Oppgaver:

4.1: 1, 3, 4, 5

4.2: 1, 2a) d), 3, 6, 9, 10

4.3: 1, 2a) c), 3a), 4, 6, 7

4.4: 1a), c) d), 2, 4, 5

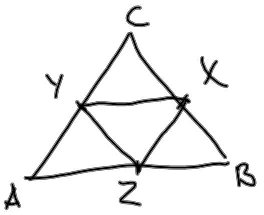
4.1.1

$$x+2y-z=3$$
$$2x+3y-3z=-1$$
$$-x+2y+3z=1$$

$$x=3-2y+z$$
$$2(3-2y+z)+3y-3z=-1$$
$$6-4y+2z+3y-3z=-1$$
$$6-y-z=-1$$
$$y=7-z$$

$$-(3-2y+z)+2(7-z)+3z=1$$
$$-3+2y-z+14-2z+3z=1$$
$$-3+2(7-z)-z+14-2z+3z=1$$
$$-3+14-2z-z+14-2z+3z=1$$
$$25-2z=1$$
$$24=2z$$
$$z=12$$
$$y=7-z=7-12=-5$$
$$x=3-2y+z=3+10+12=25$$

4.1.5:



Temprator : A = a
— // — B = b
— // — C = c

x = snittemprator av punktene C, B, Z, Y

$$=\frac{c+b+z+y}{4}$$

$$y=\frac{a+c+z+x}{4}$$

$$z=\frac{a+b+y+x}{4}$$

Temprator i X = x
— // — Y = y
— // — Z = z

$$4x=c+b+z+y$$
$$4y=a+c+z+x$$
$$4z=a+b+y+x$$

i) $4x-y-z=c+b$
ii) $-x+4y-z=a+c$
iii) $-x-y+4z=a+b$

$x=-y+4z-a-b$ fra iii)

sett inn i f.eks. i)

$$4(-y+4z-a-b)-y-z=c+b$$
$$-4y+16z-4(a+b)-y-z=c+b$$
$$-5y+15z=c+b+4(a+b)$$
$$y=c+b+4(a+b)-15z \;/\!-5$$

Sett inn i løsning ii), etc. etc.

$$x=\frac{2(b+c)+a}{5},\quad y=\frac{2(a+c)+b}{5},\quad z=\frac{2(a+b)+c}{5}$$

4.2.3:

$$x - y + 2z = 1,\quad 2x + y + z = 1,\quad -2x - y + z = 0$$

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & 1 \\ -2 & -1 & 1 & 0 \end{pmatrix} \begin{matrix} -2 \quad 2 \\ \\ \\ \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 3 & -3 & -1 \\ 0 & -3 & 5 & 2 \end{pmatrix} \begin{matrix} \\ 1 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 3 & -3 & -1 \\ 0 & 0 & 2 & 1 \end{pmatrix} \begin{matrix} \\ \frac{1}{3} \\ \frac{1}{2} \end{matrix} \sim \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1/3 \\ 0 & 0 & 1 & 1/2 \end{pmatrix}$$

$x - y + 2z = 1$, $\quad x = 1 + y - 2z = 1 + \frac{1}{6} - 1 = \frac{1}{6}$

$y - z = -\frac{1}{3}$, $\quad y = -\frac{1}{3} + z = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$

$z = \frac{1}{2}$

4.2.6:

$$x + 3y - z + 3u = 4,\quad x + 2y - 2z + 3u = 0,\quad 2x + 2y - 5z + 5u = 1$$

$$\begin{pmatrix} 1 & 3 & -1 & 3 & 4 \\ 1 & 2 & -2 & 3 & 0 \\ 2 & 2 & -5 & 5 & 1 \end{pmatrix} \begin{matrix} -1 \quad -2 \\ \\ \\ \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 3 & -1 & 3 & 4 \\ 0 & -1 & -1 & 0 & -4 \\ 0 & -4 & -3 & -1 & -7 \end{pmatrix} \begin{matrix} \\ \cdot(-1) \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 3 & -1 & 3 & 4 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & -4 & -3 & -1 & -7 \end{pmatrix} \begin{matrix} \\ 4 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 3 & -1 & 3 & 4 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 & 9 \end{pmatrix}$$

$x + 3y - z + 3u = 4$

$y + z = 4$

$z - u = 9$

u er en fri variabel

$z = 9 + u$

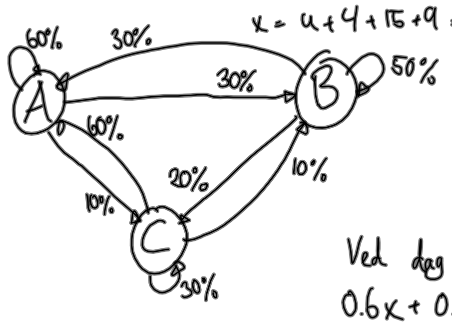
$y + z = 4$, $\quad y + 9 + u = 4$, $\quad y = -5 - u$

$x + 3(-5 - u) - 9 - u + 3u = 4$

$x - 15 - \cancel{3u} - 9 - u + \cancel{3u} = 4$

$x = u + 4 + 15 + 9 = u + 28$

4.2.10:



Anta at ved dag 0 er
x biler i A
y —//— B
z —//— C

Ved dag 1 er det
$0.6x + 0.3y + 0.6z$ biler i A
$0.3x + 0.5y + 0.1z$ —//— B
$0.1x + 0.2y + 0.3z$ —//— C

Vil ha

$$\begin{aligned} 0.6x + 0.3y + 0.6z &= x \\ 0.3x + 0.5y + 0.1z &= y \\ 0.1x + 0.2y + 0.3z &= z \\ x + y + z &= 120 \end{aligned}$$

skriv om til

$$\begin{aligned} x + y + z &= 120 \\ -0.4x + 0.3y + 0.6z &= 0 \\ 0.3x - 0.5y + 0.1z &= 0 \\ 0.1x + 0.2y - 0.7z &= 0 \end{aligned}$$

Koeffisientmatrise

$$\begin{pmatrix} 1 & 1 & 1 & 120 \\ -0.4 & 0.3 & 0.6 & 0 \\ 0.3 & -0.5 & 0.1 & 0 \\ 0.1 & 0.2 & -0.7 & 0 \end{pmatrix} \begin{matrix} \\ \cdot(-10) \\ \cdot 10 \\ \cdot 10 \end{matrix} \sim \begin{pmatrix} 1 & 1 & 1 & 120 \\ 4 & -3 & -6 & 0 \\ 3 & -5 & 1 & 0 \\ 1 & 2 & -7 & 0 \end{pmatrix} \begin{matrix} -4 \; -3 \; -1 \\ \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 1 & 1 & 120 \\ 0 & -7 & -10 & -480 \\ 0 & -8 & -2 & -360 \\ 0 & 1 & -8 & -120 \end{pmatrix} \begin{matrix} \\ \\ 9 \\ 8 \quad 7 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & -66 & -1320 \\ 0 & 0 & -66 & -1320 \end{pmatrix} \begin{matrix} \\ \\ -\frac{1}{66} \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 1 & 1 & 120 \\ 0 & 1 & -8 & -120 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x + y + z = 120$, $\quad x = 120 - 40 - 20 = 60$

$y - 8z = -120$, $\quad y = -120 + 8z = -120 + 160 = 40$

$z = 20$

$x = 60$, $y = 40$, $z = 20$

4.3.4:

$$\begin{aligned} x+2y+z&=b_1\\ 2x+4y+3z&=b_2\\ -x+3y+2z&=b_3 \end{aligned}$$

avgjør om lineæringsystemet har en entydig løsning for alle valg av b_1, b_2, b_3

Koeffisientmatrisen

$$\begin{pmatrix} 1 & 2 & 1\\ 2 & 4 & 3\\ -1 & 3 & 2 \end{pmatrix} \begin{matrix} \cdot(-2) & 1 \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 2 & 1\\ 0 & 0 & 1\\ 0 & 5 & 3 \end{pmatrix} \begin{matrix} \\ \\ \frac{1}{5} \end{matrix} \sim \begin{pmatrix} 1 & 2 & 1\\ 0 & 1 & 3/5\\ 0 & 0 & 1 \end{pmatrix}$$

siden alle søyler er pivotsøyler, fins det alltid en entydig løsning, uansett valg av b_1, b_2, b_3.

4.3.7:

$$\begin{aligned} x+y-z+2u-v&=1\\ -2x-2y+z-u+v&=2\\ 3x+3y-2u+2v&=1 \end{aligned}$$

Koeffisientmatrise (utvidet)

$$\begin{pmatrix} 1 & 1 & -1 & 2 & -1 & 1\\ -2 & -2 & 1 & -1 & 1 & 2\\ 3 & 3 & 0 & -2 & 2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 0 & 0 & 2 & 7\\ 0 & 0 & 1 & 0 & 7 & 26\\ 0 & 0 & 0 & 1 & 2 & 10 \end{pmatrix}$$

$$\begin{aligned} x+y+2v&=7\\ z+7v&=26\\ u+2v&=10 \end{aligned}$$

y og v er frie variabler

$$\begin{aligned} x&=7-y-2v\\ z&=26-7v\\ u&=10-2v \end{aligned}$$

4.4.2: Løs $Ax_1=b_1$, $Ax_2=b_2$, $A=\begin{pmatrix} 3 & 1\\ -1 & 2 \end{pmatrix}$, $b_1=\begin{pmatrix} 1\\ -2 \end{pmatrix}$, $b_2=\begin{pmatrix} -1\\ 4 \end{pmatrix}$

$[A\ b_1]$, $[A\ b_2]$,

$$[A\ b_1\ b_2] = \begin{pmatrix} 3 & 1 & 1 & -1\\ -1 & 2 & -2 & 4 \end{pmatrix} \begin{matrix} \\ (-1) \end{matrix} \sim \begin{pmatrix} 1 & -2 & 2 & -4\\ 3 & 1 & 1 & -1 \end{pmatrix} \begin{matrix} \cdot(-3) \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & -2 & 2 & -4\\ 0 & 7 & -5 & 11 \end{pmatrix} \begin{matrix} \\ \frac{1}{7} \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 2 & -4\\ 0 & 1 & -5/7 & 11/7 \end{pmatrix} \begin{matrix} \\ 2 \end{matrix} \sim \begin{pmatrix} 1 & 0 & 4/7 & -6/7\\ 0 & 1 & -5/7 & 11/7 \end{pmatrix} \qquad x_1=\begin{pmatrix} 4/7\\ -5/7 \end{pmatrix},\ x_2=\begin{pmatrix} -6/7\\ 11/7 \end{pmatrix}$$

4.4.4. a) $A=\begin{pmatrix} 0 & 1 & 2 & 0\\ 1 & 0 & -1 & 1\\ 6 & 0 & -6 & 7\\ 2 & 1 & 0 & 0 \end{pmatrix}$. b) løs $Ax=b$, hvor $b=\begin{pmatrix} 0\\ 1\\ h\\ 0 \end{pmatrix}$

$$[A\ b] = \begin{pmatrix} 0 & 1 & 2 & 0 & 0\\ 1 & 0 & -1 & 1 & 1\\ 6 & 0 & -6 & 7 & h\\ 2 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \cdot(-6) \quad -2 \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1\\ 0 & 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 1 & h-6\\ 0 & 1 & 2 & -2 & -2 \end{pmatrix} \begin{matrix} \\ \cdot(-1) \\ \\ \\ \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1\\ 0 & 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 1 & h-6\\ 0 & 0 & 0 & -2 & -2 \end{pmatrix} \begin{matrix} \\ \\ 2 \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1\\ 0 & 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 1 & h-6\\ 0 & 0 & 0 & 0 & -14+2h \end{pmatrix}$$

Gir mening kun hvis $-14+2h=0$, dvs $h=7$.

$$\begin{pmatrix} 1 & 0 & -1 & 1 & 1\\ 0 & 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \\ (-1) \\ \\ \end{matrix} \sim \begin{array}{c} \begin{pmatrix} 1 & 0 & -1 & 0 & 0\\ 0 & 1 & 2 & 0 & 0\\ 0 & 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 & \end{matrix} \end{array}$$

4.4.5:

$$C = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & a^2-a & 3 \\ -1 & 1 & -3 & a \end{pmatrix}$$

a) reduser C til trappeform

b) $b = \begin{pmatrix} 1 \\ 3 \\ a \end{pmatrix}$ for hvilke verdier av a har

$$Ax = b$$

en løsning.

$$[A \;\; b] = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & a^2-a & 3 \\ -1 & 1 & -3 & a \end{pmatrix} \begin{matrix} -2 \;\; 1 \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 1 & -2 & a+1 \end{pmatrix} \begin{matrix} -1 \\ \\ \\ \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & a^2-a-2 & 1 \\ 0 & 0 & a-a^2 & a \end{pmatrix}$$

$$x_1 + x_3 = 1$$
$$x_2 + x_3(a^2-a-2) = 1$$
$$x_3(a-a^2) = a$$

dersom $a - a^2 = 0$, mens $a \neq 0$ har systemet ingen løsning. dvs ingen løsning når $a=1$.

Hvis $a=0$ blir x_3 en fri variabel, dvs systemet har uendelig mange løsninger.

Hvis $a-a^2 \neq 0$ har systemet èn, og kun en løsning.