UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Constituent exam in: MAT1110 — Kalkulus og lineær algebra

Day of examination: Fredag 30. mars 2012

Examination hours: 15.00 – 17.00

This problem set consists of 4 pages.

Appendices: Answer sheet, formelsamling.

Permitted aids: None.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

The exam consists of 15 questions. The 10 first count for 3 points each, while the 5 last count for 4 points each, so there is a total of 50 points. There is only one correct alternative for each question. If you give no answer or a wrong answer to a question, you will be given 0 points. If you check more than one alternative, you will be given 0 points.

Unfortunately there were 2 correct answers for Question 3. Any of those answers will be credited.

Question 1. (3 points) Let $F(x,y) = (x^2y, xy^4)$. The linearization of F at the point $(1,1)$ is:

A) $\mathbf{T}_{(1,1)}(\mathrm{x},\mathrm{y}) = (2,4) + (2\mathrm{x}+\mathrm{y}, \mathrm{x}+4\mathrm{y})$

B) $\mathbf{T}_{(1,1)}(\mathrm{x},\mathrm{y}) = (-2,4) + (2\mathrm{x}+\mathrm{y}, \mathrm{x}+4\mathrm{y})$

C) $\mathbf{T}_{(1,1)}(\mathrm{x},\mathrm{y}) = (-2,-4) + (2\mathrm{x}+\mathrm{y}, \mathrm{x}+4\mathrm{y})$ **CORRECT**

D) $\mathbf{T}_{(1,1)}(\mathrm{x},\mathrm{y}) = (-2,-4) + (2\mathrm{x}+\mathrm{y}, \mathrm{x}-4\mathrm{y})$

E) $\mathbf{T}_{(1,1)}(\mathrm{x},\mathrm{y}) = (-2,-4) + (2\mathrm{x}-\mathrm{y}, \mathrm{x}+4\mathrm{y})$

Question 2. (3 points) Let $R \subset \mathbb{R}^2$ be the rectangle $R = [1,3] \times [2,4]$, and let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the affine mapping defined by $F(x,y) = (1,3) + A(x,y)$, where A is the matrix

$$A = \begin{pmatrix} 2 & 7 \\ 3 & 1 \end{pmatrix}$$

Then the area of the image $F(R)$ is

(Continued on page 2.)

A) 76 **CORRECT**
B) 42
C) 67
D) 15
E) 64

Question 3. (3 points) Which conic section does the following equation define:

$$x^2 - 10x + y^2 - 6y + 30 = 0?$$

A) A circle **CORRECT**
B) An ellipse **CORRECT**
C) A parabola
D) A hyperbola
E) None

Question 4. (3 points) Let L be a linear mapping such that $L(5 \cdot \mathbf{e}_1) = (2, 4)$ and $L(\mathbf{e}_2) = (-1, 3)$, where $\mathbf{e}_1$ is the vector $\mathbf{e}_1 = (1, 0)$ and $\mathbf{e}_2$ is the vector $\mathbf{e}_2 = (0, 1)$. Then the matrix for L is:

A) $\begin{pmatrix} 2/5 & -1 \\ 4/5 & 3 \end{pmatrix}$ **CORRECT**
B) $\begin{pmatrix} 2/5 & 4/5 \\ -1 & 3 \end{pmatrix}$
C) $\begin{pmatrix} 2/5 & 3 \\ -1 & 4/5 \end{pmatrix}$
D) $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$
E) $\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$

Question 5. (3 points) Let C be the curve in $\mathbb{R}^2$ parametrized by $\mathbf{r}(\mathrm{t}) = (2\mathrm{t}^2, \sin(\mathrm{t}))$, $t \in [1, 7]$. Then the acceleration $\mathbf{a}(\mathrm{t})$ is given by:

A) 7-1=6.
B) $\sqrt{16 + sin^2(t)}$
C)$(t, cos(t))$
D) $(4, sin(t))$
E) $(4, -sin(t))$ **CORRECT**

Question 6. (3 points) Let R be the rectangle $R = [0, 1] \times [0, 1]$ and let $f(x, y) = x^3y + 5xy^2$. Then $\int\int_R f(x, y)dxdy$ is equal to

A) 1/2
B) 24/23
C) 23/24 **CORRECT**
D) 1/7
E) 0

(Continued on page 3.)

Question 7. (3 points) Let R be the rectangle $R = [1,3] \times [1,3]$ and let $f(x,y) = 2x + 5y$. The are of the graph $\{(x,y,z) : z = f(x,y)\}$ over R is

A) $4\sqrt{15}$.

B) 4

C)$4\sqrt{30}$.**CORRECT**

D) $4\sqrt{25}$

E) 10.

Question 8. (3 points) Let $A \subset \mathbb{R}^2$ be the domain bounded by the x-axis and the graph $y = \sqrt{1-x^2}$. The integral $\int\int_A x^2 y$ is equal to:

A) 1
B) 2/15 **CORRECT**
C) 2
D) 1/7
E) 0

Question 9. (3 points) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a mapping such that $F((0,0)) = (0,0)$ and

$$F'(0,0) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Let $g : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $g'(0,0) = (2,3)$. Then the derivative of the composed function $h(x,y) = g(F(x,y))$ at the origin is equal to

A) $(1,2)$
B) $(11,16)$ **CORRECT**
C) $(0,0)$
D) $(12,14)$
E) $(13,13)$

Question 10. (3 points) Let A be the domain in $\mathbb{R}^2$ such that $x \geq 0, y \geq e^x$, and $y \leq 2e^{-x}$. The integral $\int\int_A y\,dxdy$ is equal to:

A) 1/4 **CORRECT**
B) 1/2
C) 1/3
D) 0
E) $-1/3$

Question 11. (4 points) Let $f(x,y) = x^2y + 5xy^2$ and let S be the graph of f in $\mathbb{R}^3$. The tangent plane to S at the point $(1,1,f(1,1))$ is defined by :

A) $z = 0$
B) $z = 12 + 7x + 11y$

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C) $z = -12 + 7x + 11y$ **CORRECT**
D) $z = -12 + 11x + 7y$
E) $z = -12 + 11x - 7y$

Question 12. (4 points) Let C be the curve in $\mathbb{R}^2$ parametrized by $\mathbf{r}(\mathrm{t}) = (\mathrm{t}^2, \mathrm{t}^3), \mathrm{t} \in [0, 2]$, The arc length of C is equal to:

A) $(1/27)((40)^{3/2} - 8)$ **CORRECT**
B) $(1/54)(40)^{3/2}$
C) $(2/54)((40)^{2/3} + 8)$
D) $2(40)^{3/2}$
E) 1

Question 13. (4 points) Let $C \subset \mathbb{R}^2$ be the curve parametrized by $\mathbf{r}(\mathrm{t}) = (\cos(\mathrm{t}), 3\sin(\mathrm{t})), \mathrm{t} \in [0, \pi/2]$, and let f be the function $f(x, y) = xy$. The integral $\int_C f ds$ is equal to

A) 28/3
B) 15/3
C) 26/8 **CORRECT**
D) $\pi/3$
E) 2π

Question 14. (4 points) Let C be the same curve as in the previous question, let $\phi(x, y) = x^2 + cos(xy)$, and let F be the vector field $F = \nabla\phi$ (the gradient of ϕ). Then $\int_C F \cdot dr$ is equal to :

A) 1/3

B) π

C) 1/5

D) 2π

E) -1 **CORRECT**

Question 15. (4 points) Let $C \subset \mathbb{R}^2$ be the ellipse $C = \{(x, y) : (\frac{x}{a})^2 + (\frac{y}{b})^2 = 1\}$ and let A be the domain bounded by C. Then the area of A is equal to

A) a
B) b
C) $\frac{1}{2}\int_C xdy - ydx$ **CORRECT**
D)$\frac{1}{2}\int_C xdx - ydy$
E) $\int\int_A xydxdy$

END