

STK2130: Rest of the problems for 12.02.2016

Exercise 4.23

(b) No storms in year 3.

$$P^3 = \begin{pmatrix} 0.403 & 0.597 \\ 0.398 & 0.602 \end{pmatrix}$$

$$P(N_3 = 0) = P(N_3 = 0|X_3 = G)P(X_3 = G) + P(N_3 = 0|X_3 = B)P(X_3 = B)$$

where,

$$\begin{aligned} P(X_3 = G) &= P(X_3 = G|X_0 = G)P(X_0 = G) + P(X_3 = G|X_0 = B)P(X_0 = B) = \\ &= 1 \times p_{GG}^{(3)} + 0 \times p_{BG}^{(3)} = 0.403 \end{aligned}$$

similarly,

$$\begin{aligned} P(X_3 = B) &= P(X_3 = B|X_0 = G)P(X_0 = G) + P(X_3 = B|X_0 = B)P(X_0 = B) = \\ &= 1 \times p_{GB}^{(3)} + 0 \times p_{BB}^{(3)} = 0.597 \end{aligned}$$

so,

$$P(N_3 = 0) = \left[P(N_3 = x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \right] = e^{-1}0.403 + 0.597e^{-3} = 0.17791$$

(c) Ending of (c), additionally to the slides. Thus,

$$E[N_\infty] = \pi_G + 3\pi_B = 0.4 + 3 \cdot 0.6 = 2.2$$

where π_G and π_B are the solution to the following system of equations:

$$\pi_G + \pi_B = 1$$

$$\pi_G = \pi_G p_{GG} + \pi_B p_{GB}$$