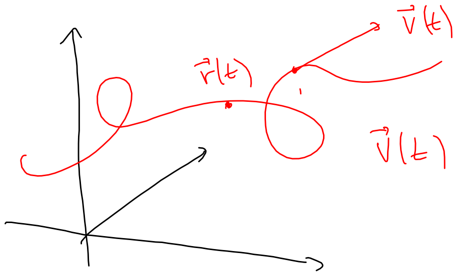
Parametriserte kurver



$$\vec{r}(t) = (x_1(t), x_2(t), \dots, x_n(t))$$

$$\vec{v}(t) = \vec{r}\,'(t) = (x_1'(t), x_2'(t), \dots, x_n'(t)) \quad \text{hastighet}$$

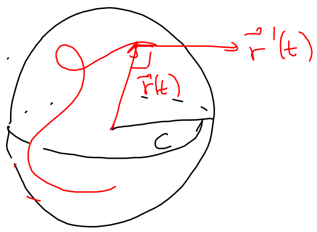
<u>Setning</u>: Hvis $|\vec{r}(t)|$ er konstant, så står $\vec{r}(t)$ og $\vec{r}\,'(t)$ normalt på hverandre.

<u>Bevis</u>: La $C = |\vec{r}(t)|$, da $C^2 = |\vec{r}(t)|^2 = \underbrace{\vec{r}(t)\cdot\vec{r}(t)}$. Derivér:

$$0 = \vec{r}\,'(t)\cdot\vec{r}(t) + \vec{r}(t)\cdot\vec{r}\,'(t) = 2\,\vec{r}(t)\cdot\vec{r}\,'(t)$$

som viser $\vec{r}(t) \perp \vec{r}\,'(t)$.

<u>Geometrien:</u>



$\vec{r}(t)$ <u>posisjon</u>, $\vec{v}(t) = \vec{r}\,'(t)$ <u>hastighet</u>

$v(t) = |\vec{v}(t)|$ <u>farten</u>.

<u>Akselerasjon</u>: $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

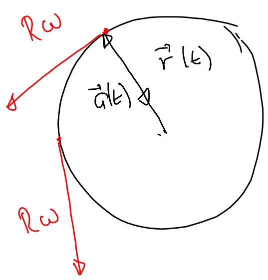
<u>Baneakselerasjon</u>: $a(t) = v'(t)$

Vanligvis $a(t) \neq |\vec{a}(t)|$!

<u>Eksempel</u>: $\vec{r}(t) = R\cos\omega t\,\vec{i} + R\sin\omega t\,\vec{j}$

$$\vec{v}(t) = \vec{r}\,'(t) = -R\omega\sin\omega t\,\vec{i} + R\omega\cos\omega t\,\vec{j}$$

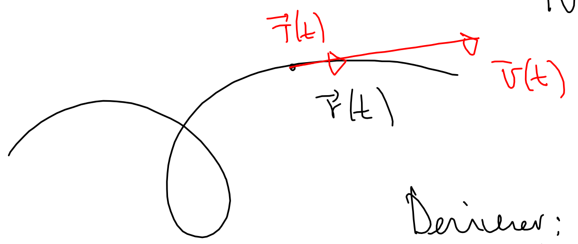
$$v(t) = \sqrt{(-R\omega\sin\omega t)^2 + (R\omega\cos\omega t)^2} = R\omega\sqrt{\sin^2\omega t + \cos^2\omega t} = R\omega.$$



$$a(t) = v'(t) = 0$$

$$\vec{a}(t) = (\vec{v}(t))' = -R\omega^2\cos\omega t\,\vec{i} - R\omega^2\sin\omega t\,\vec{j} = -\omega^2\,\vec{r}(t).$$

Enhetstangentvektor: $\vec T(t) = \dfrac{\vec v(t)}{|\vec v(t)|}$ forutsatt at $\vec v(t) \neq 0$.

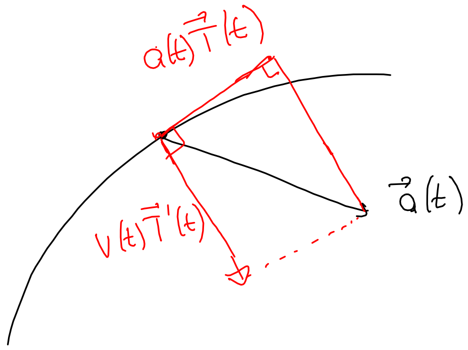


$$\vec v(t) = |\vec v(t)|\vec T(t) = v(t)\vec T(t)$$

Deriverer:

$$\vec a(t) = \vec v'(t) = \underbrace{v'(t)}_{a(t)}\vec T(t) + v(t)\underbrace{\vec T'(t)}_{\substack{\perp \vec T(t) \\ \perp \vec v(t)}}$$

$$= \underline{a(t)\vec T(t)} + \underbrace{v(t)\vec T'(t)}_{\perp v(t)}$$



3.2 Kjerneregelen for parametriserte kurver

$T(x,y,z)$ = temperaturen i punktet (x,y,z)

$\vec{r}(t)$ = flyets posisjon ved tiden t.

$$h(t) = T(\vec{r}(t)) = T(x(t),y(t),z(t))$$

Endringen i opplevd temperatur: $h'(t)$:

$$h'(t) = \frac{\partial T}{\partial x}x'(t) + \frac{\partial T}{\partial y}y'(t) + \frac{\partial T}{\partial z}z'(t)$$

$$= \nabla T(\vec{r}(t))\cdot \vec{r}\,'(t)$$

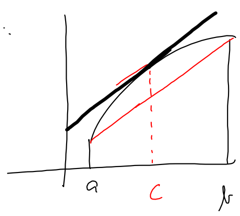
$$\left(\frac{\partial T}{\partial x}(\vec{r}(t)), \frac{\partial T}{\partial y}(\vec{r}(t)), \frac{\partial T}{\partial z}(\vec{r}(t))\right)\cdot\left(x'(t), y'(t), z'(t)\right)$$

Setning. Anta at $\vec{r}(t)$ er en deriverbar, parametrisert kurve og at f er en deriverbar funksjon. Da er den deriverte til den sammensatte $h(t) = f(\vec{r}(t))$ lik

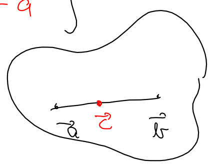
$$h'(t) = \nabla f(\vec{r}(t))\cdot\vec{r}\,'(t)$$

Middelverdisetning

Husk:



$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



Middelverdisetningen i flere variable: Anta at $f:\mathbb{R}^n\to\mathbb{R}$ er en deriverbar funksjon og at $\vec{a},\vec{b}\in\mathbb{R}^n$. Da finnes det et punkt $\vec{c}$ på linjestykket mellom $\vec{a},\vec{b}$ slik at

$$f(\vec{b}) - f(\vec{a}) = \nabla f(\vec{c})\cdot(\vec{b}-\vec{a})$$

Bevis: $\vec{r}(t) = \vec{a} + t(\vec{b}-\vec{a})$, $t\in[0,1]$

$$g(t) = f(\vec{r}(t)), \quad \boxed{g'(c) = \frac{g(1)-g(0)}{1-0} = g(1)-g(0)}$$

$$g'(t) = \nabla f(\vec{r}(t))\cdot\vec{r}\,'(t) = \nabla f(\vec{r}(t))\cdot(\vec{b}-\vec{a})$$

$$\nabla f(\underbrace{\vec{r}(c)}_{\vec{c}})\cdot(\vec{b}-\vec{a}) = g'(c) = g(1)-g(0) = f(\vec{b}) - f(\vec{a})$$