4 + z^3 - z^2 - 12z + 12$$

$$\text{Likning: } p(z) = 0$$

PRØVER OSS FRAM MED HELTALL SOM DELER 12.

$$\text{F. EKS } z=1: p(1) = 1^5 - 1^4 + 1^3 - 1^2 - 12 \cdot 1 + 12 = 0!$$

DA ER $z=1$ LØSNING OG $z-1$ ER FAKTOR I $p(z)$

$$p(z) : (z-1)$$

$$z^5 - z^4 + z^3 - z^2 - 12z + 12 : (z-1) = z^4 + z^2 - 12$$

$$\begin{array}{r} z^5 - z^4 \\ \hline 0 \end{array} \quad z^3 - z^2 - 12z + 12$$

$$\begin{array}{r} z^3 - z^2 \\ \hline -12z + 12 \end{array}$$

$$-12z + 12$$

$$-12z + 12$$

$$\hline 0$$

$$\text{så } p(z) = (z-1)(z^4 + z^2 - 12)$$

$$\text{LØSER } z^4 + z^2 - 12 = 0$$

$$(z^2)^2 + z^2 - 12 = 0$$

$$z^2 = \frac{-1 \pm \sqrt{1 - 4 \cdot (-12)}}{2} = \frac{-1 \pm \sqrt{49}}{2} = \frac{-1 \pm 7}{2} = \begin{cases} 3 \\ -4 \end{cases}$$

$$\text{så } p(z) = (z-1)(z^2-3)(z^2+4)$$

$$= (z-1)(z-\sqrt{3})(z+\sqrt{3})(z^2+4) \leftarrow \text{REEL FAKTOR-}$$

$$\text{LØSNINGER: } z=1, z=\sqrt{3}, z=-\sqrt{3}, \text{ \& } z=-2i, z=2i. \quad (z+2i)(z-2i) \text{ I SERING}$$

S_A P C

