2.7 nr 2: $f(u,v) = ue^{-v}$, $g(x,y,z) = 2xy+z$, $h(x,y,z) = 2y(z+x)$

$$= 2yz + 2yx$$

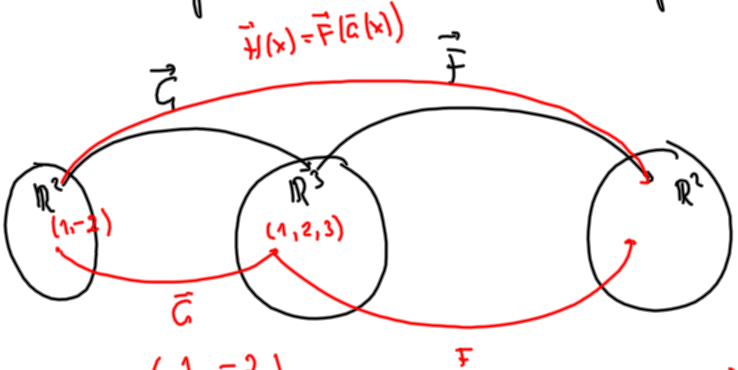
$$k(x,y,z) = f(g(x,y,z), h(x,y,z))$$

$$\frac{\partial k}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial g}{\partial x} + \frac{\partial f}{\partial v} \frac{\partial h}{\partial x}$$

$$= e^{-v} 2y + (-ue^{-v}) 2y = 2e^{-(2yz+2yx)} y - (2xy+z) e^{-(2yz+2yx)} 2y$$

$$= 2y e^{-(2yz+2yx)} (1 - 2xy - z)$$

2.7, nr 5:



Vet $\vec{G}'(1,-2) = \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}$, $\vec{F}'(1,2,3) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}$

Hva er $\vec{H}'(1,-2)$?

Kjerneregelen: $\underset{(1,-2)}{\vec{H}'(\vec{x})} = \vec{F}'(\underbrace{\vec{G}(\vec{x})}_{(1,2,3)}) \cdot \vec{G}'(\underset{(1,-2)}{\vec{x}})$

$$\vec{H}'(1,-2) = \underset{2\times 3}{\begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}} \underset{3\times 2}{\begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix}} = \underset{2\times 2}{\begin{pmatrix} 13 & -7 \\ 10 & 0 \end{pmatrix}}$$

2.7, nr 7: $E_1(p_1, p_2)$ Ved tiden t: $p_1(t)$, $p_2(t)$

$$e(t) = E_1(p_1(t), p_2(t))$$

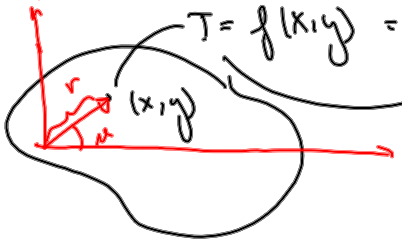
$$e'(t) = \frac{\partial E_1}{\partial p_1} p_1'(t) + \frac{\partial E_1}{\partial p_2} p_2'(t)$$

$$= \frac{\partial E_1}{\partial p_1} \frac{\partial p_1}{\partial t} + \frac{\partial E_1}{\partial p_2} \frac{\partial p_2}{\partial t}.$$

2.7, nr 8:

a)

$$T = f(x,y) = f(r\cos\theta, r\sin\theta)$$



$$\frac{\partial T}{\partial r} = \frac{\partial f}{\partial x} \cdot \frac{\partial}{\partial r}(r\cos\theta) + \frac{\partial f}{\partial y} \frac{\partial}{\partial r}(r\sin\theta) = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial f}{\partial x} \frac{\partial}{\partial \theta}(r\cos\theta) + \frac{\partial f}{\partial y} \frac{\partial}{\partial \theta}(r\sin\theta) = -\frac{\partial f}{\partial x} r\sin\theta + \frac{\partial f}{\partial y} r\cos\theta$$

b)

Temperaturen ved fuglen ved tiden t:

$$T(t) = f(g(t)\cos h(t), g(t)\sin h(t))$$

$$= T(\underset{r}{g(t)}, \underset{\theta}{h(t)})$$

$$T'(t) = \underbrace{\frac{\partial T}{\partial r}}\, g'(t) + \frac{\partial T}{\partial \theta} \cdot \underline{h'(t)}$$

$$= \left(\frac{\partial f}{\partial x}\cos(h(t)) + \frac{\partial f}{\partial y}\sin(h(t))\right) g'(t)$$

$$+ \left(-\frac{\partial f}{\partial x} g(t)\sin(h(t)) + \frac{\partial f}{\partial y} g(t)\cos(h(t))\right) h'(t)$$

1.9, nr 1 $\quad T: \mathbb{R}^3 \to \mathbb{R}^2$

Alt 1: $T(x,y,z) = \underline{\begin{pmatrix} 2x - y + z \\ -x + y - 3z \end{pmatrix}} = \underbrace{\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Alt 2: Første søyle i A:

$T(\vec{e}_1) = T(1,0,0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Andre søyle i A:

$T(\vec{e}_2) = T(0,1,0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Tredje søyle i A:

$T(\vec{e}_3) = T(0,0,1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}$

1.9, nr 3: $\quad \vec{T}: \mathbb{R}^2 \to \mathbb{R}^2 \quad \vec{T}(\vec{a}) = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \ \vec{T}(\vec{b}) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$T: \mathbb{R}^2 \to \mathbb{R}$

Hva er $\vec{T}(3\vec{a} - 2\vec{b})$?

Siden $\vec{T}$ er lineær, så

$\vec{T}(3\vec{a} - 2\vec{b}) = \vec{T}(3\vec{a}) - \vec{T}(2\vec{b}) = 3\vec{T}(\vec{a}) - 2\vec{T}(\vec{b})$

$= 3\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -6 \\ -3 \end{pmatrix}}}$

$f(x+y) = f(x) + f(y)$

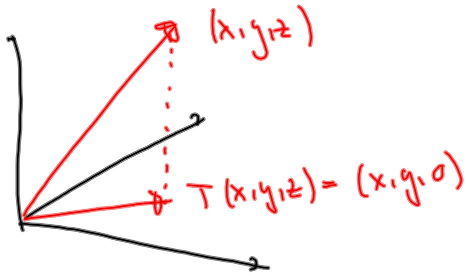
$f(ax) = af(x)$

1.9, nr 7: $T: \mathbb{R}^3 \to \mathbb{R}^3$

$T(\vec{e}_1) = T(1,0,0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

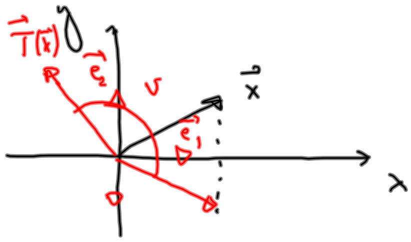
$T(\vec{e}_2) = T(0,1,0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$T(\vec{e}_3) = T(0,0,1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$



$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

1.9, nr 8: $T: \mathbb{R}^2 \to \mathbb{R}^2$ <
1. speile om x-aksen
2. rotere en vinkel ϑ



Operasjon 1: $S(\vec{e}_1) = S\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

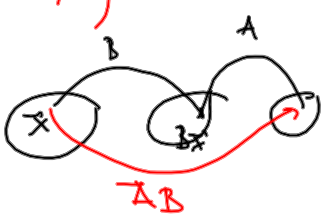
$S(\vec{e}_2) = S\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$$A_s = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Operasjon 2: $A_\vartheta = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix}$

$$A_T = A_\vartheta \cdot A_s = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \underline{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ \sin\vartheta & -\cos\vartheta \end{pmatrix}}$$



1.10, nr 1:

$$\vec F(x,y,z) = \begin{pmatrix} 2x-3y+z-7 \\ -x+z-2 \end{pmatrix}$$

$\vec b$

Affinabbildung

$$\vec F(\vec x) = \underbrace{A}_{\text{matrix}}\vec x + \underbrace{\vec b}_{\text{vektor}}$$

$$\vec F(x,y,z) = \begin{pmatrix} 2x-3y+z \\ -x+z \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \underbrace{\begin{pmatrix} 7 \\ 2 \end{pmatrix}}_{\vec b}$$

1.10, nr 2:

$$\vec r(t) = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2+t \\ -1 \\ 3+2t \end{pmatrix}$$

$$\vec F(x,y,z) = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\vec F(\vec r(t)) = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}\begin{pmatrix} 2+t \\ -1 \\ 3+2t \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+t+1+6+4t \\ -3-6-4t \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 9+5t \\ -9-4t \end{pmatrix} + \begin{pmatrix} 2 \\ -1. \end{pmatrix}$$

$$= \begin{pmatrix} 11+5t \\ -10-4t \end{pmatrix} = \begin{pmatrix} 11 \\ -10 \end{pmatrix} + t\begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

1.10, m 3: Affinabbildung $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$

$\vec{F}(0,0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\vec{F}(1,0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\vec{F}(0,1) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Generell: $\vec{F}(\vec{x}) = A\vec{x} + \vec{b} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

Vel

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \vec{F}(0,0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad \alpha = 1, \ \beta = -1$$

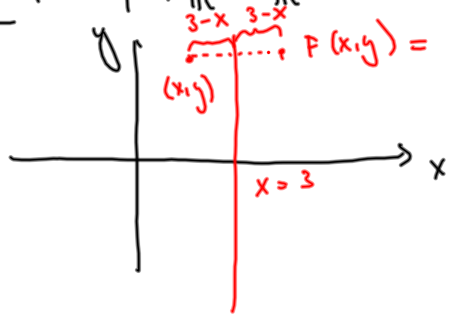
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \vec{F}(1,0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \vec{F}(0,1) = \begin{pmatrix} 1 & b \\ 4 & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad a = 1, \ c = 4$$

$$= \begin{pmatrix} b \\ d \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow b = -2, \ d = 1$$

1.10, w 5: $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$

a)



$$F(x,y) = (x + 2(3-x), y) = (6-x, y)$$

$$= \begin{pmatrix} -x \\ y \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{A}\begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{\begin{pmatrix} 6 \\ 0 \end{pmatrix}}_{\vec{b}}$$

A3, m 2: $(1, 2, 4, 8, 16, \dots, 4096) = (2^0, 2^1, 2^2, \dots, 2^{12})$

$$= 2.\wedge(0, 1, 2, \dots, 12)$$