

Iterative method SIMPLE on Eq. form (3)
 Guess (u^*, p^*) , set $u^{k-1} = u^*$

1) $(u^{k-1} \cdot \nabla) u^* = \nabla \cdot v \nabla u^* - \nabla p^* + s$ ← can be solved for u^* , but with wrong p^*
 \downarrow u^* does not satisfy $\nabla \cdot u^* = 0$

Correct p gives correct u , correct eq:

2) $(u^{k-1} \cdot \nabla) u = \nabla \cdot v \nabla u - \nabla p + s$

2) - 1) =)

3) $(u^{k-1} \cdot \nabla)(u - u^*) = \nabla \cdot v \nabla (u - u^*) - \nabla(p - p^*)$

Define

$$u = u^* + u'$$

$$p = p^* + p'$$

3) $(u^{k-1} \cdot \nabla) u' = \nabla \cdot v \nabla u' - \nabla p'$

$$\underbrace{[(u^{k-1} \cdot \nabla) - \nabla \cdot v \nabla]}_{\text{Matrix } A} u' = - \nabla p'$$

Matrix A

$$A = d - H$$

$\begin{matrix} a_p & \bar{z}_{ab} \\ \uparrow & \\ \text{diag} & - \text{off-diag} \\ a_p & \bar{z}_{ab} \end{matrix}$

$$A u' = - G p'$$

$$(d - H) u' = - G p'$$

SIMPLE

$$\Rightarrow H = 0$$

$$u' = - \frac{\nabla p'}{d}$$

$$u' = - \frac{G}{d} p'$$

Very fast, cheap comp.
 $u' = - A^{-1} G p'$
 A^{-1} expensive

You have $U = U^* + U'$

and you want $\nabla \cdot U = 0$

$\Rightarrow \nabla \cdot (U^* - \frac{1}{d} \nabla p')$

$U^{k-1} = U^*$

$\nabla \cdot \frac{\nabla p'}{d} = \nabla \cdot U^*$

Poisson eq for P
 $\nabla \cdot (\nabla P)$

Solve for p'

Update $P = P^* + P'$

$U = U^* - \frac{\nabla p'}{d}$

$U^k = U$
 $P^* = P$

Check if convergence? (P' small)



Omission of off-diagonal entries^H does not affect final solution, since $U' \rightarrow 0$
 $P' \rightarrow 0$

Underrelaxation is often required for stable convergence (as well as good initial guess)

$U^{new} = \alpha U + (1-\alpha)U^{k-1}$

$P^{new} = \alpha_p P + (1-\alpha_p)P^{k-1}$

SIMPLE Algorithm

Guess u^* , p^* , set $u^{k+1} = u^*$

Solve mom. eq. for u^*

Solve ^{simplified} pressure correction for p'
(Source $\sim \nabla \cdot (u^*)$)

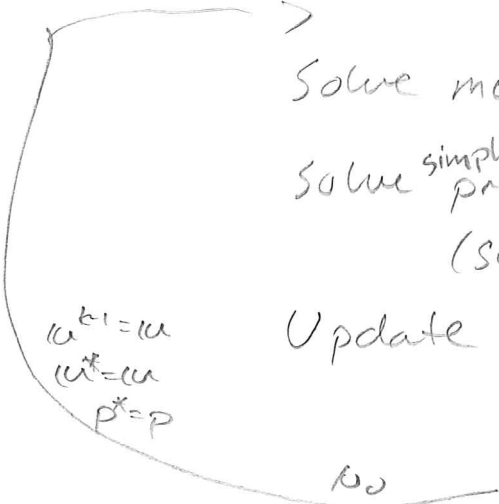
Update $u = u^* - \frac{\Delta}{a} p'$

$p = p^* + p'$

$u^{k+1} = u$
 $u^* = u$
 $p^* = p$

No

Convergence?
Yes



SIMPLER

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$$(1) \quad (d - H)u = -Gp + b$$

$$a_p - \sum a_{nb}$$

d diagonal
 H is off-diag.
 (a_{nb})

Guess u^*, p^* , compute d, H, b

$$d\hat{u} = +Hu^* + b$$

← Neglecting ∇p^*

$$\hat{u} = \frac{+Hu^* + b}{d}$$

Define pseudo-velocities

Cheap solve for \hat{u}

simplified (1)

$$d u = +Hu^* + b - Gp$$

\hat{u} gives a pressure estimate

$$u = \hat{u} - \frac{G}{d} p$$

$$\nabla \cdot u = 0$$



cheap

$$\Rightarrow \nabla \cdot \frac{G}{d} p = \nabla \cdot \hat{u}$$

Solve for p

Now you have initial guesses for u^* , $p^* = p$ as well

Solve momentum eq. (1) for new u^*

$$(d - H)u^* = -Gp^* + b$$

$$\downarrow u^*$$

Use SIMPLE approx for correction

(7)

$$u' = -\frac{\Delta}{d} p' \quad \nabla \cdot u = \nabla \cdot (u^* + u') = 0$$

Solve $\nabla \cdot \frac{\Delta}{d} p' = \nabla \cdot u^*$ for p'

Update $p = p^* + p'$

$$u = u^* + \frac{\Delta}{d} p'$$

$p^* = p$
 $u^* = u$

No

Converged?

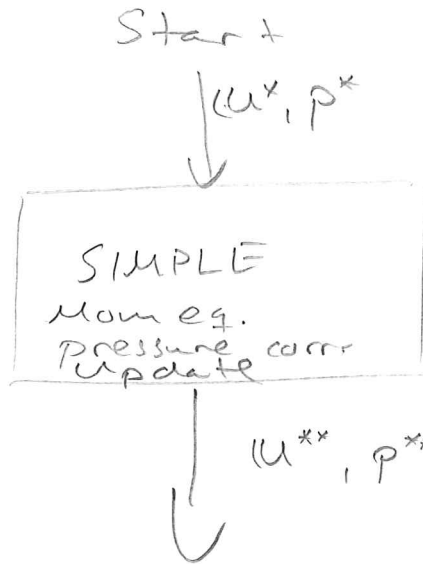
Yes

PISO

(8)

Two correction steps.

First is SIMPLE



u^{**} satisfy cont! but not momentum

2nd correction

1) New momentum eq: $d u^{***} = H u^{**} - G p^{***}$ twice corrected

2) First corr: $d u^{**} = H u^* - G p^{**}$

1) - 2) $\Rightarrow u^{***} = u^{**} + \frac{H(u^{**} - u^*)}{d} - \frac{G}{d} p''$, $p'' = p^{***} - p^{**}$

$$\nabla \cdot u^{***} = \cancel{\nabla \cdot u^{**}} + \nabla \cdot \frac{H(u^{**} - u^*)}{d} - \nabla \cdot \frac{G}{d} p'' = 0$$

Solve for p''

Update u^{***}

check for corr.

$u^* = u^{***}$

First correction eq:

1) $(d - H)u^* = -G p^*$

2) $d u^{**} = d u^* - G p^*$

1) + 2)

$\Rightarrow d u^{**} = H u^* - G p^*$

$(p^* = p^* + p'')$

This is how first corr. eq. is derived