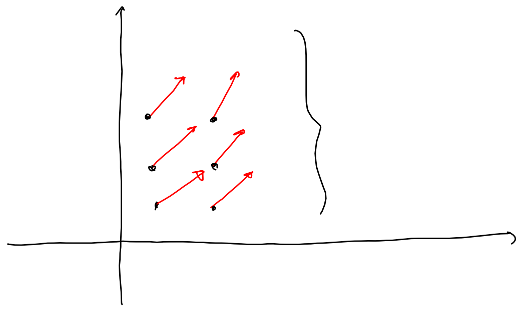
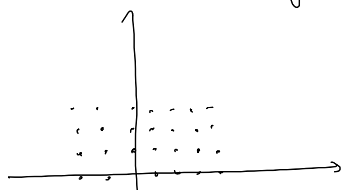
Grafiske fremstillinger av vektorfelt (3.8)

Vektorfelt: $\vec{F}:\mathbb{R}^n\to\mathbb{R}^n$, $\vec{F}:\mathbb{R}^2\to\mathbb{R}^2$, $\vec{F}:\mathbb{R}^3\to\mathbb{R}^3$



$\vec{F}(x,y)$ = vind i (x,y)

MATLAB: $\vec{F}(x,y)=x\cdot y\,\vec{i}+e^{x+y}\vec{j}$



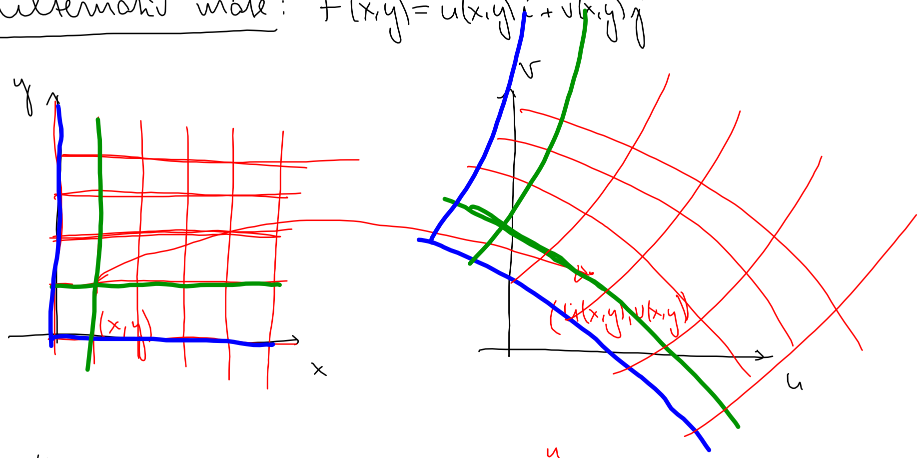
x = -5:0.5:5 u = ~~ (fartkomponenten til F)

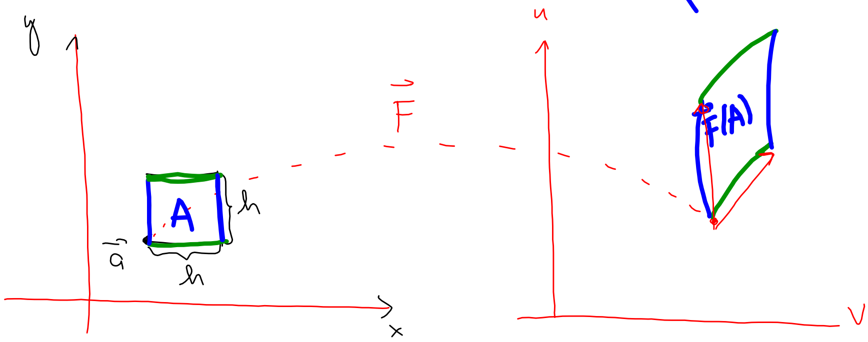
y = -5:0.5:5 v = ~ (annen — " —)

[x,y] = meshgrid(x,y) quiver(x,y,u,v) u = x.*y

v = exp(x+y)

Alternativ måte: $\vec{F}(x,y)=u(x,y)\vec{i}+v(x,y)\vec{j}$





Hva skjer med arealet: Hvordan er areal($\vec{F}(A)$) sammenlignet med areal(A)?

Svar: Hvis h er liten, vil

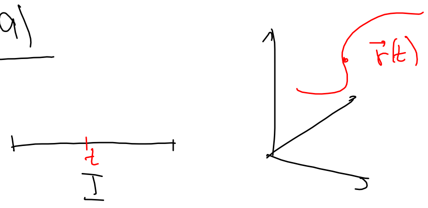
$$\text{areal}(\vec{F}(A))\approx|\det(\vec{F}'(a))|\,\text{areal}(A)$$

Moral: $|\det\vec{F}'(a)|$ er arealforstørrelsesfaktor i området rundt $\vec{a}$

Parametriserte flater (3.9)

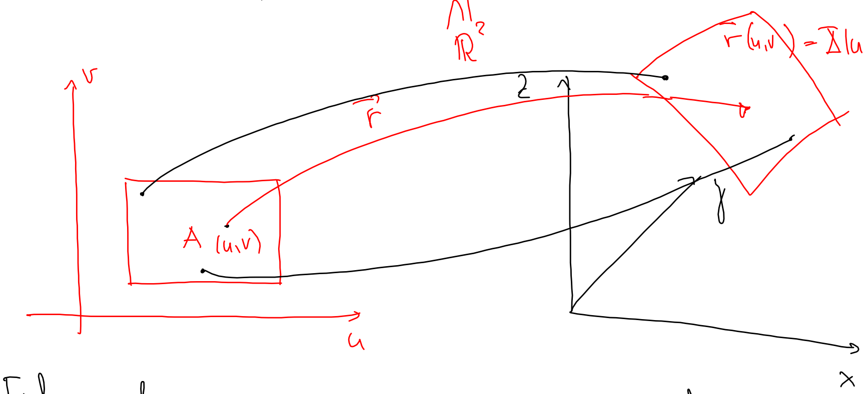
Parametrisert kurve: $\vec r : I \to \mathbb{R}^3$

$I \subseteq \mathbb{R}$



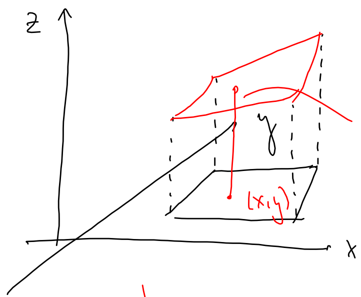
Parametrisert flate: $\vec r : A \to \mathbb{R}^3$

$A \subseteq \mathbb{R}^2$



$\vec r(u,v) = X(u,v)\vec i + Y(u,v)\vec j + Z(u,v)\vec k$

Eksempel:

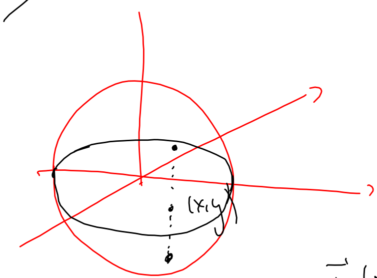


$z = f(x,y)$

$z = f(x,y)$

$\vec r(x,y) = x\vec i + y\vec j + f(x,y)\vec k$

Tröttel.



$x^2 + y^2 + z^2 = R^2$

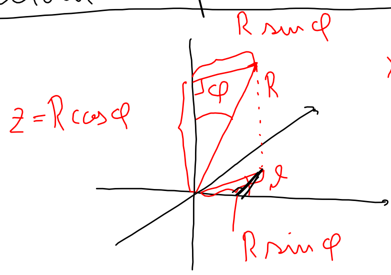
$z^2 = R^2 - x^2 - y^2$

$z = \pm\sqrt{R^2 - x^2 - y^2}$

$\vec r(x,y) = x\vec i + y\vec j + \sqrt{R^2 - x^2 - y^2}\,\vec k$

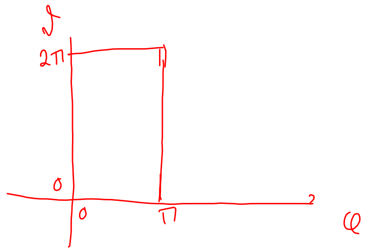
(øvre halvkule)

Alternativ parametrisering

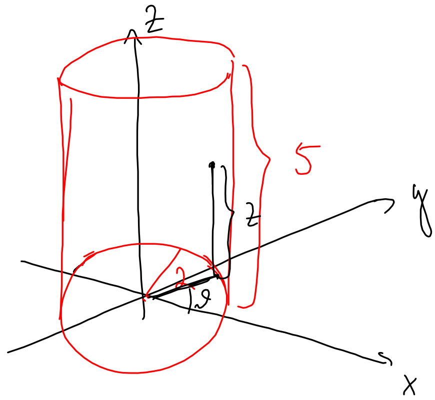


$x = R\sin\varphi\cos\theta \qquad y = R\sin\varphi\sin\theta$

$\vec r(\varphi,\theta) = R\sin\varphi\cos\theta\,\vec i + R\sin\varphi\sin\theta\,\vec j + R\cos\varphi\,\vec k$



$0 \le \varphi \le \pi, \quad 0 \le \theta \le 2\pi$

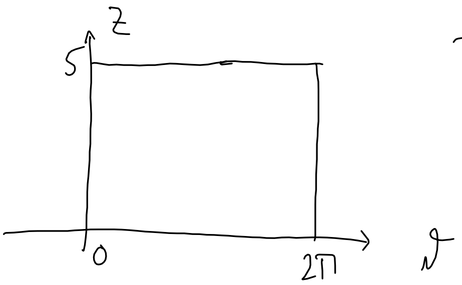


Parametrisering:

$x = 2\cos\vartheta \qquad 0 \le \vartheta \le 2\pi$

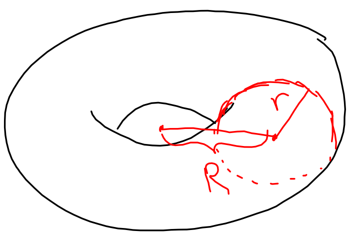
$y = 2\sin\vartheta \qquad 0 \le z \le 5$

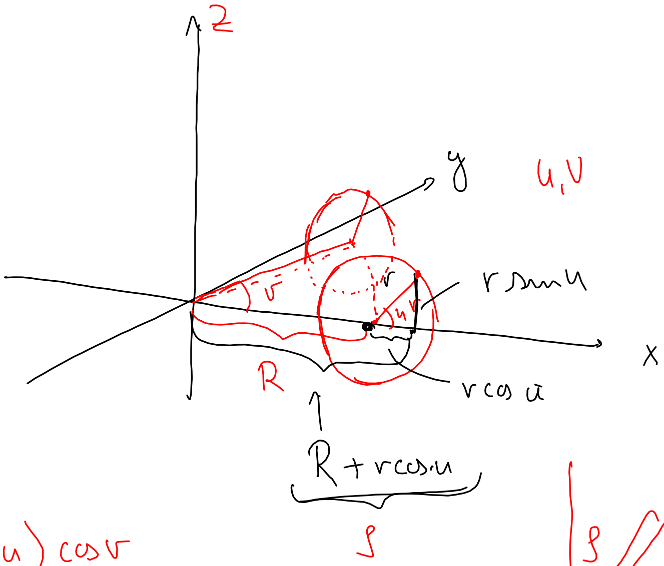
$z = z$

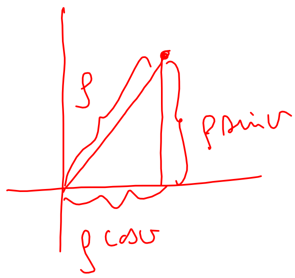


$$\vec{r}(\vartheta, z) = 2\cos\vartheta\,\vec{i} + 2\sin\vartheta\,\vec{j} + z\,\vec{k}$$

Torus (overflaten til en smultring)







$z(u,v) = r\sin u$

$x(u,v) = (R + r\cos u)\cos v$

$y(u,v) = (R + r\cos u)\sin v$

$$\vec{r}(u,v) = (R + r\cos u)\cos v\,\vec{i} + (R + r\cos u)\sin v\,\vec{j} + r\sin u\,\vec{k}$$

$0 \le u \le 2\pi$

$0 \le v \le 2\pi$

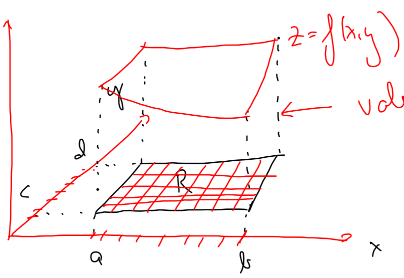
Kapittel 6

Ombytte: Onsdag: forelesning

Neste uke
Mandag: Plenum
Onsdag: Forelesning

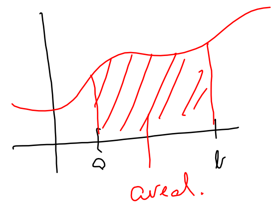
Dobbeltintegraler:

Idé:



$z = f(x,y)$

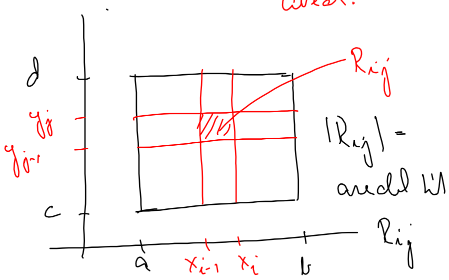
Sammenlign



Π er en partisjon av R

$a = x_0 < x_1 < x_2 < \dots < x_n = b$

$c = y_0 < y_1 < y_2 < \dots < y_n = d$



$|R_{ij}| =$ arealet til R_{ij}

$m_{ij} = \inf\{f(x,y) : (x,y) \in R_{ij}\}$

$M_{ij} = \sup\{f(x,y) : (x,y) \in R_{ij}\}$

$N(\Pi) = \sum_{i,j} m_{ij} |R_{ij}|$ nedre trappesum

Bør ha:

$N(\Pi) \leq V \leq \text{Ø}(\Pi)$

$\text{Ø}(\Pi) = \sum_{i,j} M_{ij} |R_{ij}|$ øvre trappesum.

Nedre integral: $\underline{\iint}_R f(x,y)\,dx\,dy = \sup\{N(\Pi) : \Pi \text{ en partisjon}\}$

Øvre integral: $\overline{\iint}_R f(x,y)\,dx\,dy = \inf\{\text{Ø}(\Pi) : \Pi \text{ en partisjon}\}$

Definisjon: Hvis $\underline{\iint}_R f(x,y)\,dx\,dy = \overline{\iint}_R f(x,y)\,dx\,dy$, så sier vi at f er integrerbar over R og i så fall definerer vi (dobbelt)integralet til f over R ved

$$\iint_R f(x,y)\,dx\,dy = \underline{\iint}_R f(x,y)\,dx\,dy = \overline{\iint}_R f(x,y)\,dx\,dy$$