Extra (See also 1.3.15 in EP)

We wish to solve

(*) $\frac{dy}{dx} = \sqrt{x-y}$

The "natural" substitution $v = x - y$ will do,

but

(**) $u = \sqrt{x-y}$, $\frac{du}{dx} = \frac{1-y'}{2\sqrt{x-y}} = \frac{1-y'}{2u}$, $x-y>0$,

leads to less calculations. Thus (**) yields

$$2u\frac{du}{dx} = 1 - y' = 1 - u$$

$$\frac{2u}{1-u}\,du = dx, \quad 2\frac{(u-1)+1}{1-u}\,du = dx,$$

or $2\int\left(-1+\frac{1}{1-u}\right)du = x + C,$

$$2(-u - \ln|1-u|) = x + C$$

$$-2\sqrt{x-y} - \ln(1-\sqrt{x-y})^2 = x + C$$

or

(***) $\underline{x + 2\sqrt{x-y} + \ln(1-\sqrt{x-y}) = K}$ $(K = -C)$

in which y is given <u>implicit</u> as a function of x.