Iterative method SIMPLE on Eq. form (3)

Guess u^*, p^*, set $u^{k-1} = u^*$

1) $(u^{k-1} \cdot \nabla) u^* = \nabla \cdot \nu \nabla u^* - \nabla p^* + S$ ← can be solved for u^*, but with wrong p^*

$\downarrow$ ny u^* does not satisfy $\nabla \cdot u^* = 0$

Correct p gives correct u, Correct eq:

2) $(u^{k-1} \cdot \nabla) u = \nabla \cdot \nu \nabla u - \nabla p + S$

2) − 1) ⇒

3) $(u^{k-1} \cdot \nabla)(u - u^*) = \nabla \cdot \nu \nabla (u - u^*) - \nabla (p - p^*)$

Define

$$u = u^* + u'$$
$$p = p^* + p'$$

3) $(u^{k-1} \cdot \nabla) u' = \nabla \cdot \nu \nabla u' - \nabla p'$

$$\underbrace{\left[(u^{k-1} \cdot \nabla) - \nabla \cdot \nu \nabla\right]}_{\text{Matrix } A} u' = -\nabla p'$$

$$A u' = -G p'$$

$$A = \underset{\substack{\text{diag} \\ a_p}}{\overset{a_p}{d}} - \underset{\substack{\text{off-diag} \\ \Sigma a_{nb}}}{\overset{\Sigma a_{nb}}{H}}$$

$$(d - H) u' = -G p'$$

SIMPLE

$\Rightarrow H = 0$

or $u' = -\dfrac{\nabla p'}{d}$ $\quad \boxed{u' = -\dfrac{G}{d} p'}$ ← Very fast, cheap comp.

$u' = -A^{-1} G p$

A^{-1} expensive

You have $\mathbf{u} = \mathbf{u}^* + \mathbf{u}'$

and you want $\nabla \cdot \mathbf{u} = 0$

$$\Rightarrow \quad \nabla \cdot \left(\mathbf{u}^* - \frac{1}{d}\nabla p'\right) = 0$$

$\mathbf{u}^{k-1} = \mathbf{u}^*$

$$\nabla \cdot \frac{\nabla}{d} p' = \nabla \cdot \mathbf{u}^* \quad \leftarrow \quad \text{Poisson eq for } p \quad \nabla\cdot(a\nabla p)$$

Solve for p'

Update
$$p = p^* + p'$$

$$\mathbf{u} = \mathbf{u}^* - \frac{\nabla}{d} p'$$

$\mathbf{u}^* = \mathbf{u}$
$p^* = p$

Check if convergence? (p' small)

No / yes

fin

Omisson of off-diagonal entries in H, does not affect final solution, since $\mathbf{u}' \to 0$, $p' \to 0$

Underrelaxation is often required for stable convergence (as well as good initial guess)

$$\mathbf{u}^{new} = \alpha \mathbf{u} + (1-\alpha)\mathbf{u}^{k-1}$$

$$p^{new} = \alpha_p p + (1-\alpha_p) p^{k-1}$$

SIMPLE Algorithm

Guess u^*, p^*, set $u^{k-1} = u^*$

Solve mom. eq. for u^*

Solve simplified pressure correction for p'

($\text{Source} \sim \nabla \cdot u^*$)

$u^{k-1} = u$
$u^* = u$
$p^* = p$

Update $u = u^* - \frac{\Delta}{d} \cdot p'$

$p = p^* + p'$

no

Convergence?

yes

SIMPLER

(1) $(d - H)\mathbb{u} = -Gp + b$

$a_p - \Sigma a_{nb}$

, d diagonal
H is off-diag.
(a_{nb})

Guess $\mathbb{u}^*$, p^*, compute d, H, b

$d\hat{\mathbb{u}} = +H\mathbb{u}^* + b$ ⇐ Neglecting ∇p^*

$\hat{\mathbb{u}} = \dfrac{+H\mathbb{u}^* + b^*}{d^*}$ Define pseudo-velocities

Cheap solve for $\hat{\mathbb{u}}$

simplified (1)

$d\,\mathbb{u} = +H\mathbb{u}^* + b - Gp$

$\mathbb{u} = \hat{\mathbb{u}} - \frac{G}{d}p$

$\hat{\mathbb{u}}$ gives a pressure estimate

$\nabla \cdot \mathbb{u} = 0$

$\Rightarrow \nabla \cdot \frac{G}{d}p = \nabla \cdot \hat{\mathbb{u}}$

cheap
Solve for p

Now you have initial guesses for ~~$\mathbb{u}^*$~~, $p^* = p$ as well

Solve momentum eq.$^{(1)}$ for new $\mathbb{u}^*$

$(d - H)\mathbb{u}^* = -Gp^* + b$

$\downarrow \mathbb{u}^*$

Use SIMPLE approx for correction

$$\mathbf{u}' = -\frac{\nabla}{d} p' \qquad \nabla\cdot\mathbf{u} = \nabla\cdot(\mathbf{u}^* + \mathbf{u}') = 0$$

Solve $\nabla\cdot\frac{\nabla}{d} p' = \nabla\cdot\mathbf{u}^*$ for p'

Update $p = p^* + p'$

$$\mathbf{u} = \mathbf{u}^* - \frac{\nabla}{d} p'$$

Converged?

No: $p^* = p$, $\mathbf{u}^* = \mathbf{u}$

yes

PISO

Two correction steps.

First ... SIMPLE

Start

u^*, p^*

$\boxed{\begin{array}{l}\text{SIMPLE} \\ \text{Mom eq.} \\ \text{Pressure corr} \\ \text{Update}\end{array}}$

u^{**}, p^{**} u^{**} satisfy cont! but not momentum

2nd correction

1) New momentum eq: $d u^{***} = H u^{**} - G p^{***}$ twice corrected

2) First corr: $d u^{**} = H u^{*} - G p^{**}$

1) - 2) $\Rightarrow$ $u^{***} = u^{**} + \dfrac{H(u^{**} - u^{*})}{d} - \dfrac{G}{d} p''$, $p'' = p^{***} - p^{**}$

$$\nabla \cdot u^{***} = \underbrace{\cancel{\nabla \cdot u^{**}}}_{=0} + \nabla \cdot \frac{H(u^{**} - u^{*})}{d} - \nabla \cdot \frac{G}{d} p'' = 0$$

solve for p''

Update u^{***}

check for cov.

$u^* = u^{***}$

$\boxed{\begin{array}{l}\text{First correction eq:} \\ 1)\ (d - H) u^* = -G p^* \\ 2)\ d u^{**} = d u^* - G p' \\ 1)+2) \\ \Rightarrow d u^{**} = H u^* - G p^{**} \\ \qquad (p^{**} = p^* + p') \\ \text{This is how first corr. eq. is derived}\end{array}}$