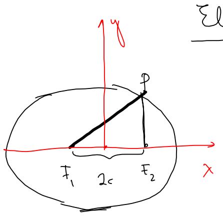
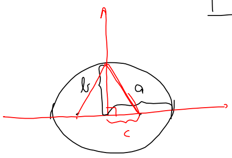
Ellipser



$|PF_1| + |PF_2| = 2a \quad : \quad a > c$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}, \quad b = \sqrt{a^2 - c^2}$$

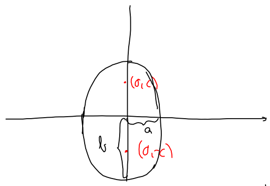
$$b^2 + c^2 = a^2$$



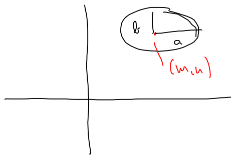
Hva skjer med ligningen

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{viss } b > a?$$

$$a^2 + c^2 = b^2$$



Annet sentrum.



$$\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1$$

Eksempel: $9x^2 + 4y^2 - 54x + 16y + 61 = 0$ Hva slags kurve er dette?

$$9x^2 - 54x + 4y^2 + 16y = -61$$

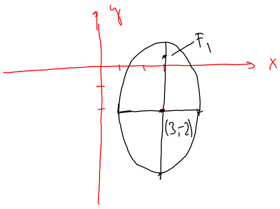
$$9\underbrace{(x^2 - 6x + 9)}_{(x-3)^2} + 4\underbrace{(y^2 + 4y + 4)}_{(y+2)^2} = -61 + 81 + 16$$

$$9(x-3)^2 + 4(y+2)^2 = 36 \quad | :36$$

$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{9} = 1$$

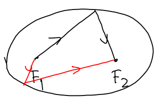
$$\frac{(x-3)^2}{2^2} + \frac{(y+2)^2}{3^2} = 1$$

Sentrum: $(3,-2)$
Halvakser: $a = 2,\ b = 3$
Brennvidden: $c = \sqrt{b^2 - a^2} = \sqrt{9-4} = \sqrt{5}$

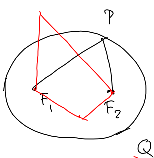


Brennpunkter: $F_1 = (3, -2+\sqrt{5})$,
$F_2 = (3, -2-\sqrt{5})$.

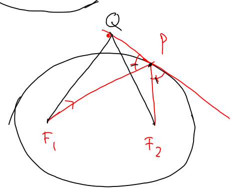
Refleksjonsegenskap



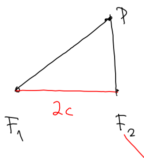
En stråle som sendes ut fra det ene brennpunktet, reflekteres gjennom det andre.



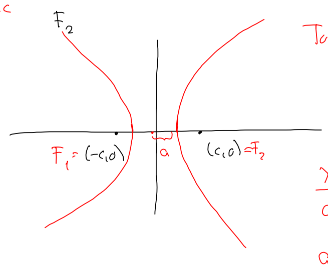
$|PF_1| + |PF_2| = 2a$ på ellipsen
$|PF_1| + |PF_2| > 2a$ utenfor ellipsen.
$|PF_1| + |PF_2| < 2a$ innenfor -"-



Hyperbler



$$\big|\,|PF_1| - |PF_2|\,\big| = 2a$$



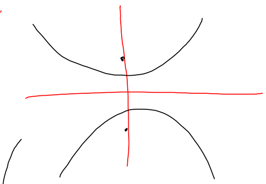
To hyperbelgrener.

Ligning:

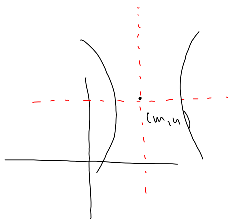
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$a^2 + b^2 = c^2, \quad b = \sqrt{c^2 - a^2}$$

Hva hvis: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$?



Flytting:



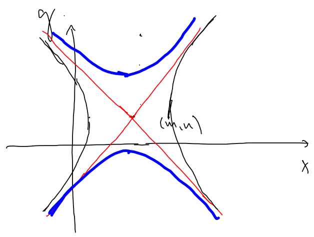
$$\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1$$

Asymptoter: Hyperblene

$$\frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1 \quad \text{og} \quad \frac{(y-n)^2}{b^2} - \frac{(x-m)^2}{a^2} = 1$$

har asymptotene

$$y - n = \pm \frac{b}{a}(x-m)$$



Eksempel: $-x^2 + 2y^2 + 2x - 8y - 1 = 0$

$$-x^2 + 2x + 2y^2 - 8y = 1$$

$$-\underbrace{(x^2 - 2x + 1)}_{(x-1)^2} + 2\underbrace{(y^2 - 4y + 4)}_{(y-2)^2} = 1 - 1 + 8$$

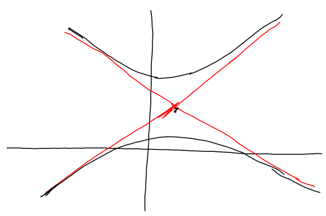
$$-(x-1)^2 + 2(y-2)^2 = 8 \quad |:8$$

$$\frac{(y-2)^2}{4} - \frac{(x-1)^2}{8} = 1$$

$$\frac{(y-2)^2}{2^2} - \frac{(x-1)^2}{(2\sqrt{2})^2} = 1$$

Hyperbel med sentrum i $(1,2)$, og $a = 2\sqrt{2}$, $b = 2$

Liggende.



$$\pm\frac{b}{a} = \pm\frac{2}{2\sqrt{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$