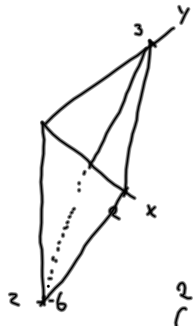
6.9: 1, 2 d) e), 6.10: 1c), 2 b), c), 3 a), c), d), e), 5, 7

6.11: 1, 3, 6, 11

6.9.2.d): A = området avgrenset av xy, yz og xz -planene og planet $3x+2y-z=6$



$$\iiint_A (3y^2-3z)\,dx\,dy\,dz$$

Sett $y,z=0$, $0\le 3x\le 6$, $0\le x\le 2$

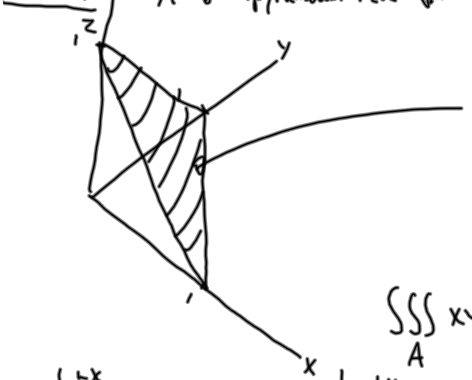
Sett $z=0$, $0\le 2y\le 6-3x$, $0\le y\le 3-\frac{3}{2}x$

$3x+2y-6\le z\le 0$

$$\int_0^2\int_0^{3-\frac{3}{2}x}\int_{3x+2y-6}^0 (3y^2-3z)\,dz\,dy\,dx = \int_0^2\int_0^{3-\frac{3}{2}x}\left(3y^2z-\tfrac{3}{2}z^2\right)\Big|_{z=3x+2y-6}^{z=0}dy\,dx$$

$$=\int_0^2\int_0^{3-\frac{3}{2}x} -3y^2(3x+2y-6)+\tfrac{3}{2}(3x+2y-6)^2\,dy\,dx$$

6.9.2.e): A er tetraeder ~~pyramiden~~ med hjørner $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$



$x+y+z=1.$

$y,z=0$, $0\le x\le 1.$

$z=0$, $0\le y\le 1-x$

$0\le z\le 1-x-y$

$$\iiint_A xy\,dx\,dy\,dz = \int_0^1\int_0^{1-x}\int_0^{1-x-y} xy\,dz\,dy\,dx$$

$$=\int_0^1\int_0^{1-x} xy(1-x-y)\,dy\,dx = \int_0^1\int_0^{1-x}(xy-x^2y-xy^2)\,dy\,dx = \int_0^1 \tfrac{1}{2}x(1-x)^2-\tfrac{x^2}{2}(1-x)^2-\tfrac{x}{3}(1-x)^3\,dx$$

$*$ $\iiint_A f(x,y,z)\,dx\,dy\,dz = \iiint_A f(r\cos\theta, r\sin\theta, z)\,r\,dr\,d\theta\,dz$ (sylinderkoordinater)

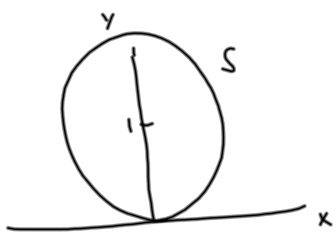
$**$ $\iiint_A f(x,y,z)\,dx\,dy\,dz = \iiint f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$ (kulekoordinater)

6.10.1c): $A=\{(x,y,z): x^2+(y-1)^2\le 1,\ 0\le z\le 2\}$

$S=\{(x,y): x^2+(y-1)^2\le 1\}$

$$\iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz = \int_0^2\left(\iint_S z\sqrt{x^2+y^2}\,dx\,dy\right)dz$$

S er sirkelen med radius 1, senter $(0,1)$



se oppg. 6.3.3a)

$$\iint_S f(x,y)\,dx\,dy = \int_0^\pi\int_0^{2\sin\theta} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Da blir

$$\iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz = \int_0^2\int_0^\pi\int_0^{2\sin\theta} z\,r\cdot r\,dr\,d\theta\,dz = \int_0^2 z\,dz\int_0^\pi\int_0^{2\sin\theta} r^2\,dr\,d\theta$$

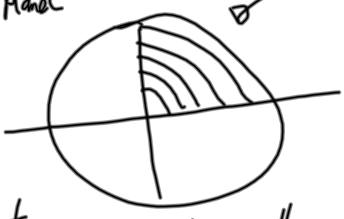
$$=2\int_0^\pi \tfrac{1}{3}2^3\sin^3\theta\,d\theta = \tfrac{16}{3}\int_0^\pi \sin^3\theta\,d\theta$$

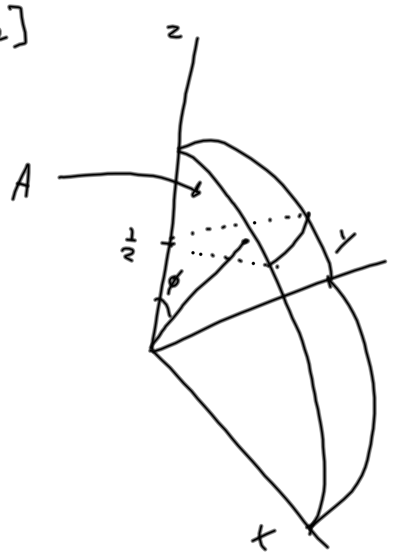
6.10.26b:

$A = \{(x,y,z) : x,y \geq 0,\ z \geq \frac{1}{2},\ x^2+y^2+z^2 \leq 1\}$

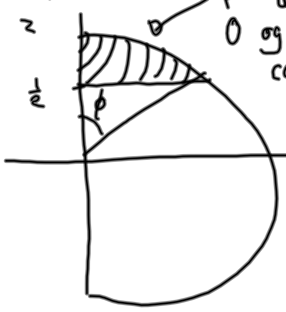
xy-planet

$\theta \in [0, \frac{\pi}{2}]$



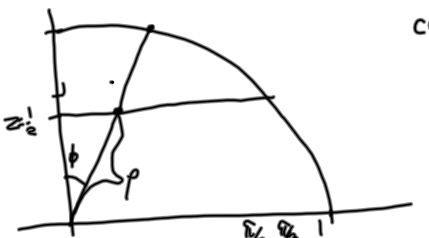


xz-planet



ϕ ligger mellom 0 og slik at $\cos\phi = \frac{1}{2}$, dvs $\phi = \frac{\pi}{3}$

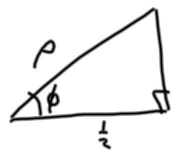
$\phi \in [0, \frac{\pi}{3}]$



$\cos\phi \cdot \rho = \frac{1}{2}$

Altså må ρ ligge i intervallet

$[\frac{1}{2\cos\phi}, 1]$



$\cos\phi\,\rho = \frac{1}{2}$

$$\iiint_A x\,dx\,dy\,dz = \int_0^{\pi/2}\int_0^{\pi/3}\int_{\frac{1}{2\cos\phi}}^{1} \rho\cos\theta\sin\phi \cdot \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{\pi/2}\cos\theta\,d\theta \int_0^{\pi/3} \sin^2\phi \cdot \frac{1}{4}\rho^4\Big|_{\rho=\frac{1}{2\cos\phi}}^{\rho=1} d\phi = \int_0^{\pi/2}\cos\theta\,d\theta \left(\frac{1}{4}\int_0^{\pi/3}\sin^2\phi\,d\phi - \frac{1}{4\cdot 2^4}\int_0^{\pi/3}\frac{\sin^2\phi}{\cos^4\phi}\,d\phi\right)$$

$$\int_0^{\pi/3}\frac{\sin^2\phi}{\cos^4\phi}\,d\phi = \int_0^{\pi/3}\frac{\tan^2\phi}{\cos^2\phi}\,d\phi.$$

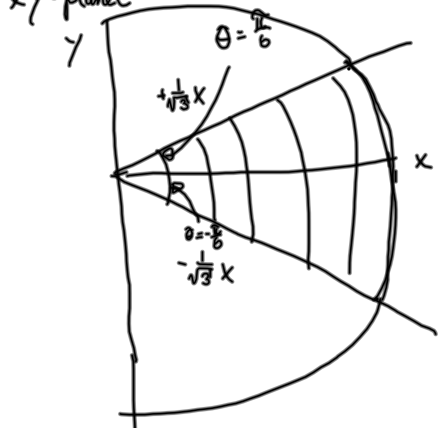
$u = \tan\phi$, $du = \frac{1}{\cos^2\phi}\,d\phi$, $u \in [0, \tan\frac{\pi}{3}]$

$$= \int_0^{\tan\frac{\pi}{3}} u^2\,du$$

6(10.2c):

$A = \{(x,y,z): z^2 \le x^2+y^2 \le 1,\ 3y^2 \le x^2,\ x \ge 0\}$

xy-planet

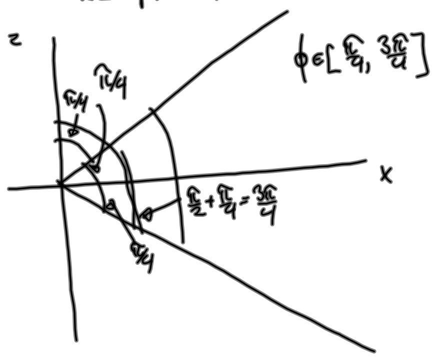


$3y^2 \le x^2$

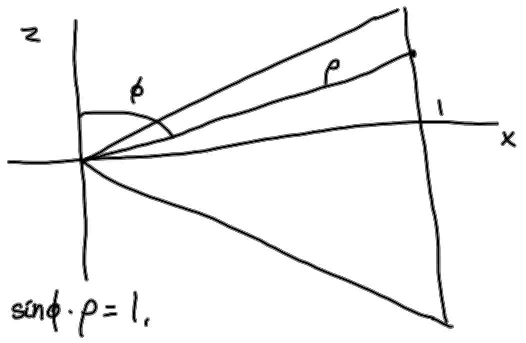
$y \le \pm\sqrt{\frac{1}{3}}\,x$

$\theta \in [-\frac{\pi}{6}, \frac{\pi}{6}]$

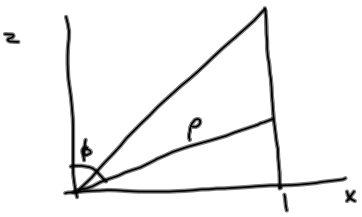
xz-planet, (a $y=0$, $z^2 \le x^2 \le 1$, $-1 \le -x \le z \le x \le 1$



$\phi \in [\frac{\pi}{4}, \frac{3\pi}{4}]$



$\sin\phi \cdot \rho = 1.$



ρ går fra 0 til ρ slik at

$\sin\phi \cdot \rho = 1.$

$\rho \in [0, \frac{1}{\sin\phi}]$

$\theta \in [-\frac{\pi}{6}, \frac{\pi}{6}]$, $\phi \in [\frac{\pi}{4}, \frac{3\pi}{4}]$, $\rho \in [0, \frac{1}{\sin\phi}]$.

$$\iiint_A 1\,dx\,dy\,dz = \int_{-\pi/6}^{\pi/6}\int_{\pi/4}^{3\pi/4}\int_0^{1/\sin\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \frac{\pi}{3}\int_{\pi/4}^{3\pi/4} \sin\phi \cdot \frac{1}{3}\rho^3 \Big|_{\rho=0}^{\rho=\frac{1}{\sin\phi}} d\phi$$

$$= \frac{\pi}{3\cdot 3}\int_{\pi/4}^{3\pi/4} \frac{\sin\phi}{\sin^3\phi}\,d\phi = \frac{\pi}{9}\int_{\pi/4}^{3\pi/4} \frac{1}{\sin^2\phi}\,d\phi = \frac{\pi}{9}(-\cot\phi)\Big|_{\phi=\pi/4}^{\phi=3\pi/4} = \frac{\sqrt{2}\,\pi}{9}$$

$$\boxed{\frac{d}{d\phi} -\cot\phi = \frac{1}{\sin^2\phi}}$$

6.10.3c): A = kulen m/radius 1 senter i origo $= \{(x,y,z) : x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi \quad \phi \in [0,\pi],\ \theta \in [0,2\pi],\ \rho \in [0,1]\}$

$$\iiint_A e^{-\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$$

$$= \int_0^1\int_0^{\pi}\int_0^{2\pi} e^{-\rho}\rho^2\sin\phi\,d\theta\,d\phi\,d\rho = 2\pi\int_0^{\pi}\sin\phi\,d\phi\int_0^1 e^{-\rho}\rho^2\,d\rho$$

$$\int_0^1 e^{-\rho}\rho^2\,d\rho = -\rho^2 e^{-\rho}\Big|_{\rho=0}^{\rho=1} + 2\int_0^1 \rho e^{-\rho}\,d\rho = -e^{-1} + 2\left[-\rho e^{-\rho}\Big|_{\rho=0}^{\rho=1} + \int_0^1 e^{-\rho}\,d\rho\right]$$

$$= -e^{-1} - 2\left(e^{-1} + (-e^{-\rho})\Big|_{\rho=0}^{\rho=1}\right) = -3e^{-1} - 2(e^{-1}+1) = 2 - 5e^{-1}$$

$u = \rho^2,\ v' = e^{-\rho}$
$u' = 2\rho,\ v = -e^{-\rho}$

$u = \rho,\ v' = e^{-\rho}$
$u' = 1,\ v = -e^{-\rho}$

6.11.1: Volum til kule m/radius R. $= \iiint_A 1\,dx\,dy\,dz = \int_0^R\int_0^{2\pi}\int_0^{\pi}\rho^2\sin\phi\,d\phi\,d\theta\,d\rho$

$$= 2\pi\int_0^R\rho^2\,d\rho\int_0^{\pi}\sin\phi\,d\phi$$

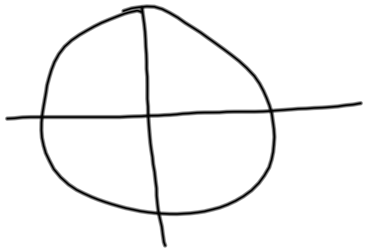
$$= \frac{2\pi R^3}{3}\underbrace{-\cos\phi\Big|_{\phi=0}^{\phi=\pi}}_{=2} = \frac{4\pi R^3}{3}$$

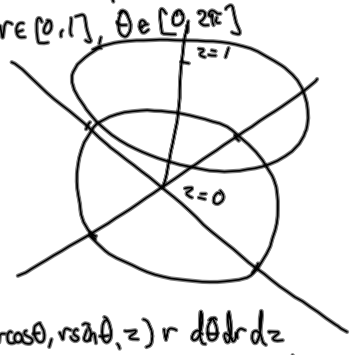
Hvor $A = \{(x,y,z) : x^2+y^2+z^2 \le 1\}$
$= \{(x,y,z) : x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi\}$
$\rho \in [0,R],\ \theta \in [0,2\pi],\ \phi \in [0,\pi]$

6.11.6:
$A = \{(x,y,z) : x^2+y^2 \le 1,\ 0 \le z \le 1\}$, har tetthet

$$f(x,y,z) = \frac{1}{x^2+y^2+z^2}$$

$x = r\cos\theta,\ y = r\sin\theta$
$r \in [0,1],\ \theta \in [0,2\pi]$





$$\iiint_A f(x,y,z)\,dx\,dy\,dz = \int_0^1\int_0^1\int_0^{2\pi} f(r\cos\theta, r\sin\theta, z)\,r\,d\theta\,dr\,dz$$

$$= \int_0^1\int_0^1\int_0^{2\pi}\frac{r}{r^2+z^2}\,d\theta\,dr\,dz = 2\pi\int_0^1\int_0^1\frac{r}{r^2+z^2}\,dr\,dz = \cancel{2}\pi\int_0^1 \cancel{\tfrac{1}{2}}\ln(r^2+z^2)\Big|_{r=0}^{r=1}\,dz$$

$$\frac{d}{dr}\ln(r^2+z^2) = \frac{1}{r^2+z^2}\cdot 2r$$

$$= \pi\Big(\underbrace{\int_0^1 \ln(1+z^2)\,dz}_{i)} - \underbrace{\int_0^1 \ln z^2\,dz}_{ii)}\Big)$$

i) $\int_0^1 \ln(1+z^2)\,dz = z\ln(1+z^2)\Big|_{z=0}^{z=1} - 2\int_0^1 \frac{z^2}{1+z^2}\,dz = \ln 2 - 2 + 2\int_0^1\frac{1}{1+z^2}\,dz$

$= \ln 2 - 2 + 2\arctan z\Big|_{z=0}^{z=1}$

$= \ln 2 - 2 + \frac{\pi}{4}$

$u = \ln(1+z^2),\ v' = 1$
$u' = \frac{2z}{1+z^2},\ v = z$

$\frac{z^2}{1+z^2} = \frac{1+z^2-1}{1+z^2} = 1 - \frac{1}{1+z^2}$

ii) $\int_0^1 \ln z^2\,dz = 2\int_0^1 \ln z\,dz = 2\left(z\ln z\Big|_{z=0}^{z=1} - 1\right)$

$u = \ln z,\ v' = 1$
$u' = \frac{1}{z},\ v = z$

$= -2.$

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$\pi\left(-2 - \left(\ln 2 - 2 + \frac{\pi}{4}\right)\right) = \frac{\pi^2}{4} - \ln 2$