4.10.7: Egenverdiene til A

$$\begin{vmatrix} \lambda - a_{11} & -a_{12} & \cdots \\ -a_{21} & \lambda - a_{22} & \cdots \\ & & \end{vmatrix} =$$

Egenverdier til A^T:

$$\begin{vmatrix} \lambda - a_{11} & -a_{21} & \cdots \\ -a_{12} & \lambda - a_{22} & \cdots \\ & & \end{vmatrix} =$$

Like siden de er transponerte av hverandre. Altså like egenverdier.

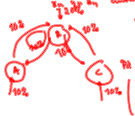
Generelt er egenvektorene ulike: $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$A\vec{v} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{v}$$

$$A^T\vec{v} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$ ikke egenvektor.

4.11.8 Tre aviser A, B, C:

x_n, y_n, z_n antall lesere



$x_{n+1} = x_n + 0.1y_n$

$y_{n+1} = 0.1x_n + y_n + 0.1z_n$

$z_{n+1} = 0.1y_n + z_n$

På matriseform

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0.1 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & 0.1 & 1 \end{pmatrix}}_{M}\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

Egenverdier:

$$\begin{vmatrix} \lambda - 1 & -0.1 & 0 \\ -0.1 & \lambda - 1 & -0.1 \\ 0 & -0.1 & \lambda - 1 \end{vmatrix}$$

$$= (\lambda - 1)\begin{vmatrix} \lambda - 1 & -0.1 \\ -0.1 & \lambda - 1 \end{vmatrix} + 0.1\begin{vmatrix} -0.1 & -0.1 \\ 0 & \lambda - 1 \end{vmatrix}$$

$$= (\lambda - 1)\left((\lambda-1)^2 - 0.01\right) - 0.01(\lambda - 1)$$ $\lambda_1 = 1$

$$= (\lambda - 1)\left[(\lambda-1)^2 - \underbrace{0.02}_{\frac{2}{100}}\right] \quad \boxed{= 0 \quad (\lambda-1)^2 = \frac{2}{100}}$$

$\lambda - 1 = \pm\frac{\sqrt{2}}{10} \Rightarrow \lambda = 1 \pm \frac{\sqrt{2}}{10}$

Egenvektor: λ_1, $\vec{v}_1 = $ [illegible] $M\vec{v}_1 = \lambda_1\vec{v}_1$

$$\begin{pmatrix} 1 & 0.1 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & 0.1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 1\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow$$

$x + 0.1y = x$, $0.1x + y + 0.1z = y$, $0.1y + z = z$ [illegible]

Får $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\lambda_2 = 1 + \frac{\sqrt{2}}{10}$, $\vec{v}_2 = $ [illegible] $M\vec{v} = \lambda_2\vec{v}$,

[illegible] $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(1 + \frac{\sqrt{2}}{10}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow$

$x + 0.1y = \left(1 + \frac{\sqrt{2}}{10}\right)x$

$0.1x + y + 0.1z = \left(1 + \frac{\sqrt{2}}{10}\right)y$

$0.1y + z = \left(1 + \frac{\sqrt{2}}{10}\right)z$

$0.1y = \frac{\sqrt{2}}{10}x \Rightarrow y = \sqrt{2}x$

[illegible]

Velger $x = 1$: $y = \sqrt{2}$, $z = 1$

$\lambda_3 = 1 - \frac{\sqrt{2}}{10}$ [illegible]

$\vec{v}_3 = \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

c) [illegible]

[illegible]

5.2.3: $\{I_n\}$ en følge av lukkede intervaller slik at

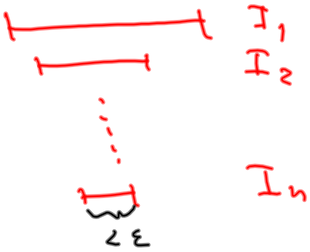
$I_{n+1} \subset I_n$ og lengde$(I_n) \to 0$.

Vis at det finnes ett og bare ett tall som ligger i alle I_n.

Galt uten betingelsen om lukkethet: $I_n = (0, \frac{1}{n})$.

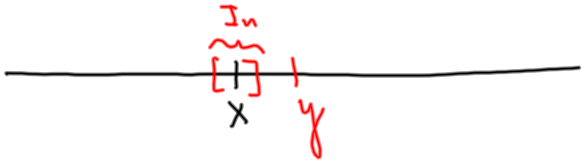
Bevis: $I_n = [a_n, b_n]$, la $x_n = \dfrac{a_n + b_n}{2}$.

Da er $\{x_n\}$ en Cauchy-følge og følgelig konvergerer den mot et punkt x.



x ligger i I_n fordi når $m \geq n$, så er $x_m \in I_m \subset I_n$

Siden I_n er lukket, må da $x \in I_n$



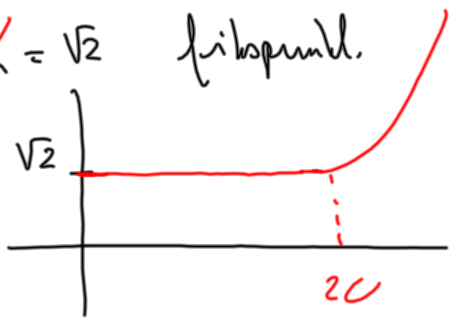
5.4.5:

$$x_{n+1} = 2.2 x_n (1 - x_n) + 0.01 x_n y_n$$

$$y_{n+1} = 3.1 y_n (1 - y_n) - 0.02 x_n y_n$$

5.4.7: $f(x) = x^2 + x - 2$

$f(\sqrt{2}) = (\cancel{\sqrt{2}})^2 + \sqrt{2} - \cancel{2} = \sqrt{2}$ fikspunkt.



$x_1 = \text{sqrt}(2)$

$x_{n+1} = f(x_n)$